# ON WEAK FORMS OF FUZZY SEMI-PREOPEN AND FUZZY SEMI-PRECLOSED FUNCTIONS

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Abstract. Our aim ,in the present paper, is to introduce two new classes of functions , called fuzzy weakly semi-preopen functions and fuzzy weakly semi-preclosed functions . Such functions are the natural generalizations of fuzzy semi-preopen and fuzzy semi-preclosed functions respectively. We investigate the fundamental properties of these new functions. Also, their characterizations and relations with other already existing functions.

## 1. INTRODUCTION

The notions of fuzzy semi-preopen sets and fuzzy semi-preclosed sets [15] play a very significant role in fuzzy topology. Also,in [8] these sets were studied as fuzzy $\beta$-open sets and fuzzy $\beta$-closed sets in fuzzy topological spaces respectively. There are many works regarding these type of sets containing their significances. This paper, is devoted to introduce and study two new classes of functions, called fuzzy weakly semi-preopen functions and fuzzy weakly semi-preclosed functions are weaker than fuzzy semi-preopen functions and fuzzy semi-preclosed functions respectively.

## 2. PRELIMINARIES

The concept of fuzzy sets and fuzzy set operations was first introduced by L.A.Zadeh in his classical paper [16 ]. Let $X$ be a non-empty set and $I$ be the unit interval [0,1] . A fuzzy set[16] in $X$ is a mapping from $X$ into $I$. The null set 0 is the mapping from $X$ into $I$ which assumes only the value 0 and the whole fuzzy set 1 is the mapping from $X$ into $I$ which takes value 1 only. A family $\tau$ of fuzzy sets of $X$ is called a fuzzy topology [5] on $X$ if 0 and 1 belong to $\tau$ and $\tau$ is closed with respect to arbitrary union and finite intersection. The members of $\tau$ are called fuzzy open sets and their complements are fuzzy closed sets.

Throughout this paper, $(X, \tau)$ and $(Y, \sigma)$ or simply $X$ and $Y$ respectively denote fuzzy topological spaces ( fts, in short) on which no separation axioms are assumed unless explicitly stated. If $\lambda$ is any fuzzy subset of a fts $X$, then Cl$\lambda$ ,Int$\lambda$ , 1-$\lambda$ denote the fuzzy closure , the fuzzy interior and the complement of fuzzy set $\lambda$ in fts $X$ respectively. Recall that a fuzzy set $\lambda$ is called fuzzy regular open ( resp. fuzzy regular closed) if $\lambda = IntCl\lambda$ (resp. $\lambda = ClInt\lambda$ ).

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**2.1 DEFINITION [9]:** A fuzzy point with singleton support $x \in X$ and the value p ( $0 < p \leq 1$ ) is denoted by $x_p$ .

**2.2 DEFINITION [9]:** We shall write $\lambda q \mu$ to mean that the fuzzy sets $\lambda$ and $\mu$ are quasi-coincident ( in short, q-coincident) if there exists a fuzzy point $x$ in $X$ such that $\lambda(x) + \mu(x) > 1$. It is known that $\lambda \quad \leq \quad \mu$ if and only if $\lambda$ and 1-$\mu$ are not q-coincident, denoted by $\lambda \bar{q}(1-\mu)$

**2.3 DEFINITION[9]:** A fuzzy point $x_p$ is said to be q-coincident with a fuzzy set $\lambda$ in $X$ denoted by $x_p q \lambda$ if $p + \lambda(x) > 1$ .

**2.4 DEFINITION[9]:** A fuzzy set $\lambda$ in a fts $X$ is called q-neighbourhood( in short q-nbd) of a fuzzy point $x_p$ if there exists a fuzzy open set $\mu$ in $X$ such that $x_p q \mu \quad \leq \quad \lambda$ .

