Fuzzy Strongly USSC-irresolute Functions *

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Abstract: *In this paper, we introduce fuzzy strongly USSC-irresolute function in fuzzy topological spaces, and study this function in relation to some other types of already known functions.*
Key words: *Fuzzy topology; Strongly semiopen set; Fuzzy strongly USSC-irresolute function*

1.Introduction and Preliminaries

The fuzzy continuity and its weaker or stronger forms plays an important role in fuzzy topological spaces. In [2,3], we introduced the concepts of fuzzy S-irresolute function and fuzzy S^* strongly semicontinuous function, respectively. In this paper, we introduce a new class of function, called fuzzy strongly USSC-irresolute function. It is the stronger forms of fuzzy S-irresolute function and fuzzy S^* strongly semicontinuous function.

Throughout the paper by (X, δ) or simply by X we mean fuzzy topological space(in the Chang's[4] sense), briefly fts. A^o, A^- and A' will denote the interior, closure and complement of fuzzy set A, respectively. A fuzzy set A in (X, δ) is called strongly semiopen if there is $B \in \delta$ such that $B \leq A \leq B^{-o}$, that is, $A \leq A^{o-o}$. A is called strongly semiclosed iff A' is strongly semiopen[1]. The family of fuzzy strongly semiopen (resp. strongly semiclosed) sets of a fts X will be denoted by $SSO(X)(resp. SSC(X))$. The $A^{\Delta} = \bigcup\{B : B \leq A, B \in SSO(X)\}$ and $A^{\sim} = \bigcap\{B : A \leq B, B \in SSC(X)\}$ are called the strong semiinterior and strong semiclosure of fuzzy set A[1], respectively.

Since the union (intersection) of any two fuzzy strongly semiclosed (semiopen) sets need not be a strongly semiclosed (semiopen) set[1], $A \in SSC(X)$ and $B \in SSC(X)$ do not necessarily lead to $A \cup B \in SSC(X)$. Let

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$USSC(X) = \{A \in SSC(X) :$ for each $B \in SSC(X), A \cup B \in SSC(X)\}$,
$ISSO(X) = \{A \in SSO(X) :$ for each $B \in SSO(X), A \cap B \in SSO(X)\}$.
Clearly, $\delta \subset ISSO(X) \subset SSO(X)$.

2. Fuzzy Strongly USSC-irresolute Functions

Definition 2.1. A function $f : (X, \delta) \to (Y, \tau)$ from an fts (X, δ) to another fts (Y, τ) is said to be fuzzy strongly USSC-irresolute if $f^{-1}(B) \in USSC(X)$ for each $B \in SSC(Y)$.

Theorem 2.2. A function $f : (X, \delta) \to (Y, \tau)$ is fuzzy strongly USSC-irresolute iff $f^{-1}(B) \in ISSO(X)$ for each $B \in SSO(Y)$.

Proof. This is immediate from Definition 2.1.

Theorem 2.3. Let $f : (X, \delta) \to (Y, \tau)$ be a fuzzy strongly USSC-irresolute function. Then:

(1) $f(A^{\sim}) \leq (f(A))^{\sim}$ for each fuzzy set A in X.

(2) $(f^{-1}(B))^{\sim} \leq f^{-1}(B^{\sim})$ for each fuzzy set B in Y.

(3) $f^{-1}(B^{\Delta}) \leq (f^{-1}(B))^{\Delta}$ for each fuzzy set B in Y.

(4) For each fuzzy point x_α in X and each $B \in SSO(Y)$ with $f(x_\alpha) \in B$, there exists an $A \in ISSO(X)$ such that $x_\alpha \in A$ and $f(A) \leq B$.

Proof. (1): Let A be a fuzzy set in X. Then $(f(A))^{\sim} \in SSC(Y)$. Since f is fuzzy strongly USSC-irresolute, $f^{-1}((f(A))^{\sim}) \in USSC(X)$, and

$$A^{\sim} \leq (f^{-1}f(A))^{\sim} \leq (f^{-1}((f(A))^{\sim}))^{\sim} = f^{-1}((f(A))^{\sim}).$$

Thus, $f(A^{\sim}) \leq ff^{-1}((f(A))^{\sim}) \leq (f(A))^{\sim}$.

