L-fuzzy J-open Sets*

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Abstract: In this paper, a new class of sets called J-open sets is introduced in L-topological space. Their properties and structures are studied.

Keywords: J-open set; J-closed set; L-topological space.

1. Introduction

Some classes of nearly open sets in L-topological space have been introduced and studied since Azad(1981) introduced fuzzy semi-open (semi-closed) sets and fuzzy regularly open (closed) sets, such as strongly semi-open sets[2], pre-semiopen sets[3], regularly pre-open sets [7]. In this paper, we give a new kind of sets called J-open sets and then study their structures and properties with other nearly open sets.

2. Preliminaries

In this paper, let X denote a non-empty general set, L a fuzzy lattice , L^X denote the set of all L-fuzzy sets on X, (L^X, δ) denote an L-topological space.

Definition 1[1-5,7]. Let (L^X, δ) be an L-topological space. $A \in L^X$, we denote the interior , closure and complement of A by Int(A) , Cl(A) ,A '. Then

(1) A is called a pre–open set if $A \leqslant Int(Cl(A))$ and A is called a pre-closed set if $Cl(Int(A)) \leqslant A$.

(2) A is called a regularly open set if A = Int (Cl(A)) and A is called a regularly closed set if A = Cl(Int(A)).

(3) A is called a regular pre-open set if $A = (A^{\wedge})^{\square}$ and A is called regular a pre-closed set if $A = (A^{\square})^{\wedge}$.

(4) A is called a semi-open set if there exits $B \in \delta$, such that $B \leqslant A \leqslant Cl(B)$ and A is called a semi-closed set if there exits $B \in \delta'$, such that $Int(B) \leqslant A \leqslant B$.

(5) A is called a semipre-open set if there exits a pre-open set B, such that $B \leqslant A \leqslant Cl(B)$ and A is called a semipre-closed set if there exits a pre-closed set B, such that

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Int (B)≤A≤B.

Definition 2[3,7]. Let (L^X, δ) be an L-topological space. $A \in L^X$, then
(1) $A^{\square} = \bigvee\{B \mid B \leq A$, A is a pre-open set$\}$ is called pre-interior of A.
(2) $A^{\wedge} = \bigwedge\{B \mid B \geqq A$, A is a pre–closed set $\}$ is called pre-closure of A.
(3) $A_{\square} = \bigvee\{B \mid B \leq A$, A is a semipre-open set$\}$ is called semipre-interior of A.
(4) $A_{\wedge} = \bigwedge\{B \mid B \geqq A$, A is a semipre–closed set $\}$ is called semipre-closure of A.

3. J-Open Set

Definition 3. Let (L^X, δ) be an L-topological space and $A \in L^X$, A is called J-open set if $A = (A_{\wedge})^{\square}$ and A is called J-closed set if $A = (A_{\square})^{\wedge}$. We will denote ω as the family of J-open sets and denote ω' as the family of the J-closed sets.

Proposition 1. Regularly pre-open set is J-open set.

Proof. This is immediate to get from the Definition 1 of [3] and Definition 3.

Proposition 2. J-open set is pre-open set.

The reverse of Propositions 1 and 2 are not hold, it can be showed by Example 1.

Example 1. Let $X=\{a, b\}$, $L=\{0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1\}$, for any $\lambda \in L$, $\lambda' = 1-\lambda$, $\delta=\{(0, 0), (\frac{1}{6}, \frac{3}{6}), (\frac{2}{6}, \frac{3}{6}), (\frac{3}{6}, \frac{5}{6}) (1, 1)\}$. Then, it is evident that (L^X, δ) is a L-topological space.

(1) Since $[(\frac{2}{6}, \frac{4}{6})_{\wedge}]^{\square} = (\frac{2}{6}, \frac{4}{6})^{\square} = (\frac{2}{6}, \frac{4}{6})$, so $(\frac{2}{6}, \frac{4}{6})$ is a J-open set by definition.

(2) Since $[(\frac{2}{6}, \frac{4}{6})^{\wedge}]^{\square} = (\frac{4}{6}, \frac{4}{6})^{\square} = (\frac{4}{6}, \frac{4}{6}) \neq (\frac{2}{6}, \frac{4}{6})$, so $(\frac{2}{6}, \frac{4}{6})$ is not a J-open set by definition.

(3) $\mathrm{Int}(\mathrm{Cl}(\frac{5}{6}, \frac{5}{6})) = \mathrm{Int}(1, 1) = (1, 1)$, so $(\frac{5}{6}, \frac{5}{6}) \leq \mathrm{Int}(\mathrm{Cl}(\frac{5}{6}, \frac{5}{6}))$, $(\frac{5}{6}, \frac{5}{6})$ is a pre-open set by definition 2, but $[(\frac{5}{6}, \frac{5}{6})_{\wedge}]^{\square} = (1, 1)^{\square} = (1, 1) \neq (\frac{5}{6}, \frac{5}{6})$, by definition, $(\frac{5}{6}, \frac{5}{6})$ is not a J-open set.

Proposition 3. J-open set is independent with open set and semi-open set. It can be showed by Example 2.

Example 2. Let X={ a, b}, L={ 0, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, 1 }, for any $\lambda \in L$, $\lambda' = 1-\lambda$, δ={ (0, 0) ($\frac{1}{6}$, $\frac{3}{6}$) ($\frac{2}{6}$, $\frac{3}{6}$), ($\frac{3}{6}$, $\frac{5}{6}$) (1, 1) }, it is evident that (L^X, δ) is a L-topological space.

