Countably Weak *SP*-compactness*

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Abstract: *In this paper, countably weak SP-compact L-subsets is introduced in L-topological spaces. It is hereditary for semi-preclosed subsets; it is finitely additive; and it is preserved under SP-irresolute mapping. Every set with finite support is countably weak SP-compact. Also the countably weak SP-compactness is described with cover form and finite intersection property.*

Key words: *L-topological spaces;Remote-neighborhood; Semi-preclosed set; Countably weak SP-compact sets*

1.Introduction and preliminaries

In general topological spaces, the concepts of semi-preopen sets and semi-preclosed sets were introduced by Andrijevic [1]. In [6], Thakur and Singh extended these concepts to L-topological spaces, where $L=[0,1]$. With these semi-preclosed sets, we have introduced the concept semi-preclosed remoted-neighborhood and studied the *SP*-compactness and weak *SP*-compactness in the general L-topological spaces in [2,3], respectively, where L is fuzzy lattice. In this paper, we introduce the concept of countably weak SP-compactness in the general L-topological space. It is defined for any L-subset, and it preserves some good properties of compactness in general topological spaces.

In this paper, L always denotes a fuzzy lattice. L^X denotes the set of all L-subsets on nonempty crisp set X. Put $pr(L)=\{e \in L$: e is a prime element[6] of L,

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and $e<1\}$ and $\varepsilon_r(A)=\{x \in X: A(x) \geqslant r\}$, And L-topological space is denoted by L-ts. $SPO(L^X)$ and $SPC(L^X)$ will always denote the family of semi-preopen sets and family of semi-preclosed sets[2,6] of an L-ts (L^X, δ), respectively.

Definition 1.1 (Bai [2,3]). Let (L^X, δ) be an L-ts and $x_\lambda \in M^*(L^X)$. $A \in SPC(L^X)$ is called a semi-preclosed remote-neighborhood, or briefly, SPC-RN of x_λ, if $x_\lambda \notin A$. The set of all SPC-RNs of x_λ is denoted by $\pi(x_\lambda)$. $\Phi \subset SPC(L^X)$ is called an α-SPC-remote neighborhood family of $A \in L^X$ (briefly α-SPC-RF of A) if for each x_α in A, there is $P \in \Phi$ such that $P \in \pi(x_\alpha)$. $\Phi \subset L^X$ is called a family with the α-finite intersection property in $A \in L^X (\alpha \in L)$, if for each finite subfamily Ψ of Φ there is an $x \in \varepsilon_\alpha(A)$ such that $(\wedge \Psi)(x) \geqslant \alpha$. $\mu \subset SPO(L^X)$ is called an r-SP-cover of $A \in L^X$ if for each $x \in \varepsilon_{r'}(A)$, there is $U \in \mu$ such that $U(x) \not\leqslant r$, $r \in pr(L)$. $A \in L^X$ is called weakly SP-compact, if for each α-net S in $A (\alpha \in M(L))$ and each $r \in \beta^*(\alpha)$, S has an SP-cluster point in A with height r. Specifically, when $A=1_X$ is weakly SP-compact, we call (L^X, δ) a weakly SP-compact space.

Theorem 1.2 (Bai [3]). *Let (L^X, δ) be an L-ts and $A \in L^X$. A is weakly SP-compact iff for each $r \in \beta^*(\alpha)$ and each r-SPC-RF Φ of A has a finite subfamily Ψ of Φ such that Ψ is an α-SPC-RF of $A (\alpha \in M(L))$.*

2. Countably weak SP-compactness

Definition 2.1. Let (L^X, δ) be an L-ts and $A \in L^X$. A is called countably weak SP-compact, if for each $r \in \beta^*(\alpha)$ and *each countable r-SPC-RF Φ of A has a finite subfamily Ψ of Φ such that Ψ is an α-SPC-RF of $A(\alpha \in M(L))$.* Specifically, when $A=1_X$ is countably weak SP-compact, we call (L^X, δ) a countably weak SP-compact space.

Lemma 2.2. *Let (L^X, δ) be an L-ts, $A \in L^X$ and $\mu \subset SPO(L^X)$. Then μ is an r-SP-cover of A iff $\mu' \subset SPC(L^X)$ is an r'-SPC-RF of A, where $r \in pr(L)$.*

Theorem 2.3. *Let (L^X, δ) be an L-ts and $A \in L^X$. Then A is countably weak SP-compact iff for each $r' \in \beta^*(\alpha)$ $(\alpha \in M(L))$ and every countable r-SP-cover μ of A, there is a finite subfamily ν of μ such that ν is an α'-SP-cover of A.*

Proof. This follows directly from Definition 2.1 and Lemma 2.2.