**2.5 DEFINITION [10]:** A fuzzy point $x_p$ in $X$ is in the fuzzy $\theta$-closure of $\lambda$ if $\lambda \Lambda$ Cl$\mu \quad \neq 0$ for every fuzzy open set $\mu$ of $X$ containing $x_p$. The fuzzy $\theta$-closure of $\lambda$ is denoted by Cl$_\theta\lambda$ .Thus, a fuzzy subset $\lambda$ of $X$ is said to be fuzzy $\theta$-closed if $\lambda$= Cl$_\theta\lambda$. The complement of a fuzzy $\theta$-closed set is called fuzzy $\theta$-open set.

**2.6 DEFINITION :** Let $\lambda$ be a fuzzy subset of a fts $X$ , then $\lambda$ is called :
(i)fuzzy semiopen if $\lambda \leq ClInt\lambda$ [1 ] ;
(ii)fuzzy semiclosed if IntCl$\lambda \leq \lambda$[1 ];
(iii)fuzzy preopen if $\lambda \leq IntCl\lambda$ [2 ] ;
(iv)fuzzy prelclosed if $ClInt\lambda \leq \lambda$ [2];
(v) fuzzy semi-preopen [15]if $\lambda \leq ClIntCl\lambda$ ;
(vi) fuzzy semi-preclosed [15] if $IntClInt\lambda \leq \lambda$ ;
(vii) fuzzy $\alpha$-open if $\lambda \leq IntClInt\lambda$ [2 ] ; (viii) fuzzy $\alpha$-closed if $ClIntCl\lambda \leq \lambda$ [ 2] .

The family of all fuzzy semi-preopen (resp. fuzzy semi-preclosed ) subsets of a fts $X$ is denoted by FSPO($X$) (resp. FSPC($X$)). The intersection of all fuzzy semi-preclosed sets containing $\lambda$ is called fuzzy semi-preclosure of $\lambda$ and is denoted by $spCl\lambda$ [15 ] and the fuzzy semi-preinterior of $\lambda$ is denoted by $spInt\lambda$[15] is the union of all fuzzy semi-preopen sets which are contained in $\lambda$.

**2.7 DEFINITION:** An fts $X$ is called fuzzy extremally disconnected (FED) [7 ], if and only if the closure of each fuzzy open subset is fuzzy open in $X$.

**2.8 DEFINITION :** Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a fts $X$ to another fts $Y$, then f is called :
(i) fuzzy weakly open [13] if $f(\lambda) \leq Int(f(Cl\lambda))$ for each fuzzy open set $\lambda$ of $X$ ;
(ii) fuzzy wekly closed (it might be new one) if $Cl(f(Int\lambda)) \leq f(\lambda)$ ;
(iii) fuzzy preopen [3]( resp. fuzzy semi-preopen[14] , fuzzy $\alpha$-open [3]) if the image of each fuzzy open subset $\lambda$ of $X$ is fuzzy preopen( resp. fuzzy semi-preopen , fuzzy $\alpha$-open ) set in $Y$;
(iv) fuzzy preclosed [3]( resp. fuzzy semi-preclosed [14] , fuzzy $\alpha$-closed [3] ) if the image of each fuzzy closed subset of $X$ is fuzzy preclosed ( resp. fuzzy semi-preclosed , fuzzy $\alpha$-closed ) set of $Y$;

(v) fuzzy almost open[12]( in the sense of Nanda)(in short, f.a.o.N) if the image of each fuzzy regular open set of $X$ is fuzzy open in $Y$;

(vi) fuzzy contra-open(resp. fuzzy contra-closed ) if the image of each fuzzy open ( resp. fuzzy closed ) set of $X$ is fuzzy closed ( resp. fuzzy open) set of Y [4].

**2.9 DEFINITION [13]:** (i). Two non-empty fuzzy sets $\lambda$ and $\beta$ in an fts $X$ are said to be fuzzy semi-pre separated if $\lambda \bar{q}\, spCl(\beta)$ and $\beta \bar{q}\, spCl(\lambda)$.

