Estimation of Origin Destination Trip Tables Based on a Partial set of Fuzzy Traffic Link volumes and Target Trip Matrix

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Abstract.
There has been a substantial interest in the development and application of methodology for estimating origin- destination (O-D) trip matrices from traffic counts. Generally the quality of an estimated O-D matrix depends much on the reliability of the input data on link volume counts and prior trip matrix. Usually the data will not be precise. Hence it becomes imperative to take imprecision of data into account while formulating the mathematical model for O-D flow estimation. Here a two phase method which can estimate the O-D flows from imprecise or fuzzy data is presented . The model is designed to get inferences about the O-D trip tables from a fuzzy data on a partial set of link counts and prior trip matrix . The objective function as well as the constraints are taken to be fuzzy in nature. The suggested methodology uses a symmetric fuzzy programming approach which does not distinguish between objective function and constraints. Max Min operator method is used for obtaining the solution in the first phase. A second phase is also considered for improving the already obtained solution if improvement is possible. The proposed method is illustrated using a numerical example.

1.Introduction.

An origin-destination trip table is a matrix whose elements represent the number of travel trips made between various O-D zone pairs located within a road network for a given region. The information contained in the table is often used to predict traffic flows on various routes and links of an urban network.Based on these predictions, decisions are made to design new roads as well as to improve existing ones to carry the expected traffic. Various transportation system management's actions also rely on such traffic volume information.

Traditional methods for collecting trip table information include home interview surveys, road side interviews, license plate surveys etc all of which are expensive and time consuming. Several inexpensive methods for estimating O-D matrices were evolved during early seventies, making use of link counts data that is easily obtainable.

The approaches prevalent in literature for estimating trip tables from link counts can be classified into two, namely, parameter calibration techniques, and matrix estimation methods.. However these techniques require zonal data for calibrating the parameters of the demand models and are therefore of limited practical use and also become outdated relatively soon. But matrix estimation methods rely only on traffic link counts and a prior information in the form of a trip table and are easy to implement. There are two types of approaches within this category, namely, standard estimation procedure and mathematical programming methods, the latter being based on maximum entropy theory or network equilibrium based techniques.[Erlander etal (1) Sherali etal(3,4)]Here a non proportional assignment approach based on traffic equilibrium principles of Wardrop(1952) is proposed. Existing methods based on these techniques estimate OD matrix using link volume counts and target trip matrices which are crisp numbers. But due to inherent inconsistencies in the link flow data, sometimes there might not exist a trip table that can exactly duplicate the link flows which are crisp numbers. In addition to that in many real situations it is rather difficult to obtain precise values for link counts and target matrix. To overcome these difficulties the proposed method uses fuzzy data. [Sushama and Revati (5)] The model is designed to handle the situations in which a partial set of target trip table and link counts are given and all of which are represented by appropriate fuzzy numbers.

The remainder of this paper is organized as follows. The next section is devoted to the introduction of notations, salient features of the problem and formulation of the model. Section3 presents a solution procedure for this model. Section 4 suggests a second phase to the solution procedure aimed at giving an improvement to the already obtained solution if there is a room to improve. A numerical example is provided in Section 5 for computational experience. Finally conclusions are given in Section 6.

Section 2. Notations and Model Formulation.

Consider an urban road network for a particular region. Let N denote the set of nodes representing either traffic intersection points where flow is conserved or zones where trips are generated and/or where trips terminate. Let A denote the set of corresponding directed links or arcs representing the roadways existing between designated pairs of nodes. Let OD denote the set of O-D pairs that comprise the trip table to be estimated, where O is the set of origin nodes and D is the set of destination nodes. The problem of reconstructing the trip table given fuzzy link volumes and target matrix values is considered here.

Let A_v represent the set of links in the traffic network for which link volume information is available and A_m represent the set of links for which link counts are missing with $A = A_v \cup A_m$. Let $\tilde{f}_a$ represent the set of available link volumes for all links $a \in A_v$. To formulate this model, the implicit enumeration of all possible n_{ij} paths between each OD pair $(i, j) \in OD$ is considered.

$$\text{Let } (p_{ij}^{k})_a = \begin{cases} 1 & \text{if the } k^{th} \text{path between } i \text{ and } j \text{ uses the link } a \\ 0 & \text{otherwise} \end{cases}$$

