ON FUZZY WEAKLY SEMICLOSED FUNCTIONS*

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Abstract

In this paper, we introduce and characterize fuzzy weakly semi-closed functions between fuzzy topological spaces and also study these functions in relation to some other types of already known functions.

1 Introduction and Preliminaries

The concept of fuzzy sets was introduced by Prof. L.A. Zadeh in his classical paper[20]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzy topology which was introduced by C.L. Chang [5] in 1968. In 1980, Ming and Ming[12], introduced the concepts of quasi-coincidence and q-neighbourhoods by which the extensions of functions in fuzzy setting can very interestingly and effectively be carried out. In 1985, D.A. Rose [16] defined weakly closed functions in a topological spaces. In this paper we introduce and discuss the concept of fuzzy weakly semiclosed function which is weaker than fuzzy semiclosed functions introduced in [1] respectively and we obtained several properties and characterizations of these functions comparing with the other functions.

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Here it is seen that fuzzy semiclosedness implies fuzzy weakly semiclosedness but not conversely. But under a certain condition the converse is also true.

Throughout this paper by (X,τ)or simply by X we mean a fuzzy topological space (fts, shorty) due to Chang [5]. A fuzzy point in X with support $x \in X$ and value $p(0 < p \leq 1)$ is denoted by x_p. Two fuzzy sets $\lambda$and β are said to be quasi-coincident (q-coincident, shorty) denoted by $\lambda q \beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ [13] and by $\bar{q}$ we denote " is not q-coincident " . It is known [13] that $\lambda \leq \beta$ if and only if $\lambda \bar{q}(1-\beta)$. A fuzzy set λ is said to be q-neighbourhood (q-nbd) of x_p if there is a fuzzy open set μ such that $x_p q \mu$, and $\mu \leq \lambda$ if $\mu(x) \leq \lambda(x)$ for all $x \in X$. The interior, closure and the complement of a fuzzy set λin X are denoted by $Int(\lambda), Cl(\lambda)$ and $1-\lambda = \lambda^c$ respectively. For definitions and results not explained in this paper, the reader is referred to [1,5,8,9,11,15,17,20] assuming them to be well known.

DEFINITIONS 1.1. A fuzzy set λ in a fts X is called,
(1) Fuzzy semiopen [1] if $\lambda \leq Cl(Int(\lambda))$.
(2) Fuzzy semiclosed [1] if $Int(Cl(\lambda)) \leq \lambda$.
(3) Fuzzy preopen [4] if $\lambda \leq Int(Cl(\lambda))$.
(4) Fuzzy preclosed [4] if $Cl(Int(\lambda)) \leq \lambda$.
(5) Fuzzy regular open [1] if $\lambda = Int(Cl(\lambda))$.
(6) Fuzzy regular closed [1] if $\lambda = Cl(Int(\lambda))$.
(7) Fuzzy α-open [4] if $\lambda \leq Int(Cl(Int(\lambda)))$.
(8) Fuzzy α-closed [4] if $Cl(Int(Cl(\lambda))) \leq \lambda$.

Recall that if, λ be a fuzzy set in a fts X then $sCl(\lambda) = \bigcap\{\beta : \beta \geq \lambda, \quad \beta$ is fuzzy semiclosed$\}$ (resp. $sInt(\lambda) = \bigcup\{\beta : \lambda \geq \beta, \quad \beta$ is fuzzy semiopen$\}$) is called a fuzzy semiclosure of λ (resp. fuzzy semi interior of λ) [17].

RESULT 1.2. A fuzzy set λ in a fts X is fuzzy semiclosed (resp. fuzzy semiopen) if and only if $\lambda = sCl(\lambda)$ (resp. $\lambda = sInt(\lambda)$)[8].

DEFINITION 1.3. Let $f : (X,\tau_1) \rightarrow (Y,\tau_2)$ be a function from a fts (X,τ_1) into a fts (X,τ_2). The function $f$is called:
(i) fuzzy semiclosed [1] if $f(\lambda)$ is a fuzzy semiclosed
set of Y for each fuzzy closed set λ in X.
(ii) fuzzy open [5] if $f(\lambda)$ is a fuzzy open set in Y

for each fuzzy open set λin X.