(ii) An fts $X$ which cannot be expressed as the union of two fuzzy semi-pre separated sets is said to be fuzzy semi-preconnected space.

**2.9 DEFINITION** [ ]A fts $X$ is called fuzzy regular iff for each $x \in X$ , $p \in [(0,1]$, $\lambda \in \tau$ with $x_p \leq \lambda$, there exists $\mu \in \tau$ such that $x_p \leq \mu \leq Cl(\mu) \leq \lambda$

## 3. FUZZY WEAKLY SEMI-PREOPEN FUNCTIONS

In this section , we define the concept of fuzzy weak semi-preopenness. We obtain several fundamental properties of this new class of functions.

**3.1 DEFINITION:** A function $f : X \to Y$ is said to be fuzzy weakly semi-preopen if $f(\lambda) \leq spInt(f(Cl(\lambda)))$ for each fuzzy open set $\lambda$ of $X$.

Clearly, every fuzzy weakly open function is fuzzy weakly semi-preopen and every fuzzy semi-preopen function is also fuzzy weakly semi-preopen function. But the converses are not true in general.

**3.2 EXAMPLE :** Let $X = \{a, b, c\}$ , $Y = \{x, y, z\}$ .Fuzzy sets $A$ ,$B$ and $H$ are defined as :

$$A(a) = 0.5, A(b) = 0.3, A(c) = 0.2;$$
$$B(x) = 0.9, B(y) = 1, B(z) = 0.7;$$
$$H(x) = 0.2, H(y) = 0.9, H(z) = 0.3.$$

Let $\tau = \{0, 1, A\}$ and $\sigma = \{0, 1, B, H\}$ then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = z, \quad f(b) = x \quad and \quad f(c) = y$ is fuzzy weakly semi-preopen but not fuzzy weakly open.

**3.3 EXAMPLE :**Let $X = \{a, b, c\}$ .Fuzzy sets $A$,$B$,$H$ and $E$ are defined as :

$$A(a) = 0.4 \quad , A(b) = 0.7 \quad , A(c) = 0.2;$$
$$B(a) = 0.3 \quad , B(b) = 0.1 \quad , B(c) = 0.6;$$
$$H(a) = 0.5 \quad , H(b) = 0.8 \quad , H(c) = 0.3;$$
$$E(a) = 0.4 \quad , E(b) = 0.2 \quad , E(c) = 0.7.$$

Let $\tau = \{0, 1, A, B, A \cup B, A \cap B\}$ and $\sigma = \{0, 1, H, E, H \cup E, H \cap E\}$. Consider the identity mapping $f : (X, \tau) \to (X, \sigma)$ . Then f is fuzzy weakly semi-preopen but not fuzzy semi-preopen.

We, characterize fuzzy weak semi-preopenness in the following.

**3.4 THEOREM :** For a function $f : X \to Y$ , the following conditions are equivalent :

(i) f is fuzzy weakly semi-preopen ;

(ii) $f(Int_\theta(\lambda)) \leq spInt(f(\lambda))$ for each fuzzy subset $\lambda$ of $X$;

(iii) $Int_\theta(f^{-1}(\mu)) \leq f^{-1}(spInt(\mu))$ for each fuzzy subset $\mu$ of $Y$ ;

(iv) $f^{-1}(spCl(\gamma)) \leq Cl_\theta(f^{-1}(\gamma))$ for each fuzzy subset $\gamma$ of $Y$ ;

(v) For each fuzzy point $x_p \in X$ and each fuzzy open set $\lambda$ of $X$ containing $x_p$, there exists a fuzzy semi-preopen set $\gamma$ in $Y$ containing $f(x_p)$ such that $\gamma \leq f(Cl\lambda)$.

Proof is easy and hence omitted.

