Weakly Fuzzy Semi-preirresolute Functions

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Abstract: *The aim of this paper is to introduce weakly fuzzy semi-preirresolute function in fuzzy topological spaces, and study its properties and its relationship with other functions.*

Key words: *Fuzzy topology; preopen set; semi-preopen set; weakly fuzzy semi-preirresolute function*

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1. Introduction

In [8], the concepts of fuzzy semi-preopen set and fuzzy semi-precontinuous function were introduced in fuzzy topological spaces. In [2,3], semi-preirresolute order-homomorphism was introduced in L-fuzzy topological spaces. In this paper, we introduce a new class of function in fuzzy topological spaces, called weakly fuzzy semi-preirresolute function. It is weaker forms of fuzzy semi-preirresolute function and fuzzy preirresolute function.

2. Preliminaries

In the paper by (X, δ) or simply by X we mean a fuzzy topological space in the Chang's[5] sense, briefly fts. A^o, A^- and A' denote the interior, closure and complement of fuzzy set A, respectively. A fuzzy set A in X is called preopen if and only if $A \leqslant A^{-o}$, and preclosed if and only if $A \geqslant A^{o-}$ [4,7]. $PO(X)$ and $PC(X)$ denote the family of preopen sets and family of preclosed sets of an fts X, respectively. The $A^{\square} = \cup\{B: B \in PO(X), B \leqslant A\}$ and $A^{\frown} = \cap\{B: B \in PC(X), A \leqslant B\}$ are called the pre-interior and pre-closure of fuzzy set A[4], respectively. A fuzzy set A in X is called semi-preopen if and only if there is a preopen set B such that $B \leqslant A \leqslant B^-$, and semi-preclosed if and only if there is a preclosed set B such that $B^o \leqslant A \leqslant B$ [8]. $SPO(X)$ and $SPC(X)$ denote the family of semi-preopen sets and family of semi-preclosed sets of an fts X, respectively. $A_{\square} = \cup\{B: B \in SPO(X), B \leqslant A\}$, and $A_{\frown} = \cap\{B: B \in SPC(X), A \leqslant B\}$ are called the semi-preinterior and semi-preclosure of A, respectively[2,3]. It is clear that every semiopen set [1] is semi-preopen and every preopen set is semi-preopen.

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None of the converses need be true [8].

Definition 1.1[2-4,7,8]. Let $f: (X, \delta) \to (Y, \tau)$ be a function from an fts (X, δ) to another fts (Y, τ). f is called:

(1) A fuzzy semi-preirresolute function if $f^{-1}(B) \in SPO(X)$ for each $B \in SPO(Y)$.

(2) A fuzzy preirresolute function if $f^{-1}(B) \in PO(X)$ for each $B \in PO(Y)$.

(3) A fuzzy precontinuous function if $f^{-1}(B) \in PO(X)$ for each $B \in \tau$.

(4) A fuzzy semi-precontinuous function $f^{-1}(B) \in SPO(X)$ for each $B \in \tau$.

3. Weakly Fuzzy Semi-preirresolute Functions

Definition 3.1. A function $f:(X, \delta) \to (Y, \tau)$ from an fts (X, δ) to another fts (Y, τ) is said to be weakly fuzzy semi-preirresolute if $f^{-1}(B) \in SPO(X)$ for each $B \in PO(Y)$.

Theorem 3.2. Let $f:(X, \delta) \to (Y, \tau)$ be a function. Then the following are equivalent:

(1) f is weakly fuzzy semi-preirresolute.

(2) $f^{-1}(B) \in SPC(X)$ for each $B \in PC(Y)$.

(3) $f(A_{\frown}) \leqslant (f(A))^{\frown}$ for each fuzzy set A in X.

(4) $(f^{-1}(B))_{\frown} \leqslant f^{-1}(B^{\frown})$ for each fuzzy set B in Y.

(5) $f^{-1}(B^{\square}) \leqslant (f^{-1}(B))_{\square}$ for each fuzzy set B in Y.

(6) For each fuzzy point x_{α} in X and each $B \in PO(Y)$ with $f(x_{\alpha}) \in B$, there exists an $A \in SPO(X)$ such that $x_{\alpha} \in A$ and $f(A) \leqslant B$.

Proof. (1)⇒(2): Obvious.

(2)⇒(3): $(f(A))^{\frown} \in PC(Y)$ for each fuzzy set A in X. *By (2)*, $f^{-1}((f(A))^{\frown}) \in SPC(X)$ and

$$A_{\frown} \leqslant (f^{-1} f(A))_{\frown} \leqslant (f^{-1}((f(A))^{\frown}))_{\frown} = f^{-1}((f(A))^{\frown}).$$

Thus, $f(A_{\frown}) \leqslant (f(A))^{\frown}$.

(3)⇒(4): Let B be fuzzy set in Y. By (3),

$$f((f^{-1}(B))_{\frown}) \leqslant (ff^{-1}(B))^{\frown} \leqslant B^{\frown}.$$

Thus, $(f^{-1}(B))_{\frown} \leqslant f^{-1}(B^{\frown})$.

