A note on fuzzy semi-preopen sets and semi-precontinuity*

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Abstract: *S.S. Thakur and Surendra Singh have introduced the concepts of fuzzy semi-preopen sets and fuzzy semi-precontinuous mappings in 1998 (Fuzzy Sets and Systems 98 (1998) 383-391). We show in this note that there are mathematical errors in some examples in this paper, and some unproper examples. Also we give some correct examples and generalize Kuratowski's 14-sets theorem.*

Key words: *I-topology; I-semi-preopen set; I-semi-precontinuity*

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1. Introduction

Fuzzy continuity and many of its weaker forms and stronger forms have been richly studied in fuzzy topological spaces. In [7], the notions of fuzzy semi-preopen sets, fuzzy semi-precontinuous mappings and fuzzy semi-preopen mappings etc. are given according to the sense of Chang-Goguen spaces. By standard terminology in [5], these are corresponding with the following *I*-topological spaces, *I*-semi-preopen sets, *I*-semi-precontinuous mappings and *I*-semi-preopen mappings etc. Here we use standard terminology in [5].

In this paper, apart from the fact that the concepts were introduced in S.S. Thakur and Surendra Singh's paper[7] without regard for whether all the examples are proper or correct or without regard for some of the results are so deep, we show in this note that there are mathematical errors in some of the examples as well. Also we show that there are some correct examples and quite important results.

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2. Preliminaries

In the paper by (X, δ) or simply by X we mean an I-topological space. $intA$, clA and A^c denote the interior, closure and complement of a subset A in X, respectively. A subset A in X is called I-preopen if and only if $A \leqslant int(clA)$, and I-preclosed if and only if $A \geqslant cl(intA)$ [3,6]. A subset A in X is called I-semiopen if and only if there is a I-open set B such that $B \leqslant A \leqslant clB$, and I-semiclosed if and only if there is a I-closed set B such that $intB \leqslant A \leqslant B$ [1]. A subset A in X is called I-semi-preopen if and only if there is a I-preopen set B such that $B \leqslant A \leqslant clB$, and I-semi-preclosed if and only if there is a I-preclosed set B such that $intB \leqslant A \leqslant B$ [1]. $FPO(X)$, $FPC(X)$, $FSO(X)$, $FSC(X)$, $FSPO(X)$ and $FSPC(X)$ denote the family of I-preopen sets, family of I-preclosed sets, family of I-semiopen sets, family of I-semiclosed sets, family of I-semi-preopen sets and family of I-semi-preclosed sets of an I-topological space X, respectively. $spintA = \cup\{B: B \in FSPO(X), B \leqslant A\}$, and $spclA = \cap\{B: B \in FSPC(X), A \leqslant B\}$ are called the I-semi-preinterior and I-semi-preclosure of A, respectively[7].

Definition. Let $f: (X, \delta) \to (Y, \tau)$ be a mapping from an I-topological space (X, δ) to another I-topological space (Y, τ). f is called:

(1) A I-semicontinuous mapping if $f^{\leftarrow}(B) \in FSO(X)$ for each $B \in \tau$ [1].

(2) A I- precontinuous mapping if $f^{\leftarrow}(B) \in FPO(X)$ for each $B \in \tau$ [3,6].

(3) A I-semi-precontinuous mapping $f^{\leftarrow}(B) \in FSPO(X)$ for each $B \in \tau$ [7].

(4) A I-semiopen mapping if $f^{\rightarrow}(A) \in FSO(Y)$ for each $A \in \delta$ [1].

(5) A I- preopen mapping if $f^{\rightarrow}(A) \in FPO(Y)$ for each $A \in \delta$ [3,6].

(6) A I-semi-preopen mapping $f^{\rightarrow}(A) \in FSPO(Y)$ for each $A \in \delta$ [7].

3. Remarks and Examples

Remark 1. The intersection of two I-semi-preopen sets need not be I-semi-preopen. Even the intersection of an I-semi-preopen set with an I-open set may fail to be I-semi-preopen.