Let x_{ij}^k denote the flow on path $k = 1,\ldots,n_{ij}$,$(i, j) \in OD$. Then to determine x_{ij}^k that reproduces the observed link counts $\tilde{f}_a$,$a \in A_v$, we need to find a solution to the system $\sum_{(i,j)\in OD} \sum_{k=1}^{n_{ij}} (p_{ij}^k)_a \, x_{ij}^k = \tilde{f}_a, \quad a \in A_v$

Note that the OD trip table entries T_{ij}^k are related to the path decomposed flows by

$$T_{ij} = \sum_{k=1}^{n_{ij}} x_{ij}^k, \qquad (i, j) \in OD$$

Based on the observed flows $\tilde{f}_a$, $a \in A_v$, link travel costs c_a can be computed. If the observed flow pattern represents a network user equilibrium solution corresponding to some interchange of traffic between the designated OD pairs then by Wardrop's first principle (1952) all routes between OD pairs that have positive flows have equal travel costs, and this cost must not exceed the travel cost of any other unused route between the OD pair.

Let C_{ij}^k denote the cost on route k between OD pair (i,j) for each $k = 1,\ldots, n_{ij}$, $(i, j) \in OD$. Let $c_{ij}^* = \text{Min}\{ C_{ij}^k , k = 1,\ldots,n_{ij} \}$.
Further let $K_{ij} = \{ k / k\in\{1, \ldots, n_{ij}\}$ and $C_{ij}^k = c_{ij}^* \}$ and $\overline{K}_{ij} = \{1, \ldots, n_{ij} \} - K_{ij}$.
Let $\overline{OD}$ represent OD pairs (i,j) with $T_{ij} > 0$ using paths containing links whose link volumes are unavailable. For each $(i, j) \in \overline{OD} \subseteq OD$. Let $\tilde{Q}_{ij}$ denote the fuzzy values of a prior trip table having associated OD flows $t_{ij} > 0$

Thus the fuzzy model of the estimation problem is

Model 1:

$$\text{Min} \sum_{(i,j)\in OD} \sum_{k=1}^{n_{ij}} C_{ij}^k x_{ij}^k$$

$$\text{s.t} \quad \sum_{(i,j)\in OD} \sum_{k=1}^{n_{ij}} (p_{ij}^k)_a x_{ij}^k = \tilde{f}_a \quad a \in A_v$$

$$\sum_{k=1}^{n_{ij}} x_{ij}^k = \tilde{Q}_{ij} \quad ,(i, j) \in \overline{OD}$$

$$\text{where } C_{ij}^k = \begin{cases} C_{ij}^* & \forall\, k \in K_{ij} \\ MC_{ij}^* & \forall\, k \in \overline{K}_{ij} \end{cases} \qquad (M > 1)$$

Given in the next section is a solution procedure for solving the above fuzzy programming problem when R.H.S values of the constraints are taken as triangular fuzzy numbers.

Section 3. Solution Procedure – Phase1.

Let the range of values of these triangular fuzzy numbers be $f_a \pm f_a^1$ and $Q_{ij} \pm Q_{ij}^1$ where f_a^1 & Q_{ij}^1 are suitable numbers that are given by $\widetilde{f}_a$ and $\widetilde{Q}_{ij}$ for each $a \in A_v$ and $(i,j) \in \overline{OD}$

Let $Z = \sum_{(i,j)\in OD} \sum_{k=1}^{n_{ij}} C_{ij}^k x_{ij}^k$, $U = \sum_{(i,j)\in OD} \sum_{k=1}^{n_{ij}} (p_{ij}^k)_a \; x_{ij}^k$

and $V = \sum_{k=1}^{n_{ij}} x_{ij}^k$ so that Z, U and V are functions of x_{ij}^k.

Let X denote the vector with components x_{ij}^k. To start with, the membership function $\mu_0(x)$ of the objective function is to be identified. This involves finding the optimum objective function values z^0 and z^1 of the following two linear programming problems as in Guu and Wu(2).

