On A Fuzzy Linear Optimization Problem

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Abstract
According to [8,12,13, 23] ,the optimization models with a linear objective function subject to fuzzy re - lation equations is decidable. Algorithms are developed to solve it. In this paper, we propose a proced ure for separating the decision variables into basic and non-basic variables .A complementary problem for the original problem has been defined .Based on the structure of the feasible domain and nature of the objective function , individual variable is restricted to become bivaued .An algorithm is proposed. Two examples are considered to explain the procedure.
Keywords: Fuzzy relation equations: feasible domain, linear function, continuous t-norm, Basic and non-basic variables.

1. Introduction

We consider the following general fuzzy linear optimization problem

minimize $Z = c_1x_1 + \ldots + c_mx_m$

subject to $\quad x \, o \, A = b \qquad (1)$

$0 \leq x_i \leq 1$

$A = [a_{ij}]$, $0 \leq a_{ij} \leq 1$, be m×n-dimensional fuzzy matrix,
$b = (b_j)$, $0 \leq b_j \leq 1$, $j \in J$, be n-dimensional vector,
$c = (c_1, \ldots, c_m) \in R^m$ be cost (or weight) vector, $x = (x_i)$, $i \in I$, be m- dimensional design vector, $I = \{1, \ldots, m\}$ and $J = \{1, \ldots, n\}$ be the index sets and 'o' is Sup-T composition, T being a continuous t-norm .More literature on Sup- T composition can be found in [2,3] .The commonly used continuous t-norms are

(i)$T(a, b) = \min(a,b)$, (2)
(ii)$T(a,b)=\text{product}(a, b) = a.b$, (3)
(iii)$T(a, b) = \max(0, a+b-1)$. (4)

Let $X(A, b) = \{ x = (x_1, \ldots, x_m) \in R^m \mid x \, o \, A = b, x_i \in [0,1] \, \forall \, i \in I\}$ be the solution set.

We are interested in finding a solution vector $x=(x_1,\ldots,x_m) \in X(A, b)$ which satisfies the constraints

$$\underset{i \in I}{Sup}\text{-}T(x_i, a_{ij}) = b_j, \quad \forall j \in J \qquad (5)$$

and minimizes the objective function Z of (1).

Now we look at the structure of X(A, b).Let $x^1, x^2 \in X(A,b)$. $x^1 \leq x^2$ if and only if $x^1_i \leq x^2_i \; \forall i \in I$. Thus, (X (A, b), ≤) becomes a lattice. Moreover, $\hat{x} \in X(A, b)$ is called maximum solution if $x \leq \hat{x}$ for all $x \in X(A, b)$. Also, $\check{x} \in X(A, b)$ is called a minimal solution, if $\check{x} \leq x$ implies $x = \check{x}, \forall x \in X(A,b)$. When X(A, b) is non- empty, it can be

completely determined by a unique maximum and a finite number of minimal solutions [1,7,8,13].
The maximum solution can be obtained by applying the following operation:

$$\hat{x} = A \Diamond b = [\underset{j\in J}{Inf}(a_{ij} \Diamond b_j)]_{i\in I} \quad (6)$$

where $\Diamond$ is inverse operator of *T*.
The inverse operators of (2), (3), (4) can be found in [22] as given below;

$$a_{ij} \Diamond b_j = \begin{cases} 1 & \text{if } a_{ij} \le b_j \\ b_j & \text{if } a_{ij} \ge b_j \end{cases} \quad (7)$$

$$a_{ij} \Diamond b_j = \begin{cases} 1 & \text{if } a_{ij} \le b_j \\ b_j / a_{ij} & \text{if } a_{ij} \ge b_j \end{cases} \quad (8)$$

$$a_{ij} \Diamond b_j = \begin{cases} 1 & \text{if } a_{ij} \le b_j \\ 1 - a_{ij} + b_j & \text{if } a_{ij} > b_j \end{cases} \quad (9)$$

Let $\breve{X}$(A, b) be the set of all minimal solutions. The X (A ,b) can be looked as X (A ,b) =

$$\bigcup_{\breve{x}\in \breve{X}(A,b)} \{x \in X \mid \breve{x} \le x \le \hat{x}\}. \quad (10)$$

Corollary 1. $\breve{X}(A, b) \subseteq X(A, b)$.
We list the following useful results established in [8,13].

Lemma 1. If $x \in X(A ,b)$, then for each $j \in J$ there exists $i_0 \in I$ such that $T(x_{i0},a_{i0j}) = b_j$ and $T(x_i, a_{ij}) \le b_j$ otherwise.

