# WEAK AND STRONG FORMS OF $\alpha$-IRRESOLUTENESS IN FUZZY TOPOLOGICAL SPACES*

Ratnesh SARAF and Govindappa NAVALAGI

**ABSTRACT :**In this paper, we introduce and study three new classes of functions using fuzzy g$\alpha$-closed sets called ap-F$\alpha$-irresolute functions, ap-F$\alpha$-closed functions and contra fuzzy $\alpha$-irresolute functions.The connections between these functions and other existing fuzzy topological functions are studied.Also, we obtain the characterizations of fuzzy $\alpha - T_{1/2}$- spaces via these defined functions.

## 1 INTRODUCTION

The concept of fuzzy sets was introduced by Prof. L.A. Zadeh in his classical paper[13]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzy topology which was introduced by C.L. Chang [3] in 1968. In 1992 M.K.Singal et al [**?**] introduced and investigated the notions of fuzzy $\alpha$-open and fuzzy $\alpha$-closed sets. Since the advent of these notions, several research papers with interesting results in different respects came to existence ([4, **?**, 7, 9, 10, 11, 12]). Recently, in 2000 Saraf and Mishra [5] have generalized the concept of fuzzy $\alpha$ - closed sets to fuzzy generalized $\alpha$-closed sets . In this paper we shall introduce a new class of generalizations of fuzzy $\alpha$-irresoluteness , called ap-fuzzy $\alpha$-irresoluteness by using fuzzy g$\alpha$-closed sets and study some of their basic properties. This definition enables us to obtain conditions under which functions and inverse functions preserve fuzzy g$\alpha$-closed sets. Alos in this paper, we present a new generalization of fuzzy $\alpha$-irresoluteness called contra fuzzy $\alpha$-irresolute. We define contra fuzzy $\alpha$-irresolute function as the inverse image of each fuzzy $\alpha$-open set is fuzzy $\alpha$-closed set.The class of contra fuzzy $\alpha$-irresolute functions is a stronger form than that of classes of ap-fuzzy $\alpha$-irresolute and ap-fuzzy $\alpha$-closed functions.

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## 2 PRELIMINARIES

Throughout this paper by $(X, \tau)$ or simply by $X$ we mean a fuzzy topological space (in brief, fts) due to Chang[3].Let $A$ be a fuzzy subset of $X$.The interior, closure and the complement of a fuzzy set $A$ in $X$ are denoted by $Int(A), Cl(A)$ and $1 - A$ respectively.

**DEFINITION 2.1:** A fuzzy set $A$ in a fts $X$ is called,

(1) Fuzzy preopen[2] if $A \leq Int(Cl(A))$.
(2) Fuzzy preclosed [2] if $Cl(Int(A)) \leq A$.
(3) Fuzzy semiopen[1] if $A \leq Cl(Int(A))$.
(4) Fuzzy semiclosed [1] if $Int(Cl(A)) \leq A$.
(5) Fuzzy regular open [1] if $A = Int(Cl(A))$.
(6) Fuzzy regular closed [1] if $A = Cl(Int(A))$.
(7) Fuzzy $\alpha$-open[8] if $A \leq Int(Cl(Int(A)))$.
(8) Fuzzy $\alpha$-closed [8] if $Cl(Int(Cl(A))) \leq A$.

Recall that if, $A$ be a fuzzy set in a fts $X$ then $\alpha Cl(A) = \bigcap\{B : B \geq A, \quad B is fuzzy \alpha - closed\}$ (resp. $\alpha Int(A) = \bigcup\{B : A \geq B, \quad B$ is fuzzy $\alpha - open\}$) is called a fuzzy $\alpha$-closure of $A$ (resp. fuzzy $\alpha$-interior of $A$ )[8].

**RESULT 2.2:** (1) A fuzzy set $A$ in a fts $X$ is fuzzy $\alpha$-closed (resp. fuzzy $\alpha$-open) if and only if $A = \alpha Cl(A)$ (resp. $A = \alpha - Int(A)$) [8].
(2) Let $A$ a fuzzy set of a fts $(X, \tau)$. Then ;
$Int(A) \leq \alpha Int(A) \leq A \leq \alpha Cl(A) \leq Cl(A)$.

**DEFINITION 2.3[5]:** A fuzzy set $A$ of $(X, \tau)$ is said to be fuzzy generalized $\alpha$-closed (in brief, $Fg\alpha$-closed) set , if $\alpha Cl(A) \leq H$ whenever $A \leq H$ and $H$ is fuzzy $\alpha$-open set in $X$.

