Z-open sets in L-topological spaces*

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Abstract: *In this paper, two classes of sets called Z-open sets and B-open sets are introduced in L-topological spaces. Then, we study their properties and establish their structure relations with known nearly open sets.*
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1. Introduction

Since semi-topological classes and semi-topological properties were introduced in Crossley and Hildebrand's article in 1972, many researchers have introduced many kind of sets, such as semi-open, regular-open, pre-open, semi-regular open, semi-regularly semiopen, strongly semi-open, semi-preopen, regularly pre-open, L-regularly pre-semiopen sets in various spaces [1-7]. Some interesting results have been obtained. Thus, research in this area has been greatly extended. In this paper we introduce Z-open sets, B-open sets, Z-closed sets and B-open sets in L-topological spaces and establish their structure relations with known nearly open sets. It is a continuation of such work.

2. Preliminaries

In this paper, X will denote non-empty set; L will denote a fuzzy lattice; L^X will denote the set of all L-fuzzy sets on X. Let (L^X, δ) be an L-topology space (simply written as L-ts) and $A \in L^X$. Call A°, A^- is the interior and the closure of A, respectively.

Definition 1 [1-7]. Let (L^X, δ) be an L-topological space, $A \in L^X$, then A is called:
(1) A semi-open set iff there is a $B \in \delta$ such that $B \leqslant A \leqslant B^-$;
A semi-closed set iff there is a $B \in \delta'$ such that $B^\circ \leqslant A \leqslant B$.

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(2) A pre-open set iff $A \leqslant A^{-\circ}$; A pre-closed set iff $A \geqslant A^{\circ -}$.

(3) A strongly semi-open set iff there is a $B \in \delta$ such that $B \leqslant A \leqslant B^{-\circ}$;
A strongly semi-closed set iff there is a $B \in \delta'$ such that $B^{\circ -} \leqslant A \leqslant B$.

(4) A pre-semiopen set iff $A \leqslant (A^{-})_{\circ}$; A pre-semiclosed set iff $A \geqslant (A^{\circ})_{-}$.

(5) A semi-regularly semi-open set iff $A = A_{-\circ}$:
A semi-regularly semi-closed set iff $A = A_{\circ -}$.

(6) A regularly semi-open set iff $A = (A^{-})_{\circ}$; A regularly semi-closed set iff $A = (A^{\circ})_{-}$.

(7) A regularly open set iff $A = A^{-\circ}$: A regularly closed set iff $A = A^{\circ -}$.

(8) A semi-preopen set iff there is a B is pre-open set such that $B \leqslant A \leqslant B^{-}$;
A semi-preclosed set iff there is a B is pre-closed set such that $B^{\circ} \leqslant A \leqslant B$.

Definition 2 [2-4,7]. Let (L^X , δ) be an L-topological space, $A \in L^X$, then

(1) $A^{\sqcup} = \bigvee \{B: B \leqslant A$, B is pre-open set $\}$.

(2) $A^{\wedge} = \bigwedge \{B: B \geqslant A$, B is pre-closed set $\}$,

(3) $A_{\sqcap} = \bigvee \{B: B \leqslant A$, B is semi-preopen set $\}$.

(4) $A_{\wedge} = \bigwedge \{B: B \geqslant A$, B is semi-preclosed set $\}$,

(5) $A_{\vee} = \bigvee \{B: B \leqslant A$, B is pre-semiopen set $\}$,

(6) $A_{\sim} = \bigwedge \{B: B \geqslant A$, B is pre-semiclosed set $\}$,

are called the preinterior, preclosure, semi-preinterior, semi-preclosure, pre-semiinterior, pre-semiclosure of A , respectively.

Definition 3 [5,6]. Let (L^X , δ) be an L-topological space, $A \in L^X$, then A is called:

(1) A regularly pre-open set iff $A = A^{\wedge\sqcap}$;A regularly pre-closed set iff $A = A^{\square\wedge}$.

(2) A regularly pre-semiopen set iff $A = A_{\sim\vee}$; A regularly pre-semiclosed set iff $A = A_{\Delta\sim}$.

Definition 4 [9]. Let (L^X , δ) be an L-topological space, $A \in L^X$, then A is called:

(1) A J-open set iff $A = (A_{\wedge})^{\sqcup}$;

(2) A J-closed set iff $A = (A_{\sqcap})^{\wedge}$.

Lemma 1 [3]. For any $A \in L^X$, the following inequalities are right.

$$A^{\circ} \leqslant A_{\circ} \leqslant A_{\sqcap} \leqslant A_{\vee} \leqslant A \leqslant A_{\sim} \leqslant A_{\wedge} \leqslant A_{-} \leqslant A^{-}.$$

3. Z-open sets and B-open sets

Definition 5. Let (L^X , δ) be an L-topological space, $A \in L^X$, then A is called:

(1) A Z-open set iff $A = A_{\wedge\sqcup}$:

(2) A Z-closed set iff $A = A_{\sqcap\wedge}$.