Proof: Since $x \, o \, A = b$, we have $\underset{i\in I}{Sup}\text{-}T(x_i,a_{ij}) = b_j$ for $j \in J$.
This means for each $j \, \varepsilon \, J$, $T(x_i ,a_{ij}) \le b_j$.
In order to satisfy the equality there exists at least one $i \in I$, say i_0, such that $T(x_{i0},a_{i0j}) = b_j$.□

Proposition 1. Let *T* be the continuous t-norm and $a,b,x \in [0,1]$, then equation $T(x,a) = b$ has a solution if and only if $b \le a$.

Definition 1. A constraint $j_0 \in J$ is called *scares or binding* constraint, if for $x \in X(A, b)$ and $i \in I$, $T(x_i , a_{ij0}) = b_{j0}$.

Definition 2. For a solution $x \in X(A, b)$ and $i_0 \in I$, x_{i0} is called *binding variable* if $T(x_{i0}, a_{i0j}) = b_j$ and $T(x_i, a_{ij}) \le b_j$, for all $i \in I$.
Let $X(A, b) \neq \varphi$. Define
$I_j = [\, i \in I \mid T(\hat{x}_i ,a_{ij}) = b_j ,a_{ij} \ge b_j \}$, for each $j \in J$. (11)

Lemma 2. If $X(A, b) \neq \varphi$, then $I_j \neq \varphi, \forall j \in J$.

Proof: Proof is consequence of lemma 1.□

Lemma 3. If $\| I_j \| = 1$, then $\hat{x}_i = \breve{x}_i = a_{ij} \Diamond b_j$ for $i \in I_j$.

Proof: Since x_i ,$i \in I_j$,is the only variable that satisfies the constraint j, it can take only one value equal to $\hat{x}_i$, determined by (6) for $i \in I_j$ and hence the lemma.□

Lemma 4. For i belonging to I_j and $I_{j'}$, $a_{ij} \Diamond b_j = a_{ij'} \Diamond b_{j'}$.

Proof: Since x_i is the only variable that satisfies the constraints j and j′,i.e. $T(x_i, a_{ij}) = b_j$ and $T(x_i, a_{ij'}) = b_{j'}$. Therefore, $x_i = a_{ij} \Diamond b_j = a_{ij'} \Diamond b_{j'}$.□

Solving fuzzy relation equations is an interesting topic of research[1,4-11,13-21,23-25].Studies on fuzzy relation equations with max-T-norm composition or generalized connectives can be found in [18] .According to Gupta and Qi [10] performance of fuzzy controllers depends upon the choice of T-operators. Pedrycz [18] provided the existence condition for max-T-norm composition. A guide line for selecting appropriate connector can be found in [24]. Extensive literatures on fuzzy relation equations with max-min composition [25] can be seen in [19]. Recently, Bourke and Fisher [4] studied a system of fuzzy relation equations with max-product composition. An efficient procedure for solving fuzzy relation equations with max-product can be found in [13].

Fang and Li [8] made seminal study on fuzzy relation equations based on max-min composition with linear objective function. They considered two sub problems of the original problem based on positive and negative costs coeff - icients. One sub problem with positive costs, after defining equivalent 0-1 integer programming problem, has been solved using branch-and-bound method with jump tracking technique. Related developments regarding this can be found in [12,15,23].Wu et.al [23] rearranged (in increasing c and b) the structure of the linear optimization problem and computed initial upper bound for equivalent 0-1 integer programming problem of original problem. They solved the 0-1 integer programmm ing problem by backward jump-tracking branch- and -bound scheme.

Solving a system of fuzzy relation equations completely is a hard problem. The total number of minimal solutions has a combinatorial nature in terms of problem size. Further more, general branch-and-bound algorithm is NP-complete. An efficient method is still required..

In this paper, we propose a procedure that takes care of the characteristics of feasible domain which shows that every variable is bounded between a minimal and the maximal values.We can reduce the problem size by removing those constraints which bound the variables(according to definition (2) Clearly, none of the variables gets increased over its maximum and gets decreased below zero (i.e. assumed minimum). These boundary values can be assigned to the variable in order to improve the value of objective function and to satisfy the functional constraints In section 2, we describe the procedure and give step by step algorithm. In section 3, we consider two numerical examples and solved by using the algorithm given in section -2. Tabular computation of algorithm is proposed.

Conclusions are given in the last.