(4)⇒(5): Let B be a fuzzy set in Y. By (4),

$$f^{-1}(B'^{\frown}) \geqslant (f^{-1}(B'))_{\frown} = (f^{-1}(B))')_{\frown}.$$

From $B'^{\frown}{}' = B^{\square}$ and $(B')_{\frown})' = B_{\square}$ we have

$$f^{-1}(B^{\square}) = f^{-1}(B'^{\frown}{}') = (f^{-1}(B'^{\frown}))' \leqslant ((f^{-1}(B'))_{\frown})' = ((f^{-1}(B))')_{\frown})' = (f^{-1}(B))_{\square}.$$

(5)⇒(1): Let $B \in PO(Y)$, then $B = B^{\square}$. By (5),

$$f^{-1}(B) = f^{-1}(B^{\square}) \leqslant (f^{-1}(B))_{\square} \leqslant f^{-1}(B).$$

i.e. $f^{-1}(B) = (f^{-1}(B))_{\square}$, and $f^{-1}(B) \in SPO(X)$. Thus, f is weakly fuzzy semi-preirresolute.

(1)⇒(6): Let f be weakly fuzzy semi-preirresolute, x_{α} be a fuzzy point in X and $B \in PO(Y)$ such that $f(x_{\alpha}) \in B$. Then $x_{\alpha} \in f^{-1}(B)$. Let $A = f^{-1}(B)$, then $A \in SPO(X)$. We have $f(A) = ff^{-1}(B) \leqslant B$.

(6)⇒(1): Let x_{α} be a fuzzy point in X and $B \in PO(Y)$ such that $f(x_{\alpha}) \in B$. Then $x_{\alpha} \in f^{-1}(B)$. By (6), there exists an $A \in SPO(X)$ such that $x_{\alpha} \in A$ and $f(A) \leqslant B$. Hence,

$x_\alpha \in A \leqslant f^{-1}f(A) \leqslant f^{-1}(B)$,

and

$x_\alpha \in A = A_\square \leqslant (f^{-1}(B))_\square$.

This implies that $f^{-1}(B) \leqslant (f^{-1}(B))_\square$, i.e. $f^{-1}(B) \in SPO(X)$. Thus, f is weakly fuzzy semi-preirresolute.

Theorem 3.3. Let $f:(X, \delta) \rightarrow (Y, \tau)$ be one-to-one and onto. Then f a weakly fuzzy semi-preirresolute function if and only if $(f(A))^\square \leqslant f(A_\square)$ for each fuzzy set A in X.

Proof. Let f be a weakly fuzzy semi-preirresolute function and A be a fuzzy set in X. Then $(f(A))^\square \in PO(Y)$ and $f^{-1}((f(A))^\square) \in SPO(X)$. By Theorem 2.===3 and the fact that f is one-to-one, we have

$f^{-1}((f(A))^\square) \leqslant (f^{-1} f(A))_\square = A_\square$.

Again, since f is onto, we have $(f(A))^\square \leqslant f(A_\square)$.

Conversely, let $B \in PO(Y)$, then $B = B^\square$. By hypothesis,

$f((f^{-1}(B))_\square) \geqslant (ff^{-1}(B))^\square = B^\square = B$.

This implies that

$(f^{-1}(B))_\square = f^{-1}f((f^{-1}(B))_\square) \geqslant f^{-1}(B)$.

i.e. $f^{-1}(B) \in SPO(X)$. Thus, f is a weakly fuzzy semi-preirresolute function.

We can easily prove the following proposition and theorem.

Proposition 3.4. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. The following statemens are valid:

(1) If f is weakly fuzzy semi-preirresolute and g is fuzzy preirresolute, then gf is weakly fuzzy semi-preirresolute.

(2) If f is fuzzy semi-preirresolute and g is weakly fuzzy semi-preirresolute, then gf is weakly fuzzy semi-preirresolute.

(3) If f is weakly fuzzy semi-preirresolute and g is fuzzy precontinuous, then gf is fuzzy semi-precontinuous.

Theorem 3.5. Let $f: X_1 \rightarrow X_2$ and $g: X_3 \rightarrow X_4$ be weakly fuzzy semi-preirresolute. Then the product $f \times g: X_1 \times X_3 \rightarrow X_2 \times X_4$ is weakly fuzzy semi-preirresolute.

4. Relations among different functions

Clearly, the following statements are valid:

fuzzy semi-preirresoluteness ↘

weakly fuzzy semi-preirresoluteness

fuzzy preirresoluteness ↗

None of the converses need to be true. We give the following examples.

Example 4.1. Let $X=\{x\}$ and A, B, C be fuzzy sets in X defined as follows:

$A(x)=0.3$; $B(x)=0.5$; $C(x)=0.2$, $D(x)=0.6$.

Then $\delta=\{0,A,B,1\}$ and $\tau=\{0,C,1\}$ are fuzzy topologies on X. Let $f: (X,\delta)\to(X,\tau)$ be an identity mapping. Clearly, f is weakly fuzzy semi-preirresolute. We can easily get that $D\in SPO(X,\tau)$ and $f^{-1}(C)=C\notin SPO(X,\delta)$. Thus, f is not fuzzy semi-preirresolute.

Example 4.2. Let $X=\{x\}$ and A, B, C be fuzzy sets in X defined as follows:

$A(x)=0.2$; $B(x)=0.4$; $C(x)=0.3$.

Then $\delta=\{0, A, 1\}$ and $\tau=\{0, B, 1\}$ are fuzzy topologies on X. Let $f: (X,\delta)\to(X,\tau)$ be an identity mapping. Clearly, f is weakly fuzzy semi-preirresolute. We can easily get that $C\in PO(X,\tau)$, but $f^{-1}(C)=C\notin PO(X,\delta)$, in fact, $C\not\leq C^{-o}=A'^{o}=A$ in (X,δ). Thus, f is not fuzzy preirresolute.

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