μ will always denote the family of Z-open sets, μ' will always denote the family of Z-closed sets in (L^X , δ).

Definition 6. Let (L^X , δ) be an L-topological space, $A \in L^X$, then A is called:

(1) A B-open set iff $A = (A^{\wedge})_{\sqcap}$;

(2) A B-closed set iff $A = (A^{\sqcap})_{\wedge}$.

Theorem 1. Let (L^X , δ) be an L-topological space, $A \in L^X$, if A is a J-open set, then A is a Z-open set .

Proof. This is easy.

Theorem 2. Let (L^X , δ) be an L-topological space, $A \in L^X$, if A is a B-open set, then A is a Z-open set .

Proof. For any $A \in L^X$, because $A=(A^{\wedge})_{\square}$,by Lemma 1, $A^{\wedge} \geqslant A_{\wedge}$,so$(A^{\wedge})_{\square} \geqslant A_{\wedge\square}$. By Definitions 1, 2, A is a semi-preopen. So $A=A_{\square} \leqslant A_{\wedge\square}$, then $A=A_{\wedge\square}$.

Theorem 3. Let (L^X , δ) be an L-topological space, $A \in L^X$, if A is a Z-open set, then A is a semi-preopen set .

Proof. This is immediate to get the conclusion from Definitions 1, 2.

Theorem 4. Let (L^X , δ) be an L-topological space, $A \in L^X$, if A is a semi-regularly semiopen set, then A is a Z-open set .

Proof. For any $A \in L^X$, if A is a semi-regularly semiopen set, then A is not only semi-open set but also semiclosed set (Bai [4]), hence A is a pre-semiopen set and pre-semiclosed set (Bai[3]), so A is a semi-preopen set and semi-preclosed set. By Definitions 1, 2, $A=A_{\square}=A_{\wedge}$, $A=A_{\square}=A_{\wedge\square}$ and $A=A_{\wedge}=A_{\square\wedge}$. So A is a Z-open set and A is a Z-closed set.

Theorem 5. Let (L^X , δ) be an L-topological space, $A \in L^X$. If A is a regularly semi-open set, then A is a B-open set.

Proof. The proof is easy and omitted.

The reverses Theorem 1-5 are not hold. It can be showed by Example1.

Example 1. Let X= {a, b}, L= $\{0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1\}$, and for any $a \in L$, $a'=1-a$. Let δ $=\{(0,0),(\frac{1}{6},\frac{3}{6}),(\frac{2}{6},\frac{3}{6}),(\frac{3}{6},\frac{5}{6}),(1,1)\}$. It is evident that (L^X , δ)is an L-ts .

(1) It is easy to get the following conclusion. Let $A=(\frac{3}{6},\frac{3}{6})$, then $A_{\wedge\square}=(\frac{3}{6},\frac{3}{6})=A$, so A is a Z-open set. But $(A_{\wedge})^{\sqcap}=(\frac{3}{6},\frac{3}{6})^{\sqcap}=(\frac{2}{6},\frac{3}{6}) \neq A$; $(A^{\wedge})_{\square}=(\frac{3}{6},\frac{4}{6})_{\square}=(\frac{3}{6},\frac{4}{6}) \neq A$, so A is a not J-open set and A is not a B-open set, either.

(2) Let $A=(\frac{5}{6},\frac{5}{6})$, it is obviously that A is a semi-preopen set. But $A_{\wedge\square}=(\frac{5}{6},\frac{5}{6})_{\wedge\square}$

$=(1,1)_{\sqcap}\neq A$, so A is not a Z-open set.

(3) Let $A=(\frac{4}{6},\frac{4}{6})$, then $A_{\wedge\sqcap}=(\frac{4}{6},\frac{4}{6})_{\sqcap}=(\frac{4}{6},\frac{4}{6})=A$, so A is a Z-open set. But $A_{-\circ}=(\frac{4}{6},\frac{4}{6})_{-\circ}=(1,1)\neq A$, so A is not a semi regularly semi-open set.

(4) Let $A=(\frac{4}{6},\frac{2}{6})$, then $(A^{\wedge})_{\sqcup}=(\frac{4}{6},\frac{2}{6})_{\sqcup}=(\frac{4}{6},\frac{2}{6})=A$, so A is a B-open set. But $(A^{-})_{\circ}=(\frac{4}{6},\frac{3}{6})_{\circ}=(\frac{4}{6},\frac{3}{6})\neq A$, so A is not a regularly semi-open set.