The complement of $Fg\alpha$-closed set is fuzzy generalized open (in brief, $Fg\alpha$-open) set [5].

**DEFINITION 2.4:** Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a fts $(X, \tau)$ into a fts $(Y, \sigma)$. Then, $f$is called:

(i) fuzzy $\alpha$-irresolute [4] if for each $V \in F\alpha O(Y)$, $f^{-1}(V) \in F\alpha O(X)$ .

(ii) fuzzy pre $\alpha$-closed [10] if $f(A)$ is a fuzzy $\alpha$-closed set of $Y$ for each fuzzy $\alpha$-closed set $A$ of $X$.

(iii) fuzzy pre $\alpha$-open [9] if $f(A)$ is a fuzzy pre $\alpha$ - open set in $Y$ for each fuzzy $\alpha$- open set $A$ of $X$.

(iv) fuzzy strongly $\alpha$-continuous [7]if inverse image each fuzzy semiopen set of $Y$ is fuzzy $\alpha$-open set in $X$

## 3 ap-F$\alpha$-IRRESOLUTE AND ap-F$\alpha$-CLOSED FUNCTIONS

**DEFINITION 3.1:** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be approximately fuzzy $\alpha$-irresolute( briefly, ap-F$\alpha$-irresolute), if $\alpha Cl(A) \leq f^{-1}(H)$ whenever $H$ is fuzzy $\alpha$-open subset of $(Y, \sigma)$,$A$ is fuzzy g$\alpha$-closed subset of $(X, \tau)$ and $A \leq f^{-1}(H)$.

**DEFINITION 3.2:** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be approximately fuzzy $\alpha$-closed (or ap-F$\alpha$-closed), if $f(B) \leq \alpha Int(A)$ whenever $A$ is fuzzy g$\alpha$-closed subset of $Y$, $B$ is fuzzy $\alpha$-closed subset of $X$ and $f(B) \leq A$.

**THEOREM 3.3:**(i) $f : (X, \tau) \to (Y, \sigma)$ is ap-F$\alpha$-irresolute if $f^{-1}(A)$ is fuzzy $\alpha$-closed in $(X, \tau)$ for every $A \in F\alpha O(Y)$

(ii) $f : (X, \tau) \to (Y, \sigma)$ is ap-F$\alpha$-closed if $f(B) \in F\alpha O(Y)$ for every fuzzy $\alpha$-closed subset $B$ of $(X, \tau)$.

**PROOF**:(i) Let $H \leq f^{-1}(A)$ where $A \in F\alpha O(Y)$ and $H$ is a fuzzy g$\alpha$-closed set of $(X, \tau)$.Therefore, $\alpha Cl(H) \leq \alpha Cl(f^{-1}(A)) = f^{-1}(A)$ .Thus, f is ap-F$\alpha$-irresolute .

(ii) Let $f(B) \leq A$ , where $B$ is fuzzy $\alpha$-closed set in $(X, \tau)$ and $A$ is fuzzy g$\alpha$-open in $(Y, \sigma)$ . Therefore, $\alpha Int(f(B)) \leq \alpha Int(A)$ .Then , $f(B) \leq \alpha Int(A)$.Thus, f is ap-F$\alpha$-closed.

Clearly, fuzzy $\alpha$-irresolute function is ap-F$\alpha$-irresolute. Also, fuzzy pre $\alpha$-closed is ap-F$\alpha$-closed .The converse implications do not hold as shown in the following example :

**EXAMPLE 3.4** : Let $X = \{a, b, c\}$ and \$Y = {x,y,z}$.Fuzzy \quad sets$ E, A, B *and* H *aredefinedas* :

$$E(a) = 0, E(b) = 0.2, E(c) = 0.2;$$

$$A(a) = 0, A(b) = 0.2, A(c) = 0.7;$$

$$H(x) = 0, H(y) = 0.2, H(z) = 0.7;$$

$$B(x) = 0, B(y) = 0.2, B(z) = 0.2.$$

Let $\tau = \{0, E, 1\}$ , $\sigma = \{0, H, 1\}$ , $\gamma = \{0, A, 1\}$ and $\nu = \{0, B, 1\}$. Then the function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$ , $f(b) = y$ and $f(c) = y$ is ap-$F\alpha$-irresolute but not fuzzy $\alpha$-irresolute and the function $h : (X, \gamma) \to (Y, \nu)$

defined by $h(a) = x$ , $h(b) = y$ and $h(c) = y$ is ap-$F\alpha$-closed but not fuzzy pre $\alpha$-closed.