2. Solution Analysis and Algorithm

Let $\hat{x} \in X(A, b) \neq \varphi$. Define

$$I_j = [\, i \in I \mid T(\hat{x}_i, a_{ij}) = b_j, a_{ij} \geq b_j \}, \quad \forall j \in J \qquad (12)$$

$$J_i = \{ j \in J \mid T(\hat{x}_i, a_{ij}) = b_j, a_{ij} \geq b_j \}, \quad \forall i \in I \qquad (13)$$

Notice that the non-negative variable

$x_i \leq \hat{x}_i, \ \forall\, i \in I$ (14)

has an upper bound.

We write (14) as

$x_i = \hat{x}_i - y_i, \ \forall\, i \in I$ (15)

and refer x_i and y_i as complem entary decision variables.

Whenever

(i) $x_i = 0$, then $y_i = \hat{x}_i$, and

(ii) $x_i = \hat{x}_i$, then $y_i = 0$.

Thus, $0 \leq x_i \leq \hat{x}_i$ implies $0 \leq y_i \leq \hat{x}_i$. Rather than taking each variable $y_i \in [0, \hat{x}_i]$, we consider that the each of y_i's takes its values from the boundary values 0 (lower bound) and/or $\hat{x}_i$ (upper bound). This reduces the problem size, also. The original problem (1)can be defined , in terms of complementary variables, as

minimize $Z = Z_0 - \sum_{i=1}^{m} c_i y_i$

subject to

$\underset{i \in I_j}{Inf}\text{-}T(y_i, a_{ij}) = 0, \ \forall\, j \in J,$ (16)

$y_i \in \{0, \hat{x}_i\} \quad \forall\, i \in I.$

where, $Z_0 = \sum_{i=1}^{m} c_i \hat{x}_i$

Lemma 5. If $a_{ij}>0$, some y_i have to become zero for solving (16).

Proof: T is continuous t-norm. $0 \leq y_i \leq \hat{x}_i$ .For $i \in I_j$ and $j \in J_i$,

$y_i = 0 \Rightarrow T(y_i, a_{ij}) = 0$

$\Rightarrow \underset{i \in I_j}{Inf}\text{-}T(y_i, a_{ij}) = 0.$

Again, $\underset{i \in I_j}{Inf}\text{-}T(y_i, a_{ij}) = 0 \Rightarrow$

$T(y_i, a_{ij}) = 0, \ \exists\, i \in I_j.$

Since $a_{ij}>0$, therefore, $y_i = 0$ for some $i \in I_j$. □

Lemma 6. If $c_i>0$, selecting $y_i = \hat{x}_i$ improves the objective function in (16).

Proof: $Z_0 \geq Z_0 - \sum_{i=1}^{m} c_i y_i = Z \geq Z_0 - \sum_{i=1}^{m} c_i \hat{x}_i \geq$ min Z..□

We call y_i, as leaving basic variable, if it takes the value $\hat{x}_i$ to improve Z_0 and we call it as entering non-basic variable, if it takes the value zero to satisfy the constraint(s).From (14), it is clear that the membership grade x_i of a fuzzy number can not exceed $\hat{x}_i$. Solution set (10) is a poset, Sanchez [21]. The o bjective of optimization problem is to find minimum value of Z. Intuitively, minimum Z can be achieved with maximally graded ($\hat{x}$) fuzzy numbers, if costs are negative, where as, at minimally graded ($\check{x}$) fuzzy numbers, if costs are positive . So, the technique is to select complementary variables y_i from the boundaries 0 and $\hat{x}_i$ so as it either improves the initial value Z_0 or satisfies the constraint(s) .Every complementary variable has to follow either of two rules; (i)Rule for selecting entering non-basic variable, i.e.choose $y_i=0$ in order to satisfy the constraints of J_i..(ii)Rule for selecting leaving basic variable, i.e. choose $y_i=\hat{x}_i$ in order to improve initial Z-value.

Procedure, adopted, is to find y^E_{NB} and y^L_B such that $y=(y^E_{NB}, y^L_B)$ and

$$y_i=\begin{cases}0 & y_i \in y^E_{NB} \\ \hat{x}_i & y_i \in y^L_B\end{cases} \quad \forall i \in I \qquad (17)$$

$y^E_{NB}=\{y_i \mid$ it satisfies the constraints of (16) for $j \in J_i$) is the set of entering non-basic variables and $y^L_B = \{y_i \mid y_i \notin y^E_{NB}\}$ is the set of leaving basic variables.