(iii) fuzzy pre semiclosed [19] if $f(\lambda)$ is a fuzzy semiclosed set of Y for each fuzzy semi-closed set λ in X.

(iv) fuzzy contra-open (resp. fuzzy contra closed) if $f(\lambda)$ is a fuzzy closed (resp. fuzzy open) set of Y for each fuzzy open (resp. closed) set λ in X.

Recall that a fuzzy point x_p is said to be a fuzzy θ-cluster point of a fuzzy set λ[13], if and only if for every fuzzy open q-nbd μ of x_p , $Cl(\mu)$ is q-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and will be denoted by $Cl_\theta(\lambda)$. A fuzzy set λ will be called θ-closed if and only if $\lambda = Cl_\theta(\lambda)$. The complement of a fuzzy θ-closed set is called of fuzzy θ-open and the θ-interior of λ denoted by $Int_\theta(\lambda)$ is defined as $Int_\theta(\lambda) = \{x_p :$ for some fuzzy open q-nbd, β of $x_p, Cl(\beta) \leq \lambda\}$.

LEMMA 1.4.[3]. Let λ be a fuzzy set in a fts X, then:
1) λ is a fuzzy θ-open if and only if $\lambda = Int_\theta(\lambda)$.
2) $1 - Int_\theta(\lambda) = Cl_\theta(1 - \lambda)$ and $Int_\theta(1 - \lambda) = 1 - Cl_\theta(\lambda)$.
3) $Cl_\theta(\lambda)$ is a fuzzy closed set but not necessarily is a fuzzy θ-closed set.

2 Fuzzy Weakly Semiclosed Functions

Now, we define the generalized form of weakly closed and semiclosed functions in fuzzy setting.

DEFINITION 2.1. A function $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be fuzzy weakly semiclosed if $sCl(f(Int(\beta))) \leq f(\beta)$ for each fuzzy closed set β in X.

Clearly, every fuzzy semiclosed function is fuzzy weakly semiclosed function since $sCl(f(Int(\beta))) \leq sCl(f(\beta)) = f(\beta)$ for every fuzzy closed subset β of X, but the converse is not generally true.For ,

EXAMPLE 2.2. Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$. Fuzzy sets A and B are defined as :

$$A(x) = 0, \quad A(y) = 0.3, \quad A(z) = 0.2 \quad ;$$

$$B(a) = 0, \quad B(b) = 0.2, \quad B(c) = 0.1.$$

Let $\tau_1 = \{0, A, 1_X\}$ and $\tau_2 = \{0, B, 1_Y\}$. Then the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = a$, $f(y) = b$ and $f(z) = c$ is fuzzy weakly semiclosed but not fuzzy semiclosed.

THEOREM 2.3. For a function $f : (X, \tau_1) \to (Y, \tau_2)$. The following conditions are equivalent.

(i) f is fuzzy weakly semiclosed ,

(ii) $sCl(f(\lambda)) \leq f(Cl(\lambda))$ for every fuzzy open set λ in X .

PROOF. (i)→(ii). Let λ be any fuzzy open subset of X. Then $sCl(f(\lambda)) = sCl(f(Int(\lambda))) \leq sCl(f(Int(Cl(\lambda))) \leq f(Cl(\lambda))$.

(ii)→(i). Let β be any fuzzy closed subset of X. Then, $sCl(f(Int(\beta))) \leq f(Cl(Int(\beta))) \leq f(Cl(\beta)) = f(\beta)$.

For the proof of the following theorem, is mostly straightforward and hence omitted.

THEOREM 2.4. For a function $f : (X, \tau_1) \to (Y, \tau_2)$ the following conditions are equivalent :

(i) f is fuzzy weakly semiclosed ,

(ii) $sCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy open set λ in X,

(iii) $sCl(f(Int(\beta))) \leq f(Cl(Int(\beta)))$ for each fuzzy closed subset β in X,

(iv) $sCl(f(Int(\beta))) \leq f(Cl(Int(\beta)))$ for each fuzzy regular closed subset β in X,

(v) $sCl(f(Int(\beta))) \leq f(\beta)$ for each fuzzy preclosed subset β in X,

(vi) $sCl(f(Int(\beta))) \leq f(\beta)$ for every fuzzy α-closed subset β in X.