Problem. 1

$$\begin{aligned} &\text{Min } Z \\ &\text{s.t. } U + f_a^1 r \ge f_a \\ &\qquad V + Q_{ij}^1 r \ge Q_{ij} \qquad \text{(P1)} \\ &\qquad x_{ij}^k \ge 0 \end{aligned}$$

Problem 2

$$\begin{aligned} &\text{Min } Z \\ &\text{s.t } U - f_a^1 r \le f_a \\ &\qquad V - Q_{ij}^1 r \le Q_{ij} \qquad \text{(P2)} \\ &\qquad x_{ij}^k \ge 0 \end{aligned}$$

The membership functions $\mu_0(x)$ of the objective function is defined as

$$\mu_0(x) = \begin{cases} \dfrac{z^0 - z}{z^0 - z^1} & \text{if } z^1 < z < z^0 \\ 0 & \text{otherwise} \end{cases}$$

Also the membership functions $\mu_a(x)$, $a \in A_v$ and $\mu_b(x)$, $b \in \overline{OD}$ of the link volume constraints and prior trip table constraints are defined as follows.

$$\mu_a(x) = \begin{cases} 0 & \text{if } U > f_a + f_a^l \\ 0 & \text{if } U < f_a - f_a^l \\ \dfrac{U - (f_a - f_a^l)}{f_a^l} & \text{if } f_a - f_a^l \le U \le f_a \\ \dfrac{(f_a + f_a^l) - U}{f_a^l} & \text{if } f_a \le U \le f_a + f_a^l \end{cases}$$

$$\mu_b(x) = \begin{cases} 0 & \text{if } V > Q_{ij} + Q_{ij}^l \\ 0 & \text{if } V < Q_{ij} - Q_{ij}^l \\ \dfrac{V - (Q_{ij} - Q_{ij}^l)}{Q_{ij}^l} & \text{if } Q_{ij} - Q_{ij}^l \le V \le Q_{ij} \\ \dfrac{(Q_{ij} + Q_{ij}^l) - V}{Q_{ij}^l} & \text{if } Q_{ij} \le V \le Q_{ij} + Q_{ij}^l \end{cases}$$

Having defined the membership functions of objective function and constraints the well known Max Min operator is used to determine

$$Max\ Min\ \{\mu_0(x), \mu_a(x)\ \forall\ a \in A_v, \mu_b(x)\ \forall\ b \in \overline{OD}\}$$

The crisp equivalent of the fuzzy model is given as

$$\begin{aligned} &\text{Max } \alpha \\ &\alpha \le \mu_0(x) \le 1 && \text{(P3)} \\ &\alpha \le \mu_a(x) \le 1 \quad \forall a \in A_v \\ &\alpha \le \mu_b(x) \le 1 \quad \forall\ b \in \overline{OD} \\ &x \ge 0 \end{aligned}$$

Let (x^*, α^*) denote the optimal solution yielded by the Max Min operator, then the T_{ij} 's are given by $T_{ij} = \sum_{k=1}^{n_{ij}} x_{ij}^k$. This provides a solution to the OD matrix estimation problem with the degree of satisfaction for the objective function and constraints as $\mu_0(x^*)$, $\mu_a(x^*)$, $a \in A_v$, $\mu_b(x^X)$, $b \in \overline{OD}$.

Section: 4 Solution Procedure-Phase 11

In this section a second phase to improve the above solution yielded by the Max Min operator is suggested. This model is a combination of the mini operator model and the arithmetical average operator model.

$$\text{Max} \sum_{j \in A_v \cup \overline{OD}} \alpha_j$$

$$\text{s.t} \quad \mu_j(x^*) \le \alpha_j \le \mu_j(x) \le 1 \qquad \forall j$$
$$x \ge 0$$

This optimum solution x** sometimes gives a better value for the objective function than that of x*. In case the objective function value remains the same, it is still a better solution in the sense that some constraints attain, if possible a higher degree of satisfaction. If the decision maker is seeking an improved solution over max-min operator's solution so that each membership degree should be improved, then it automatically is attained in the second phase , if there is room to improve.

Section 5. Numerical Example:

Link index:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Link arc :	1-7	2-7	1-5	7-8	2-6	5-8	8-5	8-6	6-8	5-3	8-9	6-4	9-3	9-4
Link cost :	40	40	120	110	110	30	40	40	50	120	120	110	40	30

Partial data of link volumes , which is link volumes on links 4,5 and 6 are given by triangular fuzzy numbers (215,20, 215,20), (265,20, 265,20)and (190, 20,190,20) Prior values of trip matrix are also given as triangular fuzzy numbers, (210,20, 210,20) , (140,20,140,20) and (135,20,135,20) for the i-j pairs 1-3, 1-4 and 2-3 respectively.
Given $f_4 = 215$, $f_5 = 265$, $f_6 = 190$ and $f_4^1 = f_5^1 = f_6^1 = 20$.