Let c^E_{NB} and c^L_B denote the costs of variables y^E_{NB} and y^L_B respectively. Thus, cost vector $c = (c^E_{NB}, c^L_B)$.

To be practical, a $y_i \in y^E_{NB}$ is selected in such a way that it has least effect on Z-function and as well as satisfies the constraints I_j, $j \in J_i$. The following steps are involved in generating the set of entering non-basic variables.

Algorithm I

(i)Compute the value set
$V=\{V_i \mid V_i= c_i\,\hat{x}_i$ for each $i \in I\}$

(ii) Generate index set
$\acute{I}=\{k \mid V_k = \min_{i\in I}(V_i)\}$

(iii) Define
$J_k=\{j \in J \mid k \in I_j\}$, $\forall k \in \acute{I}$.

(iv) Construct set $\{y_k \mid k \in \acute{I}\} \subseteq y^E_{NB}$.

(v) Select the values for y_k, $\forall k \in \acute{I}$, according to (16) .

(vi) Remove the row(s) $k \in \acute{I}$ and column(s) $j \in J_k$.

(vii) Define $\bar{I}=I \setminus \acute{I}$ and $\bar{J}=J \setminus \bigcup_k J_k$.

(viii) Set $I \leftarrow \bar{I}$ and $J \leftarrow \bar{J}$ .Go to (i).

(ix) The generated
$y^E_{NB}=\bigcup_k \{y_k \mid y_k=0\}$.

Note: 1. Since $x_i + y_i = \hat{x}_i$, $\forall i \in I$. Structure of Ij and J_i will remain unchanged

2. If $\bar{I}=\{i \mid y_i = 0, i \in I\}$,then
$\| \bar{I} \| \le \min(m,n)$.

This will help us in computing the complexity of the algorithm.

We give basic algorithm to obtain optimal solution of the problem (1)

4. The basic algorithm

Step 1: *Finding the maximum solution of system of FRE in* (1).
Consider the existence proposition 2 and compute $\hat{x}$ according to (6).
Compute

$$\hat{x} = A \Diamond b = [\mathop{Inf}_{j\in J} (a_{ij} \Diamond b_j)]_{i\in I}$$

Step 2: *Test the feasibility.*
If $\hat{x} o A = b$ then feasible.
Else, infeasible and stop!

Step 3: *Compute index sets.*
Compute
$I_j = \{ i \in I \mid \mathcal{T}(\hat{x}_i, a_{ij}) = b_j \}$, $\forall j \in J$ and $J_i = \{j \in J \mid i \in I_j\}$, $\forall i \in I$.

Step 4: *Problem transformation.*
Transform the problem(1),given in variables x,into the problem(16) in volving complementary variable y.

Step 5: *Generating entering non-basic variables.*
Generate the set $y^E_{NB} = \bigcup_k \{y_k \mid y_k=0\}$, using algorithm I.

Step 6: *Generating leaving basic variables.*
Generate the set $y^L_B=\{y_i \mid y_i \notin y^E_{NB}\}$. Set $y_i = \hat{x}_i$, $\forall y_i \in y^L_B$.

Step 7: *Generating complementary variables.*

Complementary decision vector $y^* = (y^E_{NB}, y^L_B)$.

Step 8: *Generating the decision variables.*

Compute the decision vector x^*, according to (15).

i.e. $x^*_i = \hat{x}_i - y^*_i \ \forall i \in I$.

Step 9: *Computing optimal value of objective function.*

$$Z^* = Z_0 - \sum c^L_B y^L_B$$

5. The illustration

Following two examples are considered to illustrate the procedure.

Example1. Solving problem (1) with t-norm (2) and inverse operator (7).

Let m = 6, n = 4, c=(3,4,1,1,-1,5), b=(0.85,0.6,0.5,0.1) and

$$A = \begin{bmatrix} 0.5 & 0.2 & 0.8 & 0.1 \\ 0.8 & 0.2 & 0.8 & 0.1 \\ 0.9 & 0.1 & 0.4 & 0.1 \\ 0.3 & 0.95 & 0.1 & 0.1 \\ 0.85 & 0.1 & 0.1 & 0.1 \\ 0.4 & 0.8 & 0.1 & 0.0 \end{bmatrix}$$

Step1: Finding the maximum solution.

$\hat{x}$=(0.5,0.5,0.85,0.6,1.0,0.6).

Step2: $\hat{x} o$ A = b.Solution isfeasible.