THEOREM 2.5. For a function $f : (X, \tau_1) \to (Y, \tau_2)$ the following conditions are equivalent :

(i) f is fuzzy weakly semiclosed,

(ii) $sCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy regular open subset λ of X,

(iii) For each fuzzy subset β in Y and each fuzzy open setμ in X with $f^{-1}(\beta) \leq \mu$, there exists a fuzzy semi-open set δ in Y with $\beta \leq \delta$ and $f^{-1}(\delta) \leq Cl(\mu)$,

(iv) For each fuzzy point y_p in Y and each fuzzy open set μ in X with $f^{-1}(y_p) \leq \mu$, there exists a fuzzy semi-open set δ in Y with $\beta \leq \delta$ and $f^{-1}(\delta) \leq Cl(\mu)$,

(v) $sCl(f(Int(Cl(\lambda)))) \leq f(Cl(\lambda))$for each fuzzy set λ in X,

(vi) $sCl(f(Int(Cl_\theta(\lambda)))) \leq f(Cl_\theta(\lambda))$for each fuzzy set λ in X,

(vii) $sCl(f(\lambda)) \leq f(Cl(\lambda))$for each fuzzy preopen set λ in X.

PROOF. It is clear that: $(i) \to (ii), (iii) \to (iv), (i) \to (v) \to (vii) \to (i)$ and $(i) \to (vi)$. To show that $(ii) \to (iii)$: Let β be a fuzzy subset in Y and let μ be fuzzy open in X with $f^{-1}(\beta) \leq \mu$. Then $f^{-1}(\beta)\bar{q}Cl(1_X - Cl(\mu))$ and consequently, $\beta\bar{q}f(Cl(1_X - Cl(\mu)))$. Since $1_X - Cl(\mu)$ is fuzzy regular open, $\beta\bar{q}sCl(f(1_X - Cl(\mu)))$ by (ii). Let $\delta = 1_Y - sCl(f(1_X - Cl(\mu)))$. Then δ is fuzzy semi-open with $\beta \leq \delta$ and $f^{-1}(\delta) \leq 1_X - f^{-1}(sCl(f(1_X - Cl(\mu)))) \leq 1_X - f^{-1}f(1_X - Cl(\mu)) \leq Cl(\mu)$.

$(i) \to (vi)$: Let β be fuzzy closed in $X$and let $y_p \in 1_Y - f(\beta)$. Since $f^{-1}(y_p) \leq 1_X - \beta$, there exists a fuzzy semi-open δ in $Y$with $y_p \in \delta$ and $f^{-1}(\delta) \leq Cl(1_X - \beta) = 1_X - Int(\beta)$ by (iv). Therefore $\delta\bar{q}f(Int(\beta))$, so that $y_p \in 1_Y - sCl(f(Int(\beta)))$. Thus $(iv) \to (i)$. Finally, for

$(iv) \to (vii)$: Note that $Cl_\theta(\lambda) = Cl(\lambda)$ for each fuzzy preopen subset λin X [9].

THEOREM 2.6. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiclosed, then for each fuzzy point y_p in Y and each fuzzy open q-nbd μ of $f^{-1}(y_p)$ in X, there exists a fuzzy semiopen q-nbd δ of y_p in Y, such that $f^{-1}(\delta) \leq Cl(\mu)$.

PROOF. Let μ be any fuzzy open q-nbd of $f^{-1}(y_p)$ in X. Then $\mu(x)+p > 1$ and hence there exists a positive real number α such that $\mu(x) > \alpha > 1-p$. Then μ is a fuzzy open nbd of $f^{-1}(y_\alpha)$. By Theorem 2.5(iv) there exists a fuzzy semi-open set δ containing y_α in Y such that $f^{-1}(\delta) \leq Cl(\mu)$. Now, $\delta(y) > \alpha$ and hence $\delta(y) > 1 - p$. Thus δ is a fuzzy semiopen q-nbd of y_p.