**REMARK 3.5 :** Converse of the Theorem -3.3 do not hold. For,

**EXAMPLE 3.6 :** The function $f$ defined in Example 3.4 is ap-F$\alpha$-irresolute. Fuzzy set $K(x)0.8$ , $K(y) = 0.7$ and $K(z) = 0.2$ is fuzzy $\alpha$-open in $(Y, \sigma)$ but $f^{-1}(K)$ is not fuzzy $\alpha$-closed in $(X, \tau)$ and hence $f$ is not fuzzy $\alpha$-irresolute. The function $h$ defined in Example-3.4 is ap-F$\alpha$-closed and the fuzzy set $W(a) = 0.2, W(b) = 0.3, W(c) = 0.2$ is fuzzy $\alpha$-closed in $(X, \gamma)$ but $h(W)$ is not fuzzy $\alpha$-open in $(Y, \nu)$ and hence $h$ is not fuzzy pre $\alpha$-closed.

In the following theorem, we get under certain conditions the converse of the theorem 3.3 is true:

**THEOREM 3.7 :**Let $f : (X, \tau) \to (Y, \sigma)$ be a function:

(i) Let all fuzzy subsets of $(X, \tau)$ be fuzzy clopen, then $f$ is ap-F$\alpha$-irresolute if and only if $f^{-1}(H)$ is fuzzy $\alpha$-closed in $(X, \tau)$ for each $H \in F\alpha O(Y)$ .

(ii) Let all fuzzy subsets of $(Y, \sigma)$ be fuzzy clopen, then $f$ is ap-F$\alpha$-closed if and only if $f(B) \in F\alpha O(Y)$ for each fuzzy $\alpha$-closed subset $B$ of $(X, \tau)$ .

**PROOF :** (i) . The **sufficiency** is stated in Theorem- 3.2.

**Necessity** : Assume $f$ is ap-F$\alpha$-irresolute . Let $A$ be an arbitrary fuzzy subset of $(X, \tau)$ such that $A \leq Q$ where $Q \in F\alpha O(X)$. Then by hypothesis $\alpha Cl(A) \leq \alpha Cl(Q) = Q$ . Therefore , all fuzzy subsets of $(X, \tau)$ are fuzzy $g\alpha$-closed ( and hence all are fuzzy $g\alpha$-open ). So for any $H \in F\alpha O(Y), f^{-1}(H)$ is fuzzy $g\alpha$-closed in $X$ . Since $f$ is ap-F$\alpha$-irresolute $\alpha Cl(f^{-1}(H) \leq f^{-1}(H)$ . Therefore, $\alpha Cl(f^{-1}(H) = f^{-1}(H)$ , i.e. $f^{-1}(H)$ is fuzzy $\alpha$-closed in $(X, \tau)$ .

(ii) The **sufficiency** is clear by Theorem -3.2 .

**Necessity:** Assume $f$ is ap-F$\alpha$-closed . As in (i) , we obtain that all fuzzy subsets of $(Y, \sigma)$ are fuzzy $\alpha$-open. Therefore, for any fuzzy $\alpha$-closed subset $B$ of $(X, \tau)$ , $f(B)$ is fuzzy $g\alpha$-open in $Y$. Since $f$ is ap-F$\alpha$-closed ,$f(B) \leq \alpha Int(f(B))$ . Therefore, $f(B) = \alpha Int(f(B))$ i.e. $f(B)$ is fuzzy $\alpha$-open .

As an immediate consequence of Theorem -3.7,we have the following:

**COROLLARY 3.8 :** Let $f : (X, \tau) \to (Y, \sigma)$ be a function :

(i) Let all fuzzy subsets of $(X, \tau)$ be fuzzy clopen , then $f$ is ap-$F\alpha$-irresolute if and only if $f$ is fuzzy $\alpha$-irresolute .

(ii) Let all fuzzy subsets of $(Y, \sigma)$ be fuzzy clopen , then $f$ is ap-$F\alpha$-closed if and only if $f$ is fuzzy pre $\alpha$-closed.

# 4 CONTRA F$\alpha$-IRRESOLUTE MAPS

**DEFINITION 4.1:** A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be contra fuzzy $\alpha$-irresolute, if $f^{-1}(A)$ is fuzzy $\alpha$-closed in $(X,\tau)$ for each $A\in F\alpha O(Y)$.