(5) Let $A=(\frac{2}{6},\frac{4}{6})$, then $(A_{\wedge})^{\sqcup}=(\frac{2}{6},\frac{4}{6})^{\sqcup}=(\frac{2}{6},\frac{4}{6})=A$, so A is a J-open set. But $A_{-\circ}=(\frac{2}{6},\frac{4}{6})_{-\circ}=(1,1)_{\circ}=(1,1)\neq A$, so A is not a semi-regularly semi-open set. Because $(A^{\wedge})_{\sqcap}=((\frac{2}{6},\frac{4}{6})^{\wedge})_{\sqcap}=(\frac{4}{6},\frac{4}{6})_{\sqcap}=(\frac{4}{6},\frac{4}{6})\neq A$, A is not a B-open set, either.

(6) Suppose that $A=(\frac{3}{6},\frac{3}{6})$, then $A_{-\circ}=(\frac{3}{6},\frac{3}{6})_{-\circ}=(\frac{4}{6},\frac{3}{6})_{\circ}=(\frac{3}{6},\frac{3}{6})=A$, so A is a semi-regularly semi-open set. By Example1 (1), so A is not a J-open set and A is not a B-open set, either.

(7) Let $A=(\frac{4}{6},\frac{2}{6})$, then $(A^{\wedge})_{\sqcup}=(\frac{4}{6},\frac{2}{6})_{\sqcup}=(\frac{4}{6},\frac{2}{6})=A$, so A is a B-open set. But $A_{-\circ}=(\frac{4}{6},\frac{2}{6})_{-\circ}=(\frac{4}{6},\frac{3}{6})_{\circ}=(\frac{4}{6},\frac{3}{6})\neq A$, so A is not a semi-regularly semi-open set. Because $(A_{\wedge})^{\sqcup}=((\frac{4}{6},\frac{2}{6})_{\wedge})^{\sqcup}=(\frac{4}{6},\frac{2}{6})^{\sqcup}\neq A$, A is not a J-open set.

From Example 1 we can obtain the following proposition.

Proposition 1. J-open set, semi-regularly semi-open set and B-open set are independent respectively.

Remark 1. Let (L^X , δ) be an L-topological space, $A\in L^X$. If A is a regularly pre-semiopen set, then A need not be Z-open set.

Example 2. In Example 1, let $A=(\frac{4}{6},\frac{1}{6})$, then $A_{-\Delta}=(\frac{4}{6},\frac{1}{6})_{-\Delta}=(\frac{4}{6},\frac{1}{6})_{\Delta}=(\frac{4}{6},\frac{1}{6})=A$, so A is a regularly pre-semiopen set. But $A_{\wedge\sqcap}=(\frac{4}{6},\frac{1}{6})_{\wedge\sqcap}=(\frac{4}{6},\frac{1}{6})_{\sqcap}=(0,0)\neq A$, so A is not a Z-open .

Remark 2. The intersection (union) of any two Z-open sets (B-open sets) need not be a Z-open set (B-open set). This is shown by the follow example.

Example 3. In Example 1, let $A=(0,\frac{5}{6})$, $B=(1,\frac{1}{6})$, then $A=A_{\wedge\square}$, $B=B_{\wedge\square}$, but $(A\vee B)_{\wedge\square}=(1,\frac{5}{6})_{\wedge\square}=(1,1)\neq(1,\frac{5}{6})$, so $(A\vee B)$ is not a Z-open set. Because $(A\wedge B)_{\wedge\square}=(0,\frac{1}{6})_{\wedge\square}=(0,0)\neq(0,\frac{1}{6})$, $(A\wedge B)$ is not a Z-open set, either. Similarly B-open set has the conclusion.

Proposition 2. Let (L^X, δ) be an L-topological space, $A\in L^X$, then $A\in\mu$ iff $A'\in\mu'$.

Proof. By Definition 4, it is immediate to get the conclusion.

Theorem 6. Let (L^X, δ) be an L-topological space, $A\in L^X$.
(1) If $A=A_{\square}$, then $A_{\wedge}=A_{\wedge\square}$;
(2) If $A=A_{\wedge}$, then $A_{\square}=A_{\wedge\square}$.

Proof. By Definitions 1, 2 and 4, it is immediate to get the conclusion.

Theorem 7. Let X_1 and X_2 be L-topological spaces such that X_1 is product-related to X_2. Then the product $A\times B$ of a Z-open set(B-open set) A of X_1 and a Z-open set(B-open set) B of X_2 is a Z-open set (B-open set)of the Product space $X_1\times X_2$.

Proof. This follows similarly the Theorem 2.8 in [2].

4. Conclusion

In this paper we introduce the Z-open set. B-open set and Z-closed set, B-closed set in L-topological spaces, and establish their structure relations with known nearly open sets. Therefore we can easily obtain the following diagram.

Semi-regularly semiopen set regularly pre-semiopen set

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