Next we investigate conditions under which fuzzy weakly semiclosed functions are fuzzy semiclosed.

THEOREM 2.7. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiclosed and if for each fuzzy closed subset β of X and each fiber $f^{-1}(y_p) \leq 1_X - \beta$ there exists a fuzzy open q-nbd μ of X such that $f^{-1}(y_p) \leq \mu \leq Cl(\mu) \leq 1_X - \beta$. Then f is fuzzy semiclosed.

PROOF. Let β is any fuzzy closed subset of X and let $y_p \in 1_Y - f(\beta)$. Then $f^{-1}(y_p)\bar{q}\beta$ and hence $f^{-1}(y_p) \leq 1_X - \beta$. By hypothesis, there exists a fuzzy open q-nbd μ of X such that $f^{-1}(y_p) \leq \mu \leq Cl(\mu) \leq 1_X - \beta$. Since f is fuzzy weakly semiclosed by Theorem 2.6, there exists a fuzzy semiopen q-nbd ν in Y with $y_p \in \nu$ and $f^{-1}(\nu) \leq Cl(\mu)$. Therefore , we obtain $f^{-1}(\nu)\bar{q}\beta$

and hence $\nu \bar{q} f(\beta)$, this shows that $y_p \notin sCl(f(\beta))$. Therefore, $f(\beta)$ is fuzzy semi-closed in Y and f is fuzzy semiclosed .

THEOREM 2.8. (i) If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy preclosed and fuzzy contra-closed, then f is fuzzy weakly semiclosed.

(ii) If $f : (X, \tau_1) \to (Y, \tau_2)$is fuzzy contra-open , then$f$ is fuzzy weakly semiclosed.

PROOF. (i) Let β be a fuzzy closed subset of X. Since f is fuzzy pre-closed $Cl(Int(f(\beta))) \leq f(\beta)$ and since$f$ is fuzzy contra-closed $f(\beta)$ is fuzzy open. Therefore $sCl(f(Int(\beta))) \leq sCl(f(\beta)) \leq Cl(Int(f(\beta))) \leq f(\beta)$.

(ii) Let β be a fuzzy closed subset of X. Then, $sCl(f(Int(\beta))) \leq f(Int(\beta)) \leq f(\beta)$.

S.Thakur and R.Saraf in [19] showed that fuzzy pre semiclosed function implies $sCl(f(\lambda)) \leq f(sCl(\lambda))$ for every fuzzy subset λ in X. Therefore every fuzzy pre-semiclosed function is fuzzy weakly semiclosed function but conversely.For ,

EXAMPLE 2.9 . Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$. Fuzzy sets A, B and H are defined as :

$$A(x) = 0, \quad A(y) = 0.3, \quad A(z) = 0.2 \quad ;$$

$$B(a) = 0, \quad B(b) = 0.3, \quad B(c) = 0.2;$$

$$H(a) = 0.9, \quad H(b) = 0.6, \quad H(c) = 0.7.$$

Let $\tau_1 = \{0, A, 1_X\}$ and $\tau_2 = \{0, B, H, 1_Y\}$. Then the function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = a$, $f(y) = b$ and $f(z) = c$ is fuzzy weakly semiclosed but not fuzzy presemiclosed.

THEOREM 2.10. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiclosed , then for every fuzzy subset β in Y and every fuzzy open set λ in X with $f^{-1}(\beta) \leq \lambda$, there exists a fuzzy semi-closed set δ in Y such that $\beta \leq \delta$ and $f^{-1}(\delta) \leq Cl(\lambda)$.

PROOF. Let β be a fuzzy subset of Y and let λ be a fuzzy open subset of X with $f^{-1}(\beta) \leq \lambda$. Put $\delta = sCl(f(Int(Cl(\lambda)))$, then δ is a fuzzy semi-closed set of Y such that $\beta \leq \delta$ since $\beta \leq f(\lambda) \leq f(Int(Cl(\lambda)) \leq sCl(f(Int(Cl(\lambda))) = \delta$. And since f is fuzzy weakly semiclosed, $f^{-1}(\delta) \leq Cl(\lambda)$.