Example 2.2 in [7], which meant to show this statement, is incorrect. We rewrite this as follows:

Example 2.2 [7]. Let $X=\{x, y\}$ and A, B be subsets of X defined as follows:

$A(x)=0.3, A(y)=0.7; \quad B(x)=0.7, B(y)=0.4.$

Let $\tau=\{0,A,1\}$ be an I-topology on X. Then A is I-preopen (I-open) and B is I-semi-preopen but their intersection is not I-semi-preopen.

Where $A \cap B(x)=0.3$, $A \cap B(y)=0.4$ is I-semi-preopen. In fact, $A \cap B \leqslant \mathrm{IntCl}(A \cap B) = \mathrm{Int}1=1$, i.e. $A \cap B$ is I-preopen. Hence, $A \cap B$ is I-semi-preopen.

The next example shows that Remark 1 is valid.

Example 1. Let $X=\{x, y, z\}$ and A, B be subsets of X defined as follows:

$A(x)=0.4$, $A(y)=0.5$, $A(z)=0.6$;

$B(x)=0.8$, $B(y)=0.7$, $B(z)=0.3$.

Then $\tau=\{0,A,1\}$ is an I-topology on X. Clearly, B is I-semi-preopen. But $A\cap B$ is not I-semi-preopen. In fact, for every $D\neq 0$ and $D\leqslant(A\cap B)$, by easy computations it follows that $IntCl(D)=Int(A^c)=0$, ie., $D\not\leqslant IntCl(D)$. Hence, D is not I-preopen. The above shows that only 0 is an I-preopen set contained in $(A\cap B)$. Thus, $A\cap B$ is not I-semi-preopen.

Remark 2. Every I-semiopen set and every I-preopen set is I-semi-preopen. But the converses may not be true.

Example 2.1 in [7], which meant to show this statement, is improper. We rewrite this as follows:

Example 2.1 [7]. Let $X=\{x, y\}$ and A, B, C be subsets of X defined as follows:

$A(x)=0.3, A(y)=0.4$; $B(x)=0.7, B(y)=0.8$; $C(x)=0.6, C(y)=0.5$.

Let $\tau=\{0,A,1\}$ be an I-topology on X, then (1) B is I-semi-preopen but not I-semiopen. (2) C is I-semi-preopen but not I-preopen.

Clearly, B is I-preopen, so B is surely I-semi-preopen; C is I-semiopen, so C is surely I-semi-preopen. Thus, this example actually shows that I-semiopen set and I-preopen set are independent notions.

Here we give a far more proper example to show that Remark 2 is valid.

Example 2. Let $X=\{x, y, z\}$ and A, B, C be subsets of X defined as follows:

$A(x)=0.2$, $A(y)=0.4$, $A(z)=0.5$;

$B(x)=0.3$, $B(y)=0.2$, $B(z)=0.4$;

$C(x)=0.1$, $C(y)=0.2$, $C(z)=0.3$.

Then $\tau=\{0,A,1\}$ is an I-topology on X. Clearly, C is I-preopen, such that $C\leqslant B\leqslant Cl(C)=A^c$. So B is I-semi-preopen. But B is not an I-semiopen set neither an I-preopen set. In fact, because 0 is the only I-open set contained in B, B is not I-semiopen. And because $B\not\leqslant IntCl(B)=Int(A^c)=A$, B is not I-preopen.

Remark 3. Every I-semicontinuous (resp. I-precontinuous) mapping is I-semi-precontinuous. But the converse may not be true.

Example 3.1 in [7], which meant to show this statement, is improper. We rewrite this as follows:

Example 3.1 [7]. Let $X=\{a, b\}$, $Y=\{x, y\}$ and A, B, C be subsets of X defined as follows:

$A(a)=0.3, A(b)=0.4$; $B(x)=0.7, B(y)=0.8$; $C(x)=0.6, C(y)=0.5$.