**DEFINITION 4.2 :** A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be approximately fuzzy pre $\alpha$-closed , if $f(B)\in F\alpha O(Y)$ for each fuzzy $\alpha$-closed set $B$ of $(X,\tau)$.

**REMARK 4.3**:The concepts of contra fuzzy $\alpha$-irresoluteness and fuzzy $\alpha$-irresoluteness are independent notions.For,

**EXAMPLE 4.4 :**Let $X=\{a,b,c\}$ , $Y=\{x,y,z\}$ and fuzzy sets $A$ ,$E$,$H$ and $B$ are defined as :

$$A(a)=0, A(b)=0.2, A(c)=0.7;$$

$$E(a)=0, E(b)=0.6, E(c)=0.7;$$

$$H(x)=0, H(y)=0.8, H(z)=0.9;$$

$$B(x)=0, B(y)=0.2, B(z)=0.2.$$

Let $\tau=\{0,A,1\}$ , $\Gamma=\{0,E,1\}$ ,$\nu=\{0,H,1\}$ and $\sigma=\{0,B,1\}$. Then the function $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by $f(a0=x$ , $f(b)=y$ , $f(c)=z$ is contra fuzzy $\alpha$-irresolute but not fuzzy $\alpha$-irresolute. And the function $h:(X,\Gamma)\rightarrow(Y,\nu)$ defined by $h(a)=x$ , $h(b)=y$ and $h(c)=z$ is fuzzy $\alpha$-irresolute but not contra fuzzy $\alpha$-irresolute.

In the same manner one can prove that, contra fuzzy $\alpha$-closed functions and fuzzy pre-$\alpha$-closed functions are independent notions.

**DEFINITION 4.5:**A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is contra fuzzy $\alpha$-continuous if $f^{-1}(A)$ is fuzzy $\alpha$-closed in $(X,\tau)$ for each open set $A$ of $(Y,\sigma)$.

**REMARK 4.6 :** Every contra fuzzy $\alpha$-irresolute function is contra fuzzy $\alpha$-continuous, but not conversely. For,

**EXAMPLE 4.7:** Let $X=\{a,b\}$ and $Y=\{x,y\}$. Fuzzy sets $A$ and $B$ are defined as :

$$A(a)=0.3, A(b)=0.4;$$

$$B(x)=0.7, B(y)=0.6.$$

Let $\tau=\{0,A,1\}$ and $\sigma=\{0,B,1\}$. Then the function $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by $f(a)=x$ , $f(b)=y$ is contra fuzzy $\alpha$-continuous but not contra fuzzy $\alpha$-irresolute.

The following result can be easily verified . Its proof is straightforward.

**THEOREM 4.8:** Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following conditions are equivalent :
(i) f is contra fuzzy $\alpha$-irresolute,
(ii)The inverse image of each fuzzy $\alpha$-closed set in $Y$ is fuzzy $\alpha$-open in $X$.

**REMARK 4.9:** By Theorem 3.2 , we have that every contra fuzzy $\alpha$-irresolute function is ap-F$\alpha$-irresolute and every contra fuzzy $\alpha$-closed function is ap-F$\alpha$- closed, the converse implication do not hold (see Remark 3.5)

We define the following.

**DEFINITION 4.10:** A function $f : (X, \tau) \to (Y, \sigma)$ is called perfectly contra fuzzy $\alpha$-irresolute if, the inverse image of each fuzzy $\alpha$-open set in $Y$ is fuzzy $\alpha$-clopen in $X$ .

**LEMMA 4.11:**The following properties are equivalent for a fuzzy set $A$ of a fts $(X, \tau)$ :
(i) $A$ is fuzzy clopen ;
(ii) $A$ is fuzzy $\alpha$-closed and fuzzy $\alpha$-open ;
(iii) $A$ is fuzzy $\alpha$-closed and fuzzy preopen.

**PROOF:** We only show that the implication (iii) $\Rightarrow$ (ii) , the proof of the other being obvious. Let $A$ be fuzzy $\alpha$-closed and fuzzy preopen in $X$ . Then, we have $Int(Cl(A)) \leq Cl(Int(Cl(A))) \leq A \leq Int(Cl(A)) \leq Cl(Int(Cl(A)))$. Therefore , $A$ is fuzzy regular open and fuzzy regular closed and hence $A$ is fuzzy clopen in $X$.