(1) Since ($\frac{3}{6}$, $\frac{5}{6}$) ∈δ, but [($\frac{3}{6}$, $\frac{5}{6}$) ^]□= (1, 1) □≠ ($\frac{3}{6}$, $\frac{5}{6}$), so ($\frac{3}{6}$, $\frac{5}{6}$), is an open set not a J-open set.

(2) Since ($\frac{2}{6}$, $\frac{4}{6}$) is a J-open set by (1) of example 1, but ($\frac{2}{6}$, $\frac{4}{6}$) ∉δ, ($\frac{2}{6}$, $\frac{4}{6}$) is a J-open set not an open set.

(3) Since ($\frac{2}{6}$, $\frac{4}{6}$) is a J-open set, by simple computation, we find there exit not an open set B such that B≤ ($\frac{2}{6}$, $\frac{4}{6}$) ≤ Cl (B) , so ($\frac{2}{6}$, $\frac{4}{6}$) is not a semi-open set.

(4) Since ($\frac{1}{6}$, $\frac{3}{6}$) ∈δ, it is an open set, also a semi-open set, but [($\frac{3}{6}$, $\frac{5}{6}$) ^]□ = ($\frac{2}{6}$, $\frac{3}{6}$) □= ($\frac{2}{6}$, $\frac{3}{6}$) ≠ ($\frac{1}{6}$, $\frac{3}{6}$), ($\frac{1}{6}$, $\frac{3}{6}$) is not a J-open set.

Proposition 4. Let (L^X, δ) be a L-topological space and A∈L^X, then (A^) □ ∈**ω** and ($A_□$) ^ ∈ω'.

Lemma 1. Let (L^X, δ) be a L-topological space, A∈L^X,then:
(1)Int (Cl(A)= [Cl(A)]□.
(2)Cl(Int(A)) =[Int(A)]^.

Lemma2 [3]. (1) Regularly pre-open set is pre-open set.
(2) Regularly open set is regular pre-open set.

Lemma3 [4]. Let (L^X, δ) be a L-topological space, Y≠X, Y⊂X, δ|Y is the restriction of δ on Y, for any A∈L^X, A|Y is the restriction of A on Y, then:
(1) IntA|Y≤Int(A|Y).
(2) Cl(A) |Y≤Cl(A|Y).
(3) A '|Y≤(A|Y) '.

Lemma 4 [5]. Let (L^X, δ) be a L-topological space. Y⊂X,Y≠X, so (L^Y, δ|Y) is a subspace of (L^X, δ) , A∈L^X, A is a pre-open(closed) set, then A|Y is a pre-open(closed) set in subspace.

Theorm 1. Let (L^X, δ) be a L-topological space. Y⊂X,Y≠X, so (L^Y, δ|Y) is a subspace of (L^X, δ) , A∈L^X, A is a J-open(closed) set in (L^X, δ) , and A^|Y ≤

$(A|Y)^{\wedge}$, then $A|Y$ is a J-open(closed) set in subspace.

Proof. $A=(A^{\wedge})^{\square}$, so $A|Y=(A^{\wedge})^{\square}|Y$, we need to prove $(A^{\wedge})^{\square}|Y=[(A|Y)^{\wedge})]^{\square}$
First, we prove $(A^{\wedge}|Y)^{\square}=(A^{\wedge})^{\square}|Y$, since $(A^{\wedge})^{\square}\leqslant A^{\wedge}$, so $(A^{\wedge})^{\square}|Y\leqslant A^{\wedge}|Y$, by proposition 2, $(A^{\wedge})^{\square}$ is a pre-open set, by lemma 4, $(A^{\wedge})^{\square}|Y$ is a pre-open set, thus

$$(A^{\wedge})^{\square}|Y=[(A^{\wedge})^{\square}|Y]^{\square}\leqslant(A^{\wedge}|Y)^{\square}.$$

On the other hand, since $(A^{\wedge}|Y)^{\square}\leqslant(A^{\wedge})^{\square}$, so $(A^{\wedge}|Y)^{\square}=(A^{\wedge}|Y)^{\square}|Y\leqslant(A^{\wedge})^{\square}|Y$. thus

$$(A^{\wedge}|Y)^{\square}=(A^{\wedge})^{\square}|Y.$$

Second, we prove $A^{\wedge}|Y=(A|Y)^{\wedge}$. Since $A\leqslant Cl(A)$, so $A|Y\leqslant Cl(A)|Y$, and $A^{\wedge}$ is a pre-closed set, by lemma 4, so $A^{\wedge}|Y$ is a pre-closed set, then $(A|Y)^{\wedge}\leqslant(A^{\wedge}|Y)^{\wedge}=A^{\wedge}|Y$. and by suppose $A^{\wedge}|Y\leqslant(A|Y)^{\wedge}$, we have $A^{\wedge}|Y=(A|Y)^{\wedge}$.
Then, $(A^{\wedge})^{\square}|Y=(A^{\wedge}|Y)^{\square}=[(A|Y)^{\wedge})]^{\square}$.

Now, we can obtain the relation between J-open set and some other nearly open set, showed by following diagram:

Regularly open set →Regularly pre-open set →J-open set → Pre-open set → Semipre-open set →Presemi-open set.

4. References

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