The PS-Lindelöf L-set *

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Abstract: *In this paper, continuing research of PS-compactness and countable PS-compactness, we introduce a new notion of the PS-Lindelöf L-set in L-topological spaces, where L is a complete De Morgan algebra and it does not require any distributivity. An L-set is PS-compact iff it is countably PS-compact and the PS-Lindelöf.*
Key words: *L-topological space; Pre-semi-open L-set; PS-compactness; PS-Lindelöf L-set*

1.Introduction

Compactness and its stronger and weaker forms play very important roles in topology. In [2,3], we introduced the concepts of PS-compactness, countable PS-compactness and PS-Lindelöf L-sets in L-topological spaces respectively. In [4,5], following the lines of [10], we introduced new concepts of PS-compactness and countable PS-compactness in L-topological spaces by means of pre-semi-open L-sets and their inequality, where L is a complete de Morgan algebra. They do not depend on the structure of basis lattice L and L does not require any distributivity. They are the further generalization of concepts in [2,3].

Based on [4,5], the purpose of this paper is to do the research of the PS-Lindelöf L-set in L-topological spaces.

*NNSFs of China, NSF of Guangdong Province and STF of Jiangmen City(No. 60473009, 60542001, 021358, [2005]102).
MR(2000) Subject Classification 54A40; 54A20

2. Preliminaries

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete De Morgan algebra, X a nonempty set. L^X is the set of all L-fuzzy sets (or L-sets for short) on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$. The set of nonunit prime elements[6] in L is denoted by $P(L)$. The set of nonzero co-prime elements[6] in L is denoted by $M(L)$. The greatest minimal family of $a \in L$ is denoted by $\beta(a)$. The greatest maximal family of $a \in L$ is denoted by $\alpha(a)$[7,11]. Moreover for $b \in L$, define $\beta^*(b) = \beta(b) \cap M(L)$ and $\alpha^*(b) = \alpha(b) \cap P(L)$. For $a \in L$ and $A \in L^X$, we define $A^{(a)} = \{x \in X | A(x) \not\leq a\}$ and $A_{(a)} = \{x \in X | a \in \beta(A(x))\}$. For a subfamily $\psi \subseteq L^X$, $2^{[\psi]}$ denotes the set of all countable subfamilies of ψ. An L-topological space denotes L-ts for short. An L-set A in an L-ts (X, δ) is called pre-semi-open[1] if $A \leq (A^-)_o$, and A is called pre-semi-closed[1] if A' is pre-semi-open, where A^o, A^-, A_o and A_- are the interior, closure, semi-interior and semi-closure of A, respectively. The rest unstated concepts can be referred in [5].

3. Definitions and properties

Definition 3.1. Let (X, δ) be an L-ts. $A \in L^X$ is called a PS-Lindelöf L-set (or has the PS-Lindelöf property) if for every family μ of pre-semi-open L-sets, it follows that

$$\bigwedge_{x\in X} \left(A'(x) \vee \bigvee_{B\in\mu} B(x)\right) \leq \bigvee_{\nu\in 2^{[\mu]}} \bigwedge_{x\in X} \left(A'(x) \vee \bigvee_{B\in\nu} B(x)\right).$$

(X, δ) is called PS-Lindelöf L-ts (or has the SR-Lindelöf property) if $\underline{1}$ is the PS-Lindelöf L-set.

Remark 3.2. Since every semi-open L-set is pre-semi-open[1], every PS-Lindelöf L-set is a semi-Lindelöf L-set[9].

From definitions of PS-compactness and countable PS-compactness in [4,5], we have the following theorem.

Theorem 3.3. Let (X, δ) be an L-ts. $A \in L^X$ is a PS-compact L-set iff A is a countably PS-compact L-set and a PS-Lindelöf L-set.

From Definition 3.1 we can obtain the following theorem by using quasi-complement.

Theorem 3.4. Let (X, δ) be an L-ts. $A \in L^X$ is the PS-Lindelöf L-set iff for every family μ of pre-semi-closed L-sets, it follows that

$$\bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \mu} B(x)) \geq \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \nu} B(x)).$$

From Definition 3.1 and Theorem 3.4 we immediately obtain the following result.

Theorem 3.5. Let (X, δ) be an L-ts and $A \in L^X$. Then the following conditions are equivalent.

(1) A is a PS-Lindelöf L-set.

(2) For any $a \in L \setminus \{1\}$, each pre-semi-open S-a-shading μ of A has a countable subfamily which is an S-a-shading of A.

(3) For any $a \in L \setminus \{0\}$, each pre-semi-closed S-a-R-NF ψ of A has a countable subfamily which is an S-a-R-NF of A.

(4) For any $a \in L \setminus \{0\}$, each family of pre-semi-closed L-sets which has the countable weak a-intersection property in A has weak a-nonempty intersection in A.

We can obtain the following Theorems 3.6,3.8-3.12 by using similar way of the proof in [4].

Theorem 3.6. If C is a PS-Lindelöf L-set and D a is pre-semi-closed L-set, then $C \wedge D$ is a PS-Lindelöf L-set.

corollary 3.7. Let (X, δ) be a PS-Lindelöf L-ts and $D \in L^X$ is a pre-semi-closed L-set. Then D is a PS-Lindelöf L-set.