**LEMMA 4.12:** $f : (X, \tau) \to (Y, \sigma)$ is perfectly contra fuzzy $\alpha$-irresolute function iff the inverse image of every fuzzy $\alpha$-open set in $Y$ is fuzzy clopen in $X$.

**PROOF :** It is suffices to apply Lemma -4.11 above.

Clearly, every perfectly contra fuzzy $\alpha$-irresolute function is contra fuzzy $\alpha$-irresolute and fuzzy $\alpha$-irresolute function . But the converses may not be true . For,

**EXAMPLE 4.13 :** The functions f and g defined in Example -4.4 are contra fuzzy $\alpha$-irresolute and fuzzy $\alpha$-irresolute respectively but not perfectly contra fuzzy $\alpha$-irresolute.

Clearly, we have the following implications :
(i)Perfectly contra fuzzy $\alpha$-irresoluteness $\Rightarrow ap - F\alpha$-irresolute .

(ii)Perfectly contra fuzzy $\alpha$-irresoluteness $\Rightarrow$ contra fuzzy $\alpha$ -irresoluteness as well as fuzzy $\alpha$ -irresolute.

(iii)Both contra fuzzy $\alpha$ -irresoluteness as well as fuzzy $\alpha$ -irresolute $\Rightarrow ap-F\alpha$-irresolute.

**Note** that the none of above implications is reversible.

The following Theorem is a decomposition of perfectly contra fuzzy $\alpha$-irresoluteness.

**THEOREM 4.14 :** For a function $f:(X,\tau)\rightarrow(Y,\sigma)$ the following are equivalent :

(i) f is perfectly contra $\alpha$-irresolute .

(ii) f is contra fuzzy $\alpha$-irresolute and fuzzy $\alpha$-irresolute.

**THEOREM 4.15 :** If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is ap-fuzzy $\alpha$-irresolute and fuzzy pre-$\alpha$-closed , then for every $Fg\alpha$ -closed set $A$ of $(X,\tau)$ , $f(A)$ is $Fg\alpha$-closed in $(Y,\sigma)$.

**PROOF :** Let $A$ be a $Fg\alpha$-closed subset of $(X,\tau)$ . Let $f(A)\leq B$ where $B\in F\alpha O(Y)$ . Then $A\leq f^{-1}(B)$ holds. Since f is $ap-F\alpha$-irresolute , $\alpha Cl(A)\leq f^{-1}(B)$ and hence $f(\alpha C(A))\leq B$ . Therefore , we have $\alpha Cl(f(A))\leq \alpha Cl(f(\alpha Cl(A)))=f(\alpha Cl(A))\leq B$. Hence, $f(A)$ is $Fg\alpha$-closed in $(Y,\sigma)$.

Now, we obtain that the composition of two contra fuzzy $\alpha$-irresolute functions need not be contra fuzzy $\alpha$-irresolute.

However, the following theorem holds. The proof is easy and hence omitted.

**THEOREM 4.16 :** Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\gamma)$ be two functions . Then , the following properties hold :

(i) If g is fuzzy $\alpha$-irresolute and f is contra fuzzy $alpha$-irresolute ,then $gof:(X,\tau)\rightarrow(Z,\gamma)$ is contra fuzzy $\alpha$-irresolute.

(ii) If g is contra fuzzy $\alpha$-irresolute and f is fuzzy $\alpha$-irresolute ,then $gof:(X,\tau)\rightarrow(Z,\gamma)$ is contra fuzzy $\alpha$-irresolute.

In an analogous way, we have the following.

**THEOREM 4.17 :** Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\gamma)$ be two functions . Then , the following properties hold :

(i) If f is fuzzy pre-$\alpha$-closed and g is ap-$F\alpha$-closed , then $gof:(X,\tau)\rightarrow(Z,\gamma)$ is ap-$F\alpha$-closed.

(ii) If f is ap-$F\alpha$-closed and g is fuzzy pre-$\alpha$-open and $g^{-1}$ preserves $Fg\alpha$-open sets ,then $gof:(X,\tau)\rightarrow(Z,\gamma)$ is ap-$F\alpha$-closed.

(iii) If f is ap-$F\alpha$-irresolute and g is fuzzy $\alpha$-irresolute, then $gof:(X,\tau)\rightarrow(Z,\gamma)$ is ap-$F\alpha$-irresolute.