Let $\tau_1=\{0,A,1\}$, $\tau_2=\{0,B,1\}$ and $\tau_3=\{0,C,1\}$. Then the mapping $f:(X, \tau_1)\to(Y, \tau_2)$

defined by $f^{\to}(a)=x$, $f^{\to}(b)=y$ is *I*-semi-precontinuous but not *I*-semicontinuous, and $g:(X, \tau_1)\to(Y, \tau_3)$ defined by $g^{\to}(a)=x$, $g^{\to}(b)=y$ is *I*-semi-precontinuous but not *I*-precontinuous.

The problem here is analogous the Example 2.1 in [7]. Clearly, f is *I*-precontinuous and g is *I*-semicontinuous. Thus, this example actually shows that *I*-semicontinuous mapping and *I*-precontinuous mapping is independent notions.

Here we give a far more proper example to show that Remark 3 is valid.

Example 3. Considering Example 2 and let $\delta=\{0,B,1\}$. Then $\delta=\{0,B,1\}$ is an *I*-topology on X. Considering the identity mapping $f:(X, \tau)\to(X, \delta)$. Then f is *I*-semi-precontinuous. But f is not an *I*-semicontinuous mapping neither an *I*-precontinuous mapping.

Remark 4. Every *I*-semiopen (resp. *I*-preopen) mapping is *I*-semi-preopen. But the converse may not be true.

In [7], Example 4.1 is the analogous problem of Example 3.1 and omitted. The next example shows that Remark 4 is valid.

Example 4. Refer to Example 3. Then $f^{\leftarrow}$ is an *I*-semi-preopen mapping. But $f^{\leftarrow}$ is not an *I*-semiopen mapping neither an *I*-preopen mapping.

The Kuratowski's 14-sets theorem is one of the most important properties in topology. Here Kuratowski's 14-sets theorem is generalized as the following. Firstly we show two lemmas.

Lemma 1. *Let A be a subset of I-topological space X. Then*

(1) $(spintA)^c=spcl(A^c)$.

(2) $(spclA)^c=spint(A^c)$.

Proof. We prove only (1). Since $spintA\leqslant A$ and $spintA\in FSPO(X)$, $A^c\leqslant(spintA)^c$ and $(spintA)^c\in FSPC(X)$. Hence, $spcl(A^c)\leqslant(spintA)^c$. Conversely, by $A^c\leqslant spcl(A^c)$ and $spcl(A^c)\in FSPC(X)$, we have $(spcl(A^c))^c\leqslant A$ and $(spcl(A^c))^c\in FSPO(X)$, so that $(spcl(A^c))^c\leqslant spintA$ and hence $(spintA)^c\leqslant spcl(A^c)$. Thus, $(spintA)^c=spcl(A^c)$.

Lemma 2. *Let A be a subset of I-topological space X. Then*

(1) $spint(spcl(spint(spclA)))=spint(spclA)$.

(2) $spcl(spint(spcl(spintA)))=spcl(spintA)$.

Proof. We prove only (1). Since $spint(spclA)\in FSPO(X)$ and

$$spint(spclA)\leqslant spcl(spint(spclA)),$$

$$spint(spclA)=spint(spint(spclA))\leqslant spint(spcl(spint(spclA))).$$

Conversely, by $spint(spclA) \leqslant spclA$ and $spclA \in FSPC(X)$, we have

$$spcl(spint(spclA)) \leqslant spcl(spclA) = spclA,$$

so $spint(spcl(spint(spclA))) \leqslant spint(spclA)$.

Thus, $spint(spcl(spint(spclA))) = spint(spclA)$.

Theorem 1. *Let A be a subset of I-topological space X. If we subject A to three operations: semi-preinterior, semi-preclosure and complemen, we can have at most 14 different sets in X.*

Proof. This can be proved easily as in general topological spaces by using Lemma 1 and 2.

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