COROLLARY 2.11. If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is fuzzy weakly semi-closed , then for every fuzzy point y_p in $Y$and every fuzzy open set λ in X with$f^{-1}(y_p) \leq \lambda$, there exists a fuzzy semi-closed set δ in Y containing y_p such that $f^{-1}(\delta) \leq Cl(\lambda)$.

A fuzzy set β in a fts X is fuzzy θ-compact if for each cover Ω of β by fuzzy open q-nbd $\mu_{x_p}$with $x_p \in \mu_{x_p}$ in X, there is a finite family $\{\mu_1, ..., \mu_n\}$ in Ω such that $\beta \leq Int(\cup\{Cl(\mu_i) : i = 1, 2, ..., n\})$.

THEOREM 2.12. If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is fuzzy weakly semiclosed with all fibers fuzzy θ-closed, then$f(\beta)$ is fuzzy semi-closed for each fuzzy θ-compact β in$X$.

PROOF. Let β be fuzzy θ-compact and let $y_p \in 1_Y - f(\beta)$. Then $f^{-1}(y_p)\bar{q}\beta$ and for each $x_p \in \beta$ there is a fuzzy open q-nbd μ_{x_p} containing x_p in X and $Cl(\mu_{x_p})\bar{q}f^{-1}(y_p)$. Clearly $\Omega = \{\mu_{x_p} : x_p \in \beta\}$ is a fuzzy open q-nbd cover of β and since β is fuzzy θ-compact, there is a finite family $\{\mu_{x_1}, ..., \mu_{x_n}\}$ in Ω such that $\beta \leq Int(\lambda)$, where $\lambda = \cup\{Cl(\mu_{x_i}) : i = 1, ..., n\}$. Since f is fuzzy weakly semiclosed by Theorem 2.6 there exists a fuzzy semi-open q-nbd δ in Y with $f^{-1}(y_p) \leq f^{-1}(\delta) \leq Cl(1_X - \lambda) = 1_X - Int(\lambda) \leq 1_X - \beta$. Therefore $y_p \in \delta$ and $\delta\bar{q}f(\beta)$. Thus $y_p \in 1_Y - sCl(f(\beta))$.This shows that $f(\beta)$ is fuzzy semi-closed.

Two non empty fuzzy subsets λ and β in X are strongly fuzzy separated if there exist fuzzy open sets μ and ν in X with $\lambda \leq \mu$ and $\beta \leq \nu$ and $Cl(\mu)\bar{q}Cl(\nu)$. If λ and β are fuzzy singleton sets we may speak of fuzzy points being strongly fuzzy separated . We will use the fact that in a fuzzy normal space, disjoint fuzzy closed sets are strongly fuzzy separated.

Recall that space$X$ is fuzzy $semi - T_2$ [7] if for every pair of fuzzy points x_p and x_q with different supports, there exist two fuzzy semi-open sets λ and β such that $x_p \leq \lambda \leq x_q^c$ and $x_q \leq \beta \leq x_p^c$ and $\lambda\bar{q}\beta$.

THEOREM 2.13.If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiclosed surjection and all pairs of disjoint fibers are strongly fuzzy separated, then Y is fuzzy $semi - T_2$.

PROOF. Let y_p and y_q two fuzzy points in Y. Let μ and ν be fuzzy open sets in X such that $f^{-1}(y_p) \leq \mu$ and $f^{-1}(y_q) \leq \nu$ respectively with $Cl(\mu)\bar{q}Cl(\nu)$. By fuzzy weak semiclosedness (Theorem 2.5(iv)) there are fuzzy semi-open sets λ and β in Y such that $y_p \leq \lambda$ and $y_q \leq \beta$, $f^{-1}(\lambda) \leq Cl(\mu)$ and $f^{-1}(\beta) \leq Cl(\nu)$. Therefore $\lambda\bar{q}\beta$, because $Cl(\mu)\bar{q}Cl(\nu)$ and $f$surjective. Then Y is fuzzy $semi - T_2$.

COROLLARY 2.14. If $f : (X, \tau_1) \to (Y, \tau_2)$ is weakly fuzzy semiclosed surjection with all fuzzy closed fibers and X is fuzzy normal, then Y is fuzzy $semi - T_2$.