**PROOF :**(i) Suppose $B$ is fuzzy $\alpha$-closed subset in $(X,\tau)$ and $A$ is $Fg\alpha$-open subset of $(Z,\gamma)$ for each $(gof)(B)\leq A$. Then $f(B)$ is $F\alpha$-closed in $(Y,\sigma)$

since f is fuzzy pre-$\alpha$-closed. As g is ap-$F\alpha$-closed , $g(f(B)) \leq \alpha Int(A)$ . This implies that $gof$ is ap-$F\alpha$-closed.

(ii) Suppose $B$ is fuzzy $\alpha$-closed subset of $(X, \tau)$ and $A$ is fuzzy $\alpha$-open in $(Z, \gamma)$ for which $(gof)(B) \leq A$ . Hence, $f(B) \leq g^{-1}(A)$. Then, $f(B) \leq \alpha Int(g^{-1}(A))$ because $g^{-1}(A)$ is $Fg\alpha$-open and f is ap-$F\alpha$-closed . Thus, $(gof)(B) = g(f(B)) \leq g(\alpha Int(g^{-1}(A))) \leq \alpha Int(gg^{-1}(A)) \leq \alpha Int(A)$. This shows that gof is ap-$F\alpha$-closed.

(iii) Suppose $H$ is $Fg\alpha$-closed subset $(X, \tau)$ and $E \in F\alpha O(Z)$ for which $H \leq (gof)^{-1}(E)$ . Then $g^{-1}(E) \in F\alpha O(E)$ . Then, $g^{-1}(E) \in F\alpha O(Y)$ because g is fuzzy $\alpha$ -irresolute. Since f is ap-$F\alpha$-irresolute , $\alpha Cl(A) \leq f^{-1](g^{-1}(E))=(gof)^{-1}(E)}$ . This shows that gof is ap-$F\alpha$-irresolute.

In the following we recall the $F\alpha - T_{1/2}$ spaces [5], to obtain the new characterizations of ap-$F\alpha$-irresoluteness and ap-$F\alpha$-closedness.

**DEFINITION 4.18 [5]:** A space $(X, \tau)$ is said to be $F\alpha T_{1/2}$-space if every $Fg\alpha$-closed set is fuzzy $\alpha$-closed in it.

**THEOREM 4.19 :** Let $(X, \tau)$ be a space. Then the following statement are equivalent:

(i) $(X, \tau)$ is $F\alpha T_{1/2}$-space,

(ii) f is ap-$F\alpha$-irresolute, for every space $(Y, \sigma)$ and every function $f : (X, \tau) \to (Y, \sigma)$ .

**PROOF :**(i)$\Rightarrow$ (ii) : Let $E$ be a $Fg\alpha$-closed subset of $(X, \tau)$ and $E \leq f^{-1}(H)$ where $H \in F\alpha O(Y)$. Since $(X, \tau)$ is $F\alpha T_{1/2}$-space, $E$ is fuzzy $\alpha$-closed ( i.e. $E = \alpha Cl(E)$) . Therefore, $\alpha Cl(E) \leq f^{-1}(H)$ and hence f is ap-$F\alpha$-irresolute function.

(ii) $\Rightarrow$ (i) : Let $B$ be a $Fg\alpha$-closed subset of $(X, \tau)$ and $Y$ be the set $X$ with topology $\tau = \{0, B, 1\}$ . Finally, let $f : (X, \tau) \to (Y, \sigma)$ be the identity function and by assumption f is an ap-$F\alpha$-irresolute function. Since $B$ is $Fg\alpha$-closed in $(X, \tau)$ and fuzzy $\alpha$-open in $(Y, \sigma)$ with $B \leq f^{-1}(B)$, it follows that $\alpha Cl(B) \leq f^{-1}(B) = B$ . Thus, $B$ is fuzzy $\alpha$-closed in $X$ and hence the space $X$ is $F\alpha T_{1/2}$-space.

**THEOREM 4.20 :** Let $(Y, \sigma)$ be a space. Then the following are equivalent:

(i) $(Y, \sigma)$ is $F\alpha T_{1/2}$-space ,

(ii) For every $(X, \tau)$ and every function $f : (X, \tau) \to (Y, \sigma)$ , f is ap -$F\alpha$-closed.

Proof is analogous to Theorem-4.19 above.

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ADDRESSES :

R.K.Saraf
Department of Mathematics
Govt.K.N.G.College, DAMOH(M.P.)-470661,
INDIA

G.B.Navalagi
Department of Mathematics
KLE Societys
G.H.College,HAVERI-581110,
Karnataka,INDIA
e-mail: gnavalagi@hotmail.com