Theorem 3.8. Let L be a complete Heyting algebra. If both C and D are the PS-Lindelöf L-sets, then $C \vee D$ a PS-Lindelöf L-set.

Theorem 3.9. Let L be a complete Heyting algebra and $f : (X, \delta) \to (Y, \tau)$ be a PS-irresolute map. If A is a PS-Lindelöf L-set in (X, δ), then so is $f_L^{\to}(A)$ in

(Y,τ).

Theorem 3.10. Let L be a complete Heyting algebra and $f:(X,\delta)\to(Y,\tau)$ be a pre-semi-continuous map. If A is a PS-Lindelöf L-set in (X,δ), then $f_L^{\rightarrow}(A)$ is a Lindelöf L-set in (Y,τ).

Theorem 3.11. Let L be a complete Heyting algebra and $f:(X,\delta)\to(Y,\tau)$ be a strongly pre-semi-continuous map. If A is a PS-Lindelöf L-set in (X,δ), then $f_L^{\rightarrow}(A)$ is a semi-Lindelöf L-set in (Y,τ).

Theorem 3.12. Let L be a complete Heyting algebra and $f:(X,\delta)\to(Y,\tau)$ be a strongly pre-semi-irresolute map. If A is a semi-Lindelöf L-set in (X,δ), then $f_L^{\rightarrow}(A)$ is a PS-Lindelöf L-set in (Y,τ).

4. Other characterizations

In this section and the next section, we assume that L is a completely distributive De Morgan algebra.

Theorem 4.1. Let (X,δ) be an L-ts and $A\in L^X$. Then the following conditions are equivalent.

(1) A a PS-Lindelöf L-set.

(2) For any $a\in L\setminus\{0\}$, each pre-semi-closed S-a-R-NF ψ of A has a countable subfamily which is an S-a-R-NF of A.

(3) For any $a\in L\setminus\{0\}$, each pre-semi-closed S-a-R-NF ψ of A has a countable subfamily which is an a-R-NF of A.

(4) For any $a\in L\setminus\{0\}$ and any pre-semi-closed S-a-R-NF ψ of A, there exist a countable subfamily φ of ψ and $b\in\beta(a)$ such that φ is an S-b-R-NF of A.

(5) For any $a\in L\setminus\{0\}$ and any pre-semi-closed S-a-R-NF ψ of A, there exist a countable subfamily φ of ψ and $b\in\beta(a)$ such that φ is a b-R-NF of A.

(6) For any $a\in L\setminus\{1\}$, each pre-semi-open S-a-shading μ of A has a countable subfamily which is an S-a-shading of A.

(7) For any $a\in L\setminus\{1\}$, each pre-semi-open S-a-shading μ of A has a countable subfamily which is an a-shading of A.

(8) For any $a\in L\setminus\{1\}$ and any pre-semi-open S-a-shading μ of A, there

exist a countable subfamily ν of μ and $b \in \alpha(a)$ such that ν is an S-b-shading of A.

(9) For any $a \in L \setminus \{1\}$ and any pre-semi-open S-a-shading μ of A, there exist a countable subfamily ν of μ and $b \in \alpha(a)$ such that ν is a b-shading of A.

(10) For any $a \in L\setminus\{0\}$, each pre-semi-open S-β_a-cover μ of A has a countable subfamily which is an S-β_a-cover of A.

(11) For any $a \in L\setminus\{0\}$, each pre-semi-open S-β_a-cover μ of A has a countable subfamily which is a β_a-cover of A.

(12) For any $a \in L \setminus \{0\}$ and any pre-semi-open S-β_a-cover μ of A, there exist a countable subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is an S-β_b-cover of A.

(13) For any $a \in L \setminus \{0\}$ and any pre-semi-open S-β_a-cover μ of A, there exist a countable subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is a β_b-cover of A.

(14) For any $a \in L\setminus\{0\}$ and any $b \in \beta(a)\setminus\{0\}$, each pre-semi-open Q_a-cover μ of A has a countable subfamily which is a Q_b-cover of A.

(15) For any $a \in L\setminus\{0\}$ and any $b \in \beta(a)\setminus\{0\}$, each pre-semi-open Q_a-cover μ of A has a countable subfamily which is a β_b-cover of A.

(16) For any $a \in L\setminus\{0\}$ and any $b \in \beta(a)\setminus\{0\}$, each pre-semi-open Q_a-cover μ of A has a countable subfamily which is an S-β_b-cover of A.

Proof. This is analogous to the proof of Theorem 4.1 in [4].

Remark 4.2. In Theorem 4.1, $a \in L \setminus \{0\}$ and $b \in \beta(a)$ can be replaced by $a \in M(L)$ and $b \in \beta^*(a)$, respectively. $a \in L \setminus \{1\}$ and $b \in \alpha(a)$ can be replaced by $a \in P(L)$ and $b \in \alpha^*(a)$, respectively. Thus, we can obtain other 15 equivalent conditions about the PS-Lindelöf L-set.

5. A good extension

Lemma 5.1([4]). Let $(X, \omega_L(\tau))$ be generated topologically by (X, τ). If A is a pre-semi-open L-set in (X, τ), then χ_A is a pre-semi-open set in $(X, \omega_L(\tau))$. If B is a pre-semi-open L-set in $(X, \omega_L(\tau))$, then $B_{(a)}$ is a pre-semi-open set in (X, τ) for every $a \in L$.