(2): Let B be a fuzzy set in Y. By (1), $f((f^{-1}(B))^{\sim}) \leq (ff^{-1}(B))^{\sim} \leq B^{\sim}$. Thus, $(f^{-1}(B))^{\sim} \leq f^{-1}f((f^{-1}(B))^{\sim}) \leq f^{-1}(B^{\sim})$.

(3): Let B be a fuzzy set in Y. By (2), $f^{-1}(B'^{\sim}) \geq (f^{-1}(B'))^{\sim} = ((f^{-1}(B))')^{\sim}$. From Theorem 3.3 in [1], we have

$$f^{-1}(B^{\Delta}) = f^{-1}(B'^{\sim\prime}) = (f^{-1}(B'^{\sim}))' \leq (f^{-1}(B))'^{\sim\prime} = (f^{-1}(B))^{\Delta}.$$

(4): Let f be fuzzy strongly USSC-irresolute, x_α be a fuzzy point in X and $B \in SSO(Y)$ such that $f(x_\alpha) \in B$. Then $x_\alpha \in f^{-1}(B)$. Let $A = f^{-1}(B)$, then $A \in ISSO(X)$. We have $f(A) = ff^{-1}(B) \leq B$.

Theorem 2.4. Let $f : (X, \delta) \to (Y, \tau)$ be a fuzzy strongly USSC-irresolute function, and one-to-one and onto. Then $(f(A))^{\Delta} \leq f(A^{\Delta})$ for each fuzzy set A in X.

Proof. Let f be fuzzy strongly USSC-irresolute and A be any fuzzy set in X. Then $f^{-1}((f(A))^{\Delta}) \in ISSO(X)$. By Theorem 2.3 and the fact that f is one-to-one, we have $f^{-1}((f(A))^{\Delta}) \leq (f^{-1}f(A))^{\Delta}) = A^{\Delta}$. Again, since f is onto, we have $(f(A))^{\Delta} = ff^{-1}((f(A))^{\Delta}) \leq f(A^{\Delta})$.

The following Proposition is obvious.

Proposition 2.5. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions. The following statements are valid:

(1) If f is fuzzy strongly USSC-irresolute and g is S-irresolute, then gf is fuzzy strongly USSC-irresolute.

(2) If f is fuzzy strongly USSC-irresolute and g is strongly semicontinuous, then gf is fuzzy S^{*} strongly semicontinuous.

Theorem 2.6. Let $f : X_1 \rightarrow X_2$ and $g : X_3 \rightarrow X_4$ be fuzzy strongly USSC-irresolute. Then the product $f \times g : X_1 \times X_3 \rightarrow X_2 \times X_4$ is fuzzy S-irresolute.

Proof. Let $B = \bigcup(A_i \times B_j)$, where the $A_i \in SSO(X_2)$ and $B_j \in SSO(X_4)$. $B \in (X_2 \times X_4)$. Then

$$\begin{aligned}(f \times g)^{-1}(B) &= (f \times g)^{-1}(\bigcup(A_i \times B_j)) \\ &= \bigcup(f \times g)^{-1}(A_i \times B_j) \\ &= \bigcup(f^{-1}(A_i) \times g^{-1}(B_j)).\end{aligned}$$

That $(f \times g)^{-1}(B) \in SSO(X_1 \times X_3)$ follows from Theorem 2.8 and 2.5(1) in [1]. Thus, $f \times g$ is fuzzy S-irresolute.

Definition 2.7. A fuzzy topological space (X, δ) is called fuzzy I-compact (SR-compact) if for every cover $\{V_\alpha : V_\alpha \in ISSO(X)\}$ ($\{V_\alpha : V_\alpha \in SSO(X)\}$) of X,there exists a finite subcover of X.