**3.5 THEOREM :** For a function $f : X \to Y$ , the following conditions are equivalent :

(i) f is fuzzy weakly semi-preopen ;

(ii) $f(Int(\mu)) \leq spInt(f(\mu))$ for each fuzzy closed subset $\mu$ of $X$;

(iii) $f(Int(Cl(\lambda))) \leq spInt(f(Cl(\lambda))$ for each fuzzy open subset of $X$;

(iv) $f(\lambda) \leq spInt(f(Cl(\lambda)))$ for each fuzzy preopen subset $\lambda$ of $X$ ;

(v) $f(\lambda) \leq spInt(f(Cl(\lambda)))$ for each fuzzy $\alpha$-open subset of $X$ .

**PROOF :**(i)⇒(ii): Let $\mu$ be fuzzy closed subset of $X$ then $Int(\mu) = Int(Cl(\mu))$, which is fuzzy regular open and hence fuzzy open set in $X$. By (i), $f(Int(\mu)) = f(Int(Cl(\mu)) \leq spInt(f(Int(Cl(\mu))) = spInt(f(Cl(\mu))$. Therefore, $f(Int(\mu)) = f(Int(Cl(\mu)) \leq spInt(f(Int(Cl(\mu))) = spInt(f(Cl(\mu))$ for each fuzzy closed set $\mu$ in $X$.

(ii)⇒(iii): Clear.

(iii) ⇒(iv): Let $\lambda$ be a fuzzy preopen subset of $X$, then $\lambda \leq Int(Cl(\lambda))$.Hence, by (iii), we have $f(\lambda) \leq f(Int(Cl(\lambda)) \leq spInt(f(Cl(\lambda)))$.

(iv)⇒(v) and (v)⇒(i) are clear.

**3.6 THEOREM :** Let function $f : X \to Y$ be a bijective function.Then the following statements are equivalent :

(i) f is fuzzy weakly semi-preopen;

(ii) $spCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy open subset $\lambda$ of $X$;

(iii) $spCl(f(Int(\gamma))) \leq f(\gamma)$ for each fuzzy closed subset $\gamma$ of $X$ .

**PROOF :**(i)⇒(iii): Let $\gamma$ be any fuzzy closed set in $X$ .Then we have $f(1-\gamma) = 1-f(\gamma) \leq spInt(f(Cl(1-\gamma)))$ and so $1-f(\gamma) \leq 1-spCl(f(Int(\gamma))).Hence, spCl(f(Int(\gamma))) \leq f(\gamma)$.

(iii)⇒ (ii): Let $\lambda$ be a fuzzy open set of $X$. Since $Cl(\lambda)$ is a fuzzy closed set and $\lambda \leq Int(Cl(\lambda)$ and by (iii), we obtain that $spC(f(\lambda)) \leq spCl(f(Int(Cl(\lambda))) \leq f(Cl(\lambda))$.

(ii)⇒ (iii): Similar to (iii)⇒ (ii).

(iii) ⇒ (i) : Clear.

Now, we recall a definition of fuzzy strongly continuous function . This function when combined with fuzzy weak semi-preopenness imply fuzzy semi-preopenness.

**3.7 DEFINITION :** A function $f : X \to Y$ is said to be fuzzy strongly continuous [11 ] , if for each fuzzy subset $\lambda$ of $X$ , $f(Cl(\lambda)) \leq f(\lambda)$ .

Now, we state the following.

**3.8 THEOREM:** If function $f : X \to Y$ is fuzzy weakly semi-preopen and fuzzy strongly continuous , then f is fuzzy semi-preopen.

**PROOF :** Let $\lambda$ be any fuzzy open set of $X$.Since f is fuzzy strongly continuous ,$f(Cl(\lambda) \leq f(\lambda)$ for each fuzzy subset $\lambda$ of $X$.Again, f is fuzzy weakly semi-preopen, $f(\lambda) \leq spInt(f(Cl(\lambda))) \leq f(\lambda)$.This shows that $f(\lambda)$ is fuzzy semi-preopen set of $Y$. Thus, f is fuzzy semi-preopen function.

The following example shows that neither of this fuzzy strong continuity yield a decomposition of fuzzy semi-preopenness.