Step 3. Index sets I_j's and J_i's are $I_1=\{3,5\}, I_2=\{4,6\}, I_3=\{1,2\}, I_4=\{5\}$. $J_1=\{3\}$, $J_2 = \{3\}$, $J_3 = \{1\}$, $J_4 = \{2\}$, $J_5 = \{1,4\}$, $J_6 = \{2\}$.

Step 4: Transformed problem is

min $Z = Z_0 - 3y_1 - 4y_2 - y_3 - y_4 + y_5 - 5y_6$, $Z_0 = 6.95$, subject to

$\underset{i=1,\ldots,6}{Inf}$ -min (y_i, a_{ij}) = b_j, j=1,...,4.

$y_1 \in \{0, 0.5\}$, $y_2 \in \{0, 0.5\}$, $y_3 \in \{0. 0.85\}$, $y_4 \in \{0, 0.6\}$, $y_5 \in \{0, 1.0\}$, $y_6 \in \{0, 0.6\}$.

Step 5: Generating the set y^E_{NB}. This is shown via table.

	I_1 ↓	I_2	I_3	I_4 ↓	V
J_1			1		1.5
J_2			2		2.0
J_3	3				0.85
J_4		4			0.6
J_5	5			5	-1.0←
J_6		6			3.0

Minimum (V) = -1.0 corresponds to y_5. Setting $y_5 = 0$, satisfies the constraints of $J_5 = \{1,4\}$.Remove row 5 and columns I_1, I_4 from the table. Since J_3 becomes empty, therefore row 3 will disappear. The next table is

	I_2 ↓	I_3	V
J_1		1	1.5
J_2		2	2.0
J_4	4		0.6←
J_6	6		3.0

Minimum (V) = 0.6 corresponds to y_4. Setting y_4 =0 satisfies the constraint of $J_4 = \{2\}$. Removing row 4 and column I_2 from the table. The reduced table is

	I_3 ↓	V
J_1	1	1.5←
J_2	2	2.0

Minimum (V) =1.5 corresponds to y_1. Setting $y_1 = 0$ satisfies the constraint of $J_1 = \{3\}$.

The generated y_{NB}^{E} = (y_1, y_4, y_5) = (0, 0, 0).

Step 6: Generating the set y_B^L.

y_B^L= (y_2, y_3, y_6) = (0.5, 0.85, 0.6)

Step 7: y^* = (0, 0.5, 0.85, 0, 0, 0.6)

Step 8: x^* = (0.5, 0, 0, 0.6, 1.0, 0)

Step 9: Z^* = 6.95 -5.85 = 1.10.

Example2. Solving problem (1) with t-norm (3) and inverse operator (8). Let m = 10 and n = 8.

c = (-4, 3, 2, 3, 5, 2, 1, 2, 5, 6)

b = (0.48, 0.56, 0.72, 0.56, 0.64, 0.72, 0.42, 0.64) and

A =

$$\begin{bmatrix} 0.6 & 0.2 & 0.5 & 0.3 & 0.7 & 0.5 & 0.2 & 0.8 \\ 0.5 & 0.6 & 0.9 & 0.5 & 0.8 & 0.9 & 0.3 & 0.8 \\ 0.1 & 0.9 & 0.4 & 0.7 & 0.5 & 0.7 & 0.4 & 0.7 \\ 0.1 & 0.6 & 0.2 & 0.5 & 0.4 & 0.1 & 0.7 & 0.5 \\ 0.3 & 0.8 & 0.8 & 0.8 & 0.8 & 0.5 & 0.5 & 0.8 \\ 0.8 & 0.4 & 0.1 & 0.1 & 0.2 & 0.8 & 0.8 & 0.3 \\ 0.4 & 0.5 & 0.4 & 0.8 & 0.4 & 0.7 & 0.3 & 0.4 \\ 0.6 & 0.3 & 0.4 & 0.3 & 0.1 & 0.2 & 0.5 & 0.7 \\ 0.2 & 0.5 & 0.7 & 0.4 & 0.9 & 0.9 & 0.7 & 0.2 \\ 0.1 & 0.3 & 0.6 & 0.6 & 0.6 & 0.4 & 0.4 & 0.8 \end{bmatrix}$$

Step 1: Finding the maximum solution.

$\bar{x}$ =(0.8, 0.8, 0.622, 0.6, 0.7, 0.525, 0.7, 0.8, 0.6, 0.8).

Step 2: $\bar{x}\alpha A$ = b. Solution is feasible.