Theorem 2.8. Every surjection fuzzy strongly USSC-irresolute function image of a fuzzy I-compact space is fuzzy SR-compact.

Proof. Let $f : X \rightarrow Y$ be a surjection fuzzy strongly USSC-irresolute function of a fuzzy I-compact space X to a fuzzy topological space Y. Let $\{V_\alpha : \alpha \in J\}$ be a fuzzy strongly semiopen cover of Y. Then $f^{-1}(V_\alpha) \in ISSO(X)$ for each $\alpha \in J$, and $\phi = \{f^{-1}(V_\alpha) : \alpha \in J\}$ is a cover of X. Since X is fuzzy I-compact, there exists a finite subset J_o of J such that $\bigcup\{f^{-1}(V_\alpha) : \alpha \in J_o\} = I_X$. Now

$$\begin{aligned} I_Y = f(I_X) &= f(\bigcup\{f^{-1}(V_\alpha) : \alpha \in J_o\}) \\ &= \bigcup\{ff^{-1}(V_\alpha) : \alpha \in J_o\} \\ &\leq \bigcup\{V_\alpha : \alpha \in J_o\}. \end{aligned}$$

Therefore, Y is fuzzy SR-compact.

3. Examples

Definition 3.1[2,3]. A function $f : (X, \delta) \rightarrow (Y, \tau)$ is called

(1) fuzzy S-irresolute if $f^{-1}(B) \in SSO(X)$ for each $B \in SSO(Y)$.

(2) fuzzy S^* strongly semicontinuous if $f^{-1}(B) \in USSC(X)$ for each $B' \in \tau$.

Clearly, the following statements are valid:

fuzzy S-irresoluteness

↗

fuzzy strongly USSC-irresoluteness

↘

fuzzy S^* strong semicontinuity

None of the converses need to be true. We give the following examples.

Example 3.2. Let $X = \{x, y\}$, and A, B, C be fuzzy sets in X defined as follows:

$A(x) = 0.3, \quad A(y) = 0.7;$

$B(x) = 0.8, \quad B(y) = 0.4;$

$C(x) = 0.9, \quad C(y) = 0.5.$

Then $\delta = \{0, A, B, A \cap B, A \cup B, 1\}$ and $\tau = \{0, C, 1\}$ are fuzzy topologies on X. Let $f : (X, \delta) \rightarrow (X, \tau)$ be an identity mapping. Clearly, $SSO(X, \tau) = \{D : C \leq D\} \cup \{0\}$. We can easily get that for every $D \in SSO(X, \tau)$, $f^{-1}(D) = D$ is a fuzzy strongly semiopen set in (X, δ). Thus, f is fuzzy S-irresolute. Clearly, A and C are fuzzy strongly semiopen sets in (X, δ). Since

$$A \cap C \nleq (A \cap C)^{o-o} = (A \cap B)^{-o} = (A \cap B)'^{o} = A \cap B,$$

$A \cap C$ is not a fuzzy strongly semiopen set in (X, δ), i.e., $f^{-1}(C) = C \notin ISSO(X, \delta)$. Thus, f is not fuzzy fuzzy strongly USSC-irresolute.

Example 3.3. Let $X = [0, 1]$ and A, B be fuzzy sets in X defined as follows:

$A(x) = 0.3$, $x \in [0, 1]$;

$B(x) = 0.7$, $x \in [0, 1]$.

Then $\delta = \{0, A, B, 1\}$ and $\tau = \{0, B, 1\}$ are fuzzy topologies on X. Let $f : (X, \delta) \to (X, \tau)$ be an identity mapping. Clearly, f is fuzzy S^* strongly semicontinuous. Let $C(x) = 0.8$, $x \in [0, 1]$. Then $C \in SSO(X, \tau)$. But $f^{-1}(C) = C \notin ISSO(X, \delta)$. Thus, f is not fuzzy strongly USSC-irresolute.

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