Fuzzy IPSO-irresoluteness *

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Abstract: *The aim of this paper is to introduce fuzzy IPSO-irresolute function in fuzzy topological spaces, and study its properties and its relationship with other functions.*
Key words: *Fuzzy topology; Pre-semiopen set; IPSO-irresoluteness*

1.Introduction and Preliminaries

The theory of fuzzy continuity not only is an significantly basic theory of fuzzy topology and fuzzy analysis but also has wide applications in some other aspects. Fuzzy continuity and many of its weaker forms and stronger forms have been richly studied. In [3,2], the concepts of fuzzy pre-semi-irresolute function and fuzzy strongly pre-semicontinuous function were introduced, respectively. In this paper, we introduce a new class of function, called fuzzy IPSO-irresolute function. It is stronger forms of fuzzy pre-semi-irresolute function and fuzzy strongly pre-semicontinuous function.

Throughout the paper by (X, δ) or simply by X we mean a fuzzy topological space in the Chang's[4] sense, briefly fts. A^o, A^-, A_o, A_- and A' denote the interior, closure, semiinterior, semiclosure and complement of fuzzy set A, respectively. A fuzzy set A in X is called pre-semiopen iff $A \leq (A^-)_o$, and pre-semiclosed iff $A \geq (A^o)_-$[1]. $PSO(X)$ and $PSC(X)$ denote the family of pre-semiopen sets and family of pre-semiclosed sets of an fts X, respectively. The $A_\Delta = \bigcup\{B : B \in PSO(X), B \leq A\}$ and $A_\sim = \bigcap\{B : B \in PSC(X), A \leq B\}$ are called the pre-semiinterior and pre-semiclosure of fuzzy set A, respectively.

Since the union (intersection) of any two fuzzy pre-semiclosed (pre-semiopen) sets need not be a pre-semiclosed (pre-semiopen) set[1], $A \in PSC(X)$ and $B \in PSC(X)$ do not necessarily lead to $A \cup B \in PSC(X)$. Let

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$UPSC(X) = \{A \in PSC(X) : \text{for each } B \in PSC(X), A \cup B \in PSC(X)\}$,
$IPSO(X) = \{A \in PSO(X) : \text{for each } B \in PSO(X), A \cap B \in PSO(X)\}$.
Clearly, $\delta \subset IPSO(X) \subset PSO(X)$.

Definition 1.1[3]. A function $f : (X, \delta) \to (Y, \tau)$ is said to be fuzzy Pre-semi-irresolute if $f^{-1}(B) \in PSO(X)$ for each $B \in PSO(Y)$.

Definition 1.2[2]. A function $f : (X, \delta) \to (Y, \tau)$ is said to be fuzzy strongly pre-semicontinuous if $f^{-1}(B) \in IPSO(X)$ for each $B \in \tau$.

2. Fuzzy IPSO-irresolute Functions

Definition 2.1. A function $f : (X, \delta) \to (Y, \tau)$ from an fts (X, δ) to another fts (Y, τ) is said to be fuzzy IPSO-irresolute if $f^{-1}(B) \in IPSO(X)$ for each $B \in PSO(Y)$.

Theorem 2.2. A function $f : (X, \delta) \to (Y, \tau)$ is fuzzy IPSO-irresolute iff $f^{-1}(B) \in UPSC(X)$ for each $B \in PSC(Y)$.

Theorem 2.3. Let $f : (X, \delta) \to (Y, \tau)$ be a fuzzy IPSO-irresolute function. Then:

(1) $f(A_\sim) \le (f(A))_\sim$ for each fuzzy set A in X.

(2) $(f^{-1}(B))_\sim \le f^{-1}(B_\sim)$ for each fuzzy set B in Y.

(3) $f^{-1}(B_\Delta) \le (f^{-1}(B))_\Delta$ for each fuzzy set B in Y.

(4) For each fuzzy point x_α in X and each $B \in PSO(Y)$ with $f(x_\alpha) \in B$, there exists an $A \in IPSO(X)$ such that $x_\alpha \in A$ and $f(A) \le B$.

Proof. (1): Let A be a fuzzy set in X. Then $(f(A))_\sim \in PSC(Y)$. Since f is fuzzy IPSO-irresolute, $f^{-1}((f(A))_\sim) \in UPSC(X)$, and
$A_\sim \le (f^{-1}f(A))_\sim \le (f^{-1}((f(A))_\sim))_\sim = f^{-1}((f(A))_\sim)$.
Thus, $f(A_\sim) \le ff^{-1}((f(A))_\sim) \le (f(A))_\sim$.

(2): Let B be a fuzzy set in Y. By (1), $f((f^{-1}(B))_\sim) \le (ff^{-1}(B))_\sim \le B_\sim$. Thus, $(f^{-1}(B))_\sim \le f^{-1}f((f^{-1}(B))_\sim) \le f^{-1}(B_\sim)$.

(3): Let B be a fuzzy set in Y. By (2), $f^{-1}((B')_\sim) \ge (f^{-1}(B'))_\sim = ((f^{-1}(B))')_\sim$. Hence, $f^{-1}(B_\Delta) = f^{-1}(((B')_\sim)') = (f^{-1}((B')_\sim)' \le (((f^{-1}(B))')_\sim)' = (f^{-1}(B))_\Delta$.