Number of different paths for the O-D pairs are as follows.
k =1,2,3,4 for the O-D pair (1-3)
k = 1,2,3,4 for the O-D pair (1-4)
k = 1,2,3,4 for the O-D pair (2-3)
k = 1,2,3 for the O-D pair (2-4)

Accordingly there are 15 variables x_{ij}^k. Let these variables be renamed as $x_1, x_2, x_3, \ldots, x_{15}$ respectively.

From the fuzzy link volume and fuzzy target matrix values $f_4 = 215$, $f_5 = 265$, $f_6 = 190$ and $f_4^1 = f_5^1 = f_6^1 = 20$, $Q_{13} = 210$, $Q_{13}^1 = 20$, $Q_{14} = 140$, $Q_{14}^1 = 20$, $Q_{23} = 135$, $Q_{23}^1 = 20$

With M =10 the objective function is

$$\text{Min } 240x_1 + 2400x_2 + 2400x_3 + 2400x_4 + 300x_5 + 300x_6 + 300x_7 + 300x_8 + 3100x_9 + 3100x_{10} + 310x_{11} + 310x_{12} + 220x_{13} + 2200x_{14} + 2200x_{15}$$

Constraints are

$x_1 + x_2 + x_3 + x_4 \geq 190$
$x_1 + x_2 + x_3 + x_4 \leq 230$

$x_5 + x_6 + x_7 + x_8 \geq 120$
$x_5 + x_6 + x_7 + x_8 \leq 160$

$x_9 + x_{10} + x_{11} + x_{12} \geq 115$
$x_9 + x_{10} + x_{11} + x_{12} \leq 155$

$x_1 + x_3 + x_5 + x_6 \geq 195$
$x_1 + x_3 + x_5 + x_6 \leq 235$

$x_2 + x_4 + x_7 + x_8 + x_{11} + x_{12} + x_{15} \geq 245$
$x_2 + x_4 + x_7 + x_8 + x_{11} + x_{12} + x_{15} \leq 285$

$x_9 + x_{10} + x_{13} + x_{14} \geq 170$
$x_9 + x_{10} + x_{13} + x_{14} \leq 210$
$x_j \geq 0$

Solving the problems P1 and P2,
we get $z^0 = 158850$ with $x_1 = 195$, $x_7=130$, $x_{11}=115$, $x_{13}=170$, $x_j = 0$ for other j's
and $z^1 = 0$ with $x_j = 0$, $j = 1.......15$.

Phase I
The problem P3 now takes the form

Max α

s.t. $240x_1 + 2400x_2 + 2400x_3 + 2400x_4 + 300x_5 + 300x_6 + 300x_7 + 300x_8 + 3100x_9 + 3100x_{10} + 310x_{11} + 310x_{12} + 220x_{13} + 2200x_{14} + 2200x_{15} + 158850\alpha \leq 158850$

$240x_1 + 2400x_2 + 2400x_3 + 2400x_4 + 300x_5 + 300x_6 + 300x_7 + 300x_8 + 3100x_9 + 3100x_{10} + 310x_{11} + 310x_{12} + 220x_{13} + 2200x_{14} + 2200x_{15} \geq 0$

$x_1 + x_2 + x_3 + x_4 - 20\alpha \geq 190$
$x_1 + x_2 + x_3 + x_4 + 20\alpha \leq 230$

$x_5 + x_6 + x_7 + x_8 - 20\alpha \geq 120$
$x_5 + x_6 + x_7 + x_8 + 20\alpha \leq 160$

$x_9 + x_{10} + x_{11} + x_{12} - 20\alpha \geq 115$
$x_9 + x_{10} + x_{11} + x_{12} + 20\alpha \leq 155$

$x_1 + x_3 + x_5 + x_6 - 20\alpha \geq 195$
$x_1 + x_3 + x_5 + x_6 + 20\alpha \leq 235$

$x_2 + x_4 + x_7 + x_8 + x_{11} + x_{12} + x_{15} - 20\alpha \geq 245$
$x_2 + x_4 + x_7 + x_8 + x_{11} + x_{12} + x_{15} + 20\alpha \leq 285$

$x_9 + x_{10} + x_{13} + x_{14} - 20\alpha \geq 170$
$x_9 + x_{10} + x_{13} + x_{14} + 20\alpha \leq 210$

$0 \leq \alpha \leq 1$, $x_j \geq 0$ $j = 1.......15$

Optimum solution is $x_1 = 195$, $x_7 = 130$, $x_{11} = 115$, $x_{13} = 170$, $\alpha = 0$

Then $\mu_0 = 0$, $\mu_1 = 0.25$, $\mu_2 = 0.5$ $\mu_3 = \mu_4 = \mu_5 = \mu_6 = 0$
Accordingly the estimation problem has the following solution.
$T_{13} = 195$, $T_{14} = 130$, $T_{23} = 115$, $T_{24} = 170$.