**3.9 EXAMPLE :** Let $X = [0,1]$ and $a$ be a fixed element of $X$. Let $\tau = \{0, 1, A : \quad 1/4 \quad \leq \quad A(a) \quad \leq \quad 3/4 \quad and \quad A(x) = 0, \quad otherwise\}$ and $\sigma = \{0, 1, B : \quad B(a) = 3/4 \quad and \quad B(x) = 0, \quad otherwise\}$ . Consider the identity mapping $f : (X, \tau) \to (Y, \sigma)$. Then f is fuzzy semi-preopen but not fuzzy strongly continuous.

We, define the following .

**3.10 DEFINITION :** A function $f : X \to Y$ is said to be fuzzy relatively weakly open provided that $f(\lambda)$ is fuzzy open in $f(Cl(\lambda))$ for each fuzzy open subset $\lambda$ of $X$.

**3.11 THEOREM:** A function $f : X \to Y$ is fuzzy semi-preopen if f is fuzzy weakly semi-preopen and fuzzy relatively weakly open .
**PROOF :** Obvious .

We, define the following .

**3.12 DEFINITION:** A function $f : X \to Y$ is said to be fuzzy contra-semi-preclosed if $f(\delta)$ is a fuzzy semi-preopen set of $Y$ , for each fuzzy closed set $\delta$ of $X$.

**3.13 THEOREM:** If a function $f : X \to Y$ is fuzzy contra-semi-preclosed , then it is a fuzzy weakly semi-preopen .

**PROOF :** Obvious.

The converse of the above Theorem - 3.13 does not hold. Example 3.2 serve the purpose.

We, observe that when the range fts is fuzzy regular, then both the fuzzy weak semi-preopenness and fuzzy semi-preopenness are coincide.

It is obvious that every f.a.o.N is fuzzy weakly semi-preopen. But converse is not true. Example 3.3 serves the purpose.

We define one additional near fuzzy semi-preopen condition. This condition when combined with fuzzy weak semi-preopenness imply fuzzy semi-preopenness.

**3.14 DEFINITION :** A function $f : (X, \tau) \to (Y, \sigma)$ is said to satisfy the fuzzy weakly semi-preopen interiority condition if $spInt(f(Cl(\lambda)))) \leq f(\lambda)$ for each fuzzy open subset $\lambda$ of $X$.

Clearly, every fuzzy strongly continuous function satisfies the fuzzy weakly semi-preopen interior condition but the converse does not hold as the following exmple shows :

**3.15 EXAMPLE :** Let $X = [0,1]$ . Consider $\tau = \{0, 1, A : 1/3 \leq A(x) \leq 2/3 for some fixed element x \in X and A(x) = 0, otherwise\}$

Then the identity function $f:(X,\tau)\rightarrow(X,\tau)$ satisfies fwsp-open interiority condition which is not fuzzy strongly continuous.

Easy proof of the following theorem is omitted.

**3.16 THEOREM :** Every function that satisfies the fuzzy weakly semi-preopen interiority condition into a fuzzy discrete topological space is fuzzy strongly continuous.

**3.17 THEOREM :** If $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy weakly semi-preopen and satisfies the fuzzy weakly semi-preopen interiority condition then f is fuzzy semi-preopen.

**PROOF :** Let $\lambda$ be a fuzzy open subset of $X$.Since f is fuzzy weakly semi-preopen, $f(\lambda)\leq spInt(f(Cl(\lambda)))$.However , f is satisfies fuzzy weakly semi-preopen interiority condition, $f(\lambda)=spInt(f(Cl(\lambda)))$ and therefore $f(\lambda)$ be fuzzy semi-preopen and hence f is fuzzy semi-preopen function.

**3.18 COROLLARY :** If $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy preopen and fuzzy contra-open , then f is fuzzy weakly semi-preopen.

