Knowledge Representation and Reasoning Based on Fuzzy Petri Nets

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Abstract Petri nets are a good modeling tool for system description and analysis. But it is insufficient to describe the system containing fuzzy behavior. A kind of new fuzzy Petri net is proposed and its reachability problem is discussed. One characteristic of the fuzzy Petri net is its proper non-deterministic, convenient for analysis and the other is the model for many real problems. An illustrative example for knowledge representation and reasoning is given.

Key words Petri net, fuzzy Petri net, reachability, knowledge representation

1 Introduction

Petri nets are a graphical and mathematical modeling tool applicable to many systems. They are a promising tool for describing and studying information processing systems that are characterized as being concurrent, asynchronous, distributed, parallel, non-deterministic, and/or stochastic[8]. But the tokens of place nodes are only nonnegative integer; the firing of transition nodes has no threshold; the weight function for input and output is limited to positive integer. It is insufficient to describe the system containing fuzzy behavior. Therefore, some researcher have presented many fuzzy Petri net models[1-5] and made it solve practical problem. The existing fuzzy Petri nets are much non-deterministic[1] or limited to modeling a certain kind of problems[2]. The former is not convenient for analysis and the fuzzy Petri nets which meet typical problem is difficult to solve other one. Therefore, a kind of new fuzzy Petri nets is given. Using reachability of fuzzy Petri nets, knowledge representation and reasoning are discussed.

Basic concepts and terminologies of Petri nets which we use in this paper can be seen by References[6-8].

2 Fuzzy Petri nets

Definition 1 A fuzzy Petri net is six-tuple

$$FPN=\{P,T,W,\alpha(t),\tau(t),M_0(P)\},$$

where P is a finite set of fuzzy places and the tokens of places can be any non-negative real number;

T is a finite set of fuzzy transitions;

W is a fuzzy relation with marking on $P\times T$ or $T\times P$ and is called weight function that indicate input quantity on arc from place node to transitive node or output quantity on arc from transition node to place node;

$\alpha(t)$ and $\tau(t)$ are all non-negative real functions defined on T that indicate link strength and firing threshold of transition nodes respectively;

$M_0(P)$ is a non-negative real function on P that indicate initial marking and is also called initial resource allocation.

It is easy to know ordinary Petri nets is special case of fuzzy Petri nets in this paper. Let $M_0(P)$ and W be integer functions, $\tau(t)=0$, $\alpha(t)=1$.

Other concepts of fuzzy Petri nets is similar to ordinary Petri nets. Some formal definitions are only given.

Definition 2 (Transition rule)

A transition t is enabled at M, iff $\forall p \in {}^{\bullet}t \quad M(p) \geq W(p,t) \geq \tau(t)$. When t is enabled, we say $M[t>$

Our discussion is limited to pure net according to application of fuzzy Petri nets.

Definition 3 If a transition is enabled at M then the firing of t can transform into a new marking M',we say $M[t>M'$, i.e. M' is reachable from M and called a successor of M. $\forall p \in P$

$$M'(p)=\begin{cases} M(p)-W(p,t) & p\in {}^{\bullet}t \\ M(p)+W(t,p) & p\in t^{\bullet} \\ M(p) & otherwise \end{cases}$$

This rule defines the dynamic behavior of the fuzzy Petri net(${}^{\bullet}t$ and $t^{\bullet}$ denote the set of all the input places of and the set of all the output places of t, respectively).

Definition 4 $\forall M_1,M_2 \in [M_0>$. If there exists firing sequence $t_1, t_2,\cdots,t_n$ such that $M_1[t_1,t_2,\cdots,t_n > M_2$, we say it is reachable from M_1 to M_2.

Definition 5 The reachable marking set which can be reached with only one transition is called the reachable set with one step and denoted by $F(M)$.

3 Reachability analysis of the fuzzy Petri nets

For $\forall M_1,M_2 \in [M_0>$, consider the reachability from M_1 to M_2 and then generate reachability tree.

Step 1 Take M_1 for the root node;

Step 2 Take M_1 for the starting node. If $F(M_1)=\Phi$ or $M_2 \notin [M>$ for $\forall\ M \in F(M_1)$, then it is not reachable from M_1 to M_2;

Otherwise, if $M_2 \in F(M_1)$ and $M_1(p) \geq W(p,t) \geq \tau(t)$, then give a new node M_2 on the tree, the marking of M_2 is given by the firing rule, then it is reachable from M_1 to M_2;

Otherwise, for all $M' \in F(M_1)$, if $M_2 \in [M'>$, and $M_1(p) \geq W(p,t) \geq \tau(t)$, then give other node M' on the tree, the marking of M' is also given by the firing rule, M' is a direct successor of M_1 called non-end node;

Step 3 If there exists no non-end node , then go to Step 4, otherwise go to Step 2;

Step 4 If $M_2 \notin [M_1>$, then it is not reachable from M_1 to M_2, otherwise there exists a transition sequence t_1, $t_2,\cdots,t_n$, such that $M_1[t_1,t_2,\cdots,t_n > M_2$, that is , it is reachable from M_1 to M_2.

4 Knowledge representation and reasoning using the fuzzy Petri nets

The reachability in this paper can be used to knowledge representation and reasoning based on the fuzzy production rule. We give an illustrative example.

Example Let $p_1,p_2,\cdots,p_5$ be five propositions, fuzzy production rules are shown as follows

R_1: if p_1 then p_2 (CF=0.90)
R_2: if p_2 then p_3 (CF=0.90)
R_3: if p_3 then p_4 (CF=0.90)
R_4: if p_4 then p_5 (CF=0.80)
R_5: if p_3 then p_1 (CF=0.70)
R_6: if p_1 then p_4 (CF=0.80)

Given the threshold $\tau = 0.3$ and the believable degree of p_1 is 0.9, How many the believable degree of p_4 is ?

Solve: From algorithm 1 of [2], FPN representation of the problem is shown by Fig.1,

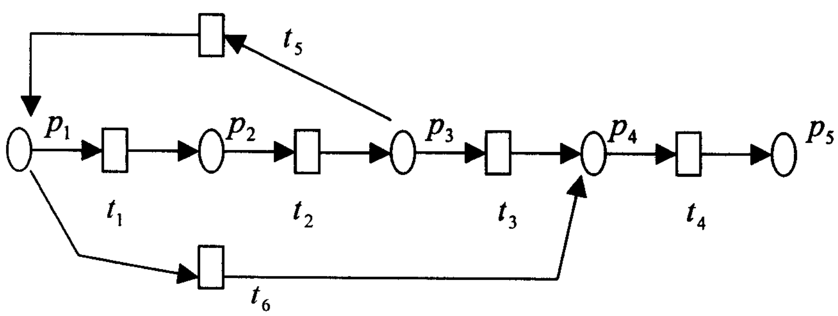

Fig. 1 FPN representation of Example

where believable degrees of propositions are converted to the markings of places, $M_0(p_1)$=0.9 (other places 0), believable degrees of the fuzzy production rules are converted to link strength of transitions, $\alpha(t_1)$ =0.90, $\alpha(t_2)$ =0.90, $\alpha(t_3)$ =0.90, $\alpha(t_4)$ =0.80, $\alpha(t_5)$ =0.80, $\alpha(t_6)$ =0.70. Take $\tau(t) = 0.3$ for $\forall\ t$. W is a changeable weight function and $W(p,t)=M(p)$, $\forall\ p \in t^{\bullet}$,$W(t,p)=\min_{p'\in t} W(p',t) * \alpha(t)$, $\forall\ p \in {}^{\bullet}t$, that is, $W(t,p)$ is a minimum product of the weight function on input arc of t and link strength of t. In the reasoning problem there are many places whose marking is 0, the marking of only one place isn't 0. Therefore make some changes in place marking vector sign and omit places whose marking is 0. For example,

$M_0 = (0.9,0,0,0,0)$ can be denoted by $M_0 = ((p_1,0.9))$, $M_0 = (p_1,0.9)$ for short. From the reachability analysis of section 2. We obtain the reasoning tree shown as Fig. 2 (all markings that p_4 isn't 0 are given) . The reasoning tree gives reachable paths from $M_0 = (p_1,0.9)$ to $M_1 = (p_4,0.66)$ and $M_2 = (p_4,0.72)$. Since marking of place is believable degree of proposition, we should choose the reasoning path which gives a bigger believable degree of the proposition. Thus the believable degree of p_4 is 0.72.

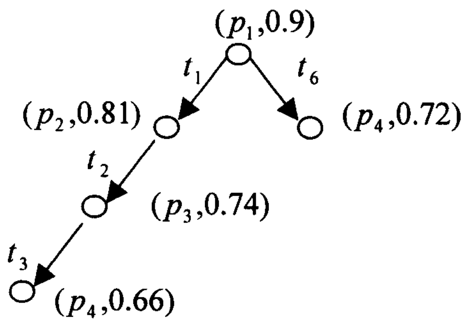

Fig. 2 Reasoning tree of Example 1

5 Conclusion

Fuzzy Petri nets have widespread application to describing and analyzing parallel and concurrent behavior of many physical and