Phase II.

$$\text{Max } \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6$$

s.t. $\alpha_1 \geq 0.25$

$\alpha_2 \geq 0.5$

$240x_1 + 2400x_2 + 2400x_3 + 2400x_4 + 300x_5 + 300x_6 + 300x_7 + 300x_8 + 3100x_9 + 3100x_{10} + 310x_{11} + 310x_{12} + 220x_{13} + 2200x_{14} + 2200x_{15} + 158850\alpha \leq 158850$

$240x_1 + 2400x_2 + 2400x_3 + 2400x_4 + 300x_5 + 300x_6 + 300x_7 + 300x_8 + 3100x_9 + 3100x_{10} + 310x_{11} + 310x_{12} + 220x_{13} + 2200x_{14} + 2200x_{15} \geq 0$

$x_1 + x_2 + x_3 + x_4 - 20\alpha_1 \geq 190$
$x_1 + x_2 + x_3 + x_4 + 20\alpha_1 \leq 230$

$x_5 + x_6 + x_7 + x_8 - 20\alpha_2 \geq 120$
$x_5 + x_6 + x_7 + x_8 + 20\alpha_2 \leq 160$

$x_9 + x_{10} + x_{11} + x_{12} - 20\alpha_3 \geq 115$
$x_9 + x_{10} + x_{11} + x_{12} + 20\alpha_3 \leq 155$

$x_1 + x_3 + x_5 + x_6 - 20\alpha_4 \geq 195$
$x_1 + x_3 + x_5 + x_6 + 20\alpha_4 \leq 235$

$x_2 + x_4 + x_7 + x_8 + x_{11} + x_{12} + x_{15} - 20\alpha_5 \geq 245$
$x_2 + x_4 + x_7 + x_8 + x_{11} + x_{12} + x_{15} + 20\alpha_5 \leq 285$

$x_9 + x_{10} + x_{13} + x_{14} - 20\alpha_6 \geq 170$
$x_9 + x_{10} + x_{13} + x_{14} + 20\alpha_6 \leq 210$

$x_j \geq 0 \quad j = 1, \ldots, 15.$

Solution of the above problem is the same as the previous one itself which indicates that further improvement in the solution is not possible.

Section 6. Conclusion.

Although several methods have been considered earlier to estimate OD matrix the proposed method has the advantage that it is capable of handling a fuzzy data which in many cases is more realistic than a crisp data. Also the solution method adopted here for solving this type of fuzzy linear programming has an advantage over methods like the minimum operator or parametric programming because it just does not pursue the highest membership degree but also pursues a better utililization of each constraint resource.

References:

1. Erlander, S., Jornsten , K.O., Lundgren , J. N., 1995. On the estimation of trip matrices in the case of missing and uncertain data. *Transportation Research.*B 19, 123-141
2. Guu.S.M. , Wu,Y.K.,1999. Two phase approach for solving the fuzzy linear programming. Fuzzy Sets and Systems. 107. 191-195.
3. Sherali. H. D. , Sivanandan , R. , Hobeika,A.G., 1999. A linear programming approach for synthesizing origin destination trip tables from link traffic volumes. *Transportation Research.*B 28 , 213-233
4. Sherali. H. D. , Sivanandan , R. Narayanan.,A., 2003. Estimation of Origin – Destination trip tables based on a partial set of traffic link volumes. *Transportation Research.* B 37, 815-836
5. Sushama. C.M. Revati, R. Estmation of Trip Matrices using Fuzzy data. *Proceedings of the International Conference on Structural and Road Transportation Engineering* I.I.T. Kharagpur. India. 2005, 873-879
6. Wilson. A.G., 1970 . *Entropy in Urban and Regional Modelling.* Pion Limited. London.
7. Zimmermann. H.J. 1996. *Fuzzy Set Theory and its Applications.* Kluwer Academic Press.