**PROOF :** Obvious.
Next, we prove the following.

**3.19 THEOREM :** If $f:X\rightarrow Y$ is an injective fuzzy weakly semi-preopen function of fts $X$ onto a fuzzy semi-preconnected space [14] $Y$, then $X$ is fuzzy connected.
**PROOF:** Obvious.

We, recall the following.

**3.20 DEFINITION [4] :** A space $X$ is said to be fuzzy hyperconnected if every non-null fuzzy open subset of $X$ is fuzzy dense in $X$.

**3.21. THEOREM :** If $X$ is a fuzzy hyperconnected space, then a function $f:X\rightarrow Y$ is fuzzy weakly semi-preopen iff $f(X)$ is fuzzy semi-preopen set in $Y$.

**PROOF :** The sufficiency is clear. For the necessary observe that for any fuzzy open subset $\lambda$ of $X$, $f(\lambda)\quad\leq f(X)=spInt(f(X))=spInt(f(Cl(\lambda)))$. This shows that f is fuzzy weakly semi-preopen .

## 4. FUZZY WEAKLY SEMI-PRECLOSED FUNCTIONS

In this section, we define the generalized form of fuzzy semi-preclosed function.

**4.1 DEFINITION :** A function $f : X \to Y$ is said to be fuzzy weakly semi-preclosed if $spCl(f(Int(\lambda))) \leq f(\lambda)$ for each fuzzy closed subset $\lambda$ of $X$.

Clearly, every fuzzy weakly closed function is fuzzy weakly semi-preclosed but converse is not true in general. Example - 3.2 serves the purpose.

The implications between fuzzy weakly semi-preclosed ( resp. fuzzy weakly semi-preopen) functions and other type of fuzzy closed (resp. fuzzy open) functions are given by the following diagram which the enlargement of the diagram of [14].

$$\begin{tikzcd}
 & & \begin{array}{c}\text{fuzzy preclosed}\\ \text{( fuzzy preopen )}\end{array} \arrow[dr] & & \\
\begin{array}{c}\text{fuzzy closed}\\ \text{(fuzzy open )}\end{array} \arrow[r] & \begin{array}{c}\text{fuzzy } \alpha\text{-closed}\\ \text{(fuzzy } \alpha\text{-open )}\end{array} \arrow[ur] \arrow[dr] & & \begin{array}{c}\text{fuzzy semipre-closed}\\ \text{(fuzzy semipre-open )}\end{array} \arrow[r] & \begin{array}{c}\text{fwsp-closed}\\ \text{(fwsp-open )}\end{array} \\
 & & \begin{array}{c}\text{fuzzy semiclosed}\\ \text{( fuzzy semiopen )}\end{array} \arrow[ur] & &
\end{tikzcd}$$

Where fwsp-closed = fuzzy weakly semi-preclosed and fwsp-open = fuzzy weakly semi-preopen functions. The converses of these statements are not necessarily true.

**4.3 THEOREM :** For a function $f : X \to Y$ , the following statements are equivalent.

(i) f is fuzzy weakly semi-preclosed ;

(ii) $spCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy open subset $\lambda$ of $X$ ;

(iii) $spCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy regular open subset $\lambda$ of $X$;

(iv) For each fuzzy subset $\mu$ in $Y$ and each fuzzy open subset $\eta$ in $X$ with $f^{-1}(\mu) \leq \eta$, there exists a fuzzy semipre-open set $\delta$ in $Y$ with $\mu \leq \delta$ and $f^{-1}(\mu) \leq Cl(\eta)$ ;

(v) For each fuzzy point $y_p$ in $Y$ and each fuzzy open subset $\eta$ in $X$ with $f^{-1}(y_p) \leq \eta$, there exists a fuzzy semi-preopen set $\delta$ in $Y$ with $y_p$ in $\delta$ and $f^{-1}(\delta) \leq Cl(\eta)$ ;

(vi) $spCl(f(Int(Cl(\lambda))) \leq f(Cl(\lambda))$ for each fuzzy subset $\lambda$ in $X$;

(vii) $spCl(f(Int(Cl_\theta(\lambda))) \leq f(Cl_\theta(\lambda))$ for each fuzzy subset $\lambda$ in $X$;

(viii) $spCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy semi-preopen subset $\lambda$ of $X$ .