Lemma 2.4. *Let (L^X , δ) be an L-ts, $A \in L^X$, $\Phi \subset SPC(L^X)$ and $\alpha \in M(L)$. Then Φ has the α-finite intersection property in A iff Ψ' is not an α'-SP-cover of A for each finite subfamily Ψ of Φ.*

Proof. It can be proved by Definition 1.1.

Theorem 2.5. *Let (L^X, δ) be an L-ts and $A \in L^X$. Then A is countably weak SP-compact iff for each $\alpha \in M(L)$ and every countable family $\Phi \subset SPC(L^X)$ which has the α-finite intersection property in A, there is $x \in \varepsilon_\alpha(A)$ and some $r' \in \beta^*(\alpha)$ such that $\wedge \Phi(x) \geqslant r'$.*

Proof. This follows directly from Theorem 2.3 and Lemma 2.4.

Theorem 2.6. *Let A be a countably weak SP-compact set in L-ts (L^X, δ). Then for each $B \in SPC(L^X)$, $A \wedge B$ is countably weak SP-compact.*

Proof. Let Φ be a countable r-SPC-RF of $A \wedge B$. Put $\Phi_1 = \Phi \cup \{B\}$, then Φ_1 is a countable r-SPC-RF of A. In fact, for each $x_r \in A$, if $x_r \in B$ then $x_r \in A \wedge B$. Hence, there is $P \in \Phi \subset \Phi_1$ such that $P \in \pi(x_r)$. If $x_r \notin B$, then $B \in \Phi_1$ and $B \in \pi(x_r)$. Thus, Φ_1 is indeed a countable r-PSC-RF of A. Since A is a countably weak SP-compact set, for each $r \in \beta^*(\alpha)$ and countable r-SPC-RF Φ_1 of A, there is finite subfamily Ψ_1 of Φ_1 such that Ψ_1 is an α-SPC-RF of A. Let $\Psi = \Psi_1 - \{B\}$, then Ψ is a finite subfamily of Φ, and Ψ is an α-SPC-RF of $A \wedge B$. In fact, $x_\alpha \in A \wedge B$, then $x_\alpha \in A$, from the definition of Ψ_1, there exists $P \in \Psi_1$ with $P \in \pi(x_\alpha)$. However, $x_\alpha \in B$ so $P \neq B$, and thus $P \in \Psi_1 - \{B\} = \Psi$. Hence, $A \wedge B$ is countably weak SP-compact.

Corollary 2.7. *Countably weak SP-compactness is hereditary for semi-preclosed subsets.*

Theorem 2.8. *Let A and B be two Countably weak SP-compact sets in an L-ts (L^X, δ). Then $A \vee B$ is also Countably weak SP-compact.*

Proof. Suppose for each $r \in \beta^*(\alpha)$ ($\alpha \in M(L)$), Φ is a countable r-SPC-RF of $A \vee B$. Then Φ is a countable r-SPC-RF of both A and B. Since A and B are both countably weak SP-compact sets, there exist finite subfamily Ψ_1 and Ψ_2 of Φ such that Ψ_1 and Ψ_2 are α-SPC-RF of A and B, respectively. Put $\Psi = \Psi_1 \cup \Psi_2$. Clearly, Ψ is a finite subfamily of Φ, and also an α-SPC-RF of $A \vee B$. Thus,

$A \vee B$ is countably weak SP-compact.

Theorem 2.9. *Let (L^X, δ) be an L-ts and $A \in L^X$. If A with finite support, then A is countably weak SP-compact.*

Proof. It is clearly.

Definition 2.10. (Bai [2]). Let $f: (L^X, \delta) \rightarrow (L^Y, \tau)$ an L-fuzzy mapping. f is called an SP-irresolute mapping if $f^{-1}(B) \in SPO(L^X)$ for each $B \in SPO(L^Y)$.

Theorem 2.11. *Let (L^X, δ) be a countably weak SP-compact space and $f: (L^X, \delta) \rightarrow (L^Y, \tau)$ an onto SP-irresolute mapping. Then (L^Y, τ) is a countably weak SP-compact space.*

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