(4): Let f be fuzzy IPSO-irresolute, x_α be a fuzzy point in X and $B \in PSO(Y)$ such that $f(x_\alpha) \in B$. Then $x_\alpha \in f^{-1}(B)$. Let $A = f^{-1}(B)$, then $A \in IPSO(X)$.

We have $f(A) = ff^{-1}(B) \leq B$.

Theorem 2.4. Let $f : (X, \delta) \to (Y, \tau)$ be a fuzzy IPSO-irresolute function, and one-to-one and onto. Then $(f(A))_\Delta \leq f(A_\Delta)$ for each fuzzy set A in X.

Proof. Let f be fuzzy IPSO-irresolute and A be any fuzzy set in X. Then $f^{-1}((f(A))_\Delta) \in IPSO(X)$. By Theorem 2.3 and the fact that f is one-to-one, we have $f^{-1}((f(A))_\Delta) \leq (f^{-1}f(A))_\Delta) = A_\Delta$. Again, since f is onto, we have $(f(A))_\Delta = ff^{-1}((f(A))_\Delta) \leq f(A_\Delta)$.

Proposition 2.5. Let $f : X \to Y$ and $g : Y \to Z$ be functions. The following statements are valid:

(1) If f is fuzzy IPSO-irresolute and g is pre-semi-irresolute, then gf is fuzzy IPSO-irresolute.

(2) If f is fuzzy IPSO-irresolute and g is pre-semicontinuous, then gf is fuzzy strongly pre-semicontinuous.

Theorem 2.6. Let $f : X_1 \to X_2$ and $g : X_3 \to X_4$ be fuzzy IPSO-irresolute. Then the product $f \times g : X_1 \times X_3 \to X_2 \times X_4$ is fuzzy PS-irresolute.

Proof. Let $B = \bigcup(A_i \times B_j)$, where the $A_i \in PSO(X_2)$ and $B_j \in PSO(X_4)$. $B \in (X_2 \times X_4)$. Then

$$\begin{aligned}(f \times g)^{-1}(B) &= (f \times g)^{-1}(\bigcup(A_i \times B_j)) \\ &= \bigcup(f \times g)^{-1}(A_i \times B_j) \\ &= \bigcup(f^{-1}(A_i) \times g^{-1}(B_j)).\end{aligned}$$

That $(f \times g)^{-1}(B) \in PSO(X_1 \times X_3)$ follows from Theorem 1.6 and 1.7 in [1]. Thus, $f \times g$ is fuzzy PS-irresolute.

Clearly, the following statements are valid:

fuzzy pre-semi-irresolutene ss

$\nearrow$

fuzzy IPSO-irresoluteness

$\searrow$

fuzzy strong pre-semicontinuity

None of the converses need to be true. We give the following examples.

Example 2.7. Let $X = \{x, y, z\}$ and A, B, C be fuzzy sets in X defined as follows:

$A(x) = 0.2, \quad A(y) = 0.4, \quad A(z) = 0.5;$
$B(x) = 0.8, \quad B(y) = 0.8, \quad B(z) = 0.6;$
$C(x) = 0.3, \quad C(y) = 0.2, \quad C(z) = 0.4.$

Then $\delta = \{0, A, B, 1\}$ and $\tau = \{0, B, 1\}$ are fuzzy topologies on X. Let $f : (X, \delta) \to (X, \tau)$ be an identity mapping. We can get that $PSO(X, \tau) = \{D : D \not< B'\} \cup \{0\}$. And for every $D \in PSO(X, \tau)$, $f^{-1}(D) = D \in PSO(X, \delta)$. Thus, f is fuzzy pre-semi-irresolute. Because $A, C \in PSO(X, \tau)$, from the above we have $f^{-1}(A) = A \in PSO(X, \delta), f^{-1}(C) = C \in PSO(X, \delta)$. But in (X, δ), by easy computations it follows that $A \cap C = B'$ and $B' \not\leq (B'^{-})_o = (B')_o = 0$, i.e, $A \cap C \notin PSO(X, \delta)$, so $f^{-1}(C) = C \notin IPSO(X, \delta)$. Thus, f is not fuzzy IPSO-irresolute.

Example 2.8. Let $X = [0, 1]$ and A, B, C, D be fuzzy sets in X defined as follows:

$A(x) = 0.1,\ x \in [0, 1];\ B(x) = 0.5,\ x \in [0, 1];$
$C(x) = 0.4,\ x \in [0, 1];\ D(x) = 0.7,\ x \in [0, 1].$

Then $\delta = \{0, A, B, 1\}$ and $\tau = \{0, C, 1\}$ are fuzzy topologies on X. Let $f : (X, \delta) \to (X, \tau)$ be an identity mapping. Clearly, f is fuzzy strongly pre-semicontinuous. We can easily get that $D \in PSO(X, \tau)$, in fact, $(D^{-})_o = 1_o = 1$, so $D \leq (D^{-})_o$, i.e., $D \in PSO(X, \tau)$. But in (X, δ), $D \not\leq (D^{-})_o = (A')_o = B$, i.e., $D \notin PSO(X, \delta)$. Hence, $f^{-1}(D) = D \notin IPSO(X, \delta)$. Thus, f is not fuzzy IPSO-irresolute.

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