Step 3: Index sets I_j's and J_i's are I_1={1,8},I_2={3,5},I_3={2},I_4={5,7}, I_5={2},I_6={2},I_7={4,6,9},I_8={1,2.10}

J_1 = {1, 8}, J_2 = {3, 5, 6, 8}, J_3 = {2},J_4 = {7}, J_5 = {2, 4}, J_6 = {7}, J_7 ={4},J_8 ={1},J_9 = {7}, J_{10} = {8}.

Step 4: Problem (1) can be transformed to become

min Z = $Z_0 + 4y_1 - 3y_2 - 2y_3 - 3y_4 - 5y_5 - 2y_6 - y_7 - 2y_8 - 5y_9 - 6y_{10}$, Z_0 = 16.894

subject to

$\underset{i \in I}{Inf}$ - (y_i . a_{ij}) = 0, $\forall$ j∈J

y_1 ∈{0, 0.8}, y_2 ∈{0, 0.8}, y_3 ∈{0. 0.622}, y_4 ∈{0, 0.6},y_5 ∈{0, 0.7}, y_6 ∈{0, 0.525}, y_7 ∈{0, 0.7}, y_8 ∈ {0, 0.8}, y_9 ∈{0. 0.6}, y_{10} ∈{0, 0.8}.

Step 5: Computing the set y_{NB}^{E}.

	I_1 ↓	I_2	I_3	I_4	I_5	I_6	I_7	I_8 ↓	V
J_1	1							1	-3.2←
J_2			2		2	2		2	2.4
J_3		3							1.244
J_4							4		1.8
J_5		5		5					3.5
J_6							6		1.05
J_7				7					0.7
J_8	8								1.6
J_9							9		3.0
J_{10}								10	4.8

Selecting y_1 = 0 satisfies the constraint of J_1 = {1,8}.After removing the rows 1,8,10 (since J_8 and J_{10} become empty) and columns 1 and 8, above table takes the following form

	I_2	I_3	I_4 ↓	I_5	I_6	I_7	V
J_2		2		2	2		2.4
J_3	3						1.244
J_4						4	1.8
J_5	5		5				3.5

J_6						6	1.05
J_7			7				0.7←
J_9						9	3.0

Setting $y_7 = 0$ satisfies the constraints of $J_7 = \{4\}$.Delete row 7 and column 4. Reduced table is

	I_2	I_3	I_5	I_6	I_7 ↓	V
J_2		2	2	2		2.4
J_3	3					1.244
J_4					4	1.8
J_5	5					3.5
J_6					6	1.05←
J_9					9	3.0

Set $y_6 = 0$.This satisfies the constraints of $J_6 = \{7\}$.Deleting the corresponding rows and column, table reduces to become

	I_2 ↓	I_3	I_5	I_6	V
J_2		2	2	2	2.4
J_3	3				1.244←
J_5	5				3.5

$y_3 = 0$ satisfies the constraint of $J_3 = \{2\}$.After removing the rows and columns, above table reduces to become

	I_3 ↓	I_5 ↓	I_6 ↓	V
J_2	2	2	2	2.4←

$y_2 = 0$ satisfies all the remaining constraints of $J_2 = \{3, 5, 6\}$.

$y^E_{NB} = (y_1, y_2, y_3, y_6, y_7) = (0, 0, 0, 0, 0)$

Step 6: $y^L_B = (y_4, y_5, y_8, y_9, y_{10})$

$= (0.6, 0.7, 0.8, 0.6, 0.8)$.

Step 7: $y^* = (0, 0, 0, 0.6, 0.7, 0, 0, 0.8, 0.6, 0.8)$.

Step 8: $x^* = (0.8, 0.8, 0.622, 0, 0, 0.525, 0.7, 0, 0, 0)$.

Step 9: $Z^* = 16.894 - 14.700 = 2.194$.

6. Conclusions

This paper studies a linear optimization problem subject to a system of fuzzy relation equations and presents a procedure to find the optimal solution. Due to non-convexity of feasible domain, traditional methods, viz, simplex method etc. can not be applied. Procedure, adopted here, finds a way of separating the set of decision variables into basic and non-basic variables and evaluates their values. Since every binding variable is bounded and has discrete behavior, because of non-convexity, they can assume only boundary values of the interval in which they lie. In terns, we define the complementary variables and hence the complementary optimization problem. Algorithm is developed to solve this complem entary problem. Significantly, the whole procedure can be presented in a table and time complexity is lesser.

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