**PROOF :** Obvious.

**4.4 THEOREM :** For a function $f : X \to Y$ , the following conditions are equivalent :

(i) f is fuzzy weakly semipre-closed ;

(ii) $spCl(f(Int(\delta)) \leq f(\delta)$ for each fuzzy semipre-closed subset $\delta$ of $X$;
(iii) $spCl(f(Int(\delta)) \leq f(\delta)$ for each fuzzy $\alpha$-closed subset $\delta$ of $X$;

**PROOF :**Obvious.
**4.5 REMARK :** In view of the above theorem-4.4 , if $f : X \to Y$ is a bijective function , then f is fuzzy weakly semi-preopen iff f is fuzzy weakly semi-preclosed.

Next, we investigate conditions under which fuzzy weakly semi-preclosed function is fuzzy semi-preclosed.

**4.6 THEOREM :** If $f : X \to Y$ is fuzzy weakly semi-preclosed and if for each fuzzy closed subset $\delta$ of $X$ and each fiber $f^{-1}(y) \leq Y - \delta, \quad y \in Y$ , there exists a fuzzy open subset $\lambda$ of $X$ such that $f^{-1}(y) \leq \lambda \leq Cl(\lambda) \leq X - \delta$.Then f is fuzzy semi-preclosed function.

Next , we define the following.

**4.7 DEFINITION :** A function $f : X \to Y$ is called fuzzy contra semi-preopen if $f()$ is a fuzzy semi-preclosed set of $Y$ , for each fuzzy open set  of $X$.

**4.8 THEOREM :** If $f : X \to Y$ is fuzzy contra semi-preopen, then f is fuzzy weakly semi-preclosed.
Easy proof is omitted.

**4.9 THEOREM :** If $f : X \to Y$ is fuzzy weakly semi-preclosed injective, then for each fuzzy subset $\delta$ of Y and each fuzzy open subset $\lambda$ in $X$ with $f^{-1}(\delta) \leq \lambda$ , there exists a fuzzy semi-preclosed subset $\mu$ of $Y$ with $\delta \leq \mu$ and $f^{-1}(\mu) \leq Cl(\lambda)$ .

We , recall the following.

**4.10 DEFINITION :** An fts $X$ is said to be fuzzy semipre-$T_2$ , if for each pair of distinct fuzzy points $X$ and y ( with different values) , there exist disjoint fuzzy semi-preopen sets $\lambda$ and $\mu$ containing $X$ and y resp.

**4.11 THEOREM :** If $f : X \to Y$ is fuzzy weakly semi-preclosed surjection and all pairs of disjoint fibers are fuzzy strongly separated , then Y is fuzzy semipre-$T_2$ space.

We, recall the following.

**4.12 DEFINITION :** An fts $X$ is called fuzzy almost compact [6] ( resp. fuzzy semi-preclosed ) if for each fuzzy open ( resp. fuzzy semi-preclosed ) cover of $X$ has a finite subfamily whose closures cover $X$.

**4.13 LEMMA [8] :** A function $f : X \to Y$ is fuzzy open if and only if for each fuzzy subset $\lambda$ of $Y$, $f^{-1}(Cl(\lambda)) \leq Cl(f^{-1}(\lambda))$ .

Next , we give the following.

**4.14 THEOREM :** Let $X$ be an fuzzy extremally disconnected space and FSPO($X$) is closed under finite intersections. Let $f : X \to Y$ is fuzzy open, fuzzy weakly semi-preclosed injective function such that $f^{-1}(y)$ is fuzzy almost compact relative to fts $X$ for each $y \in Y$. If $\lambda$ is fuzzy semi-preclosed relative to $Y$ then $f^{-1}(\lambda)$ is fuzzy almost compact .

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