The next result follows from Corollary 2.14.

COROLLARY 2.15. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy continuous and fuzzy weakly semiclosed surjection with X a fuzzy compact T_2 space and Y a fuzzy T_1 space, then Y is a fuzzy compact and a fuzzy $semi - T_2$ space.

DEFINITION 2.16. A family $\{\lambda_\alpha : \alpha \in \Omega\}$ of fuzzy open subsets of a fuzzy topological space (X, τ) is a cover if $\cup\{\lambda_\alpha : \alpha \in \Omega\} = X$. An fts X is said to be fuzzy almost compact [10,6] (resp. fuzzy c-compact) if every fuzzy open cover (resp. fuzzy semiclosed cover) contains a finite subfamily $\{\lambda_{\alpha_i} : i = 1, 2, ..., n\}$ such that $X = \overset{n}{\underset{i}{\cup}} Cl(\lambda_{\alpha_i})$. A fuzzy subset λ of a fts X is fuzzy almost compact relative to X (resp. fuzzy c-compact relative to X) if every cover of λ by fuzzy open (resp. fuzzy semiclosed) sets of X has a finite subfamily whose fuzzy closures cover X.

Recall that a fts (X, τ) is said to be fuzzy extremally disconnected if the closure of every fuzzy open set of X is fuzzy open in X [2,9].

A fts (X, τ) satisfies the property (so) if the finite intersection of semi-open sets is semi-open.

LEMMA 2.17. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy open if and only if for each fuzzy subset β in Y , $f^{-1}(Cl(\beta)) \leq Cl(f^{-1}(\beta))$ [18].

THEOREM 2.18. Let X be an fuzzy extremally disconnected space that satisfies the property (so). Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a fuzzy open and fuzzy weakly semiclosed function such that $f^{-1}(y_p)$ is fuzzy almost compact relative to X for each fuzzy point y_p in Y. If λ is fuzzy c-compact relative to Y. Then $f^{-1}(\lambda)$ is fuzzy almost compact.

PROOF. Let $\{\nu_\beta : \beta \in I\}$, I being the index set be a fuzzy open cover of $f^{-1}(\lambda)$. Then for each $y_p \in \lambda \cap f(X)$, $f^{-1}(y_p) \leq \cup\{Cl(\nu_\beta) : \beta \in I(y_p)\} = \delta_{y_p}$ for some finite subfamily $I(y_p)$ of I. Since X is fuzzy extremally disconnected each $Cl(\nu_\beta)$ is fuzzy open, hence δ_{y_p} is fuzzy open in X. So by Corollary 2.11, there exists a fuzzy semi-closed set μ_{y_p} containing y_p such that $f^{-1}(\mu_{y_p}) \leq Cl(\delta_{y_p})$. Then, $\{\mu_{y_p} : y_p \in \lambda \cap f(X)\} \cup \{1_Y - f(X)\}$ is a fuzzy semi-closed cover of λ, $\lambda \leq \cup\{Cl(\mu_{y_p}) : y_p \in K\} \cup \{Cl(1_Y - f(X))\}$ for some finite fuzzy subset K of $\lambda \cap f(X)$. Hence and by Lemma 2.17, $f^{-1}(\lambda) \leq \cup\{f^{-1}(Cl(\mu_{y_p}) : y_p \in K\} \cup \{f^{-1}(Cl(1_Y - f(X))\} \leq \cup\{Cl(f^{-1}(\mu_{y_p}) : y_p \in K\} \cup \{Cl(f^{-1}(1_Y - f(X)))\} \leq \{Cl(f^{-1}(\mu_{y_p})) : y_p \in K\}$, so $f^{-1}(\lambda) \leq \cup\{Cl(\nu_\beta) : \beta \in I(y_p), y_p \in K\}$. Therefore $f^{-1}(\lambda)$ is fuzzy almost compact.

COROLLARY 2.19. Let $f : (X, \tau_1) \to (Y, \tau_2)$ be as in Theorem 2.18. If Y is c-compact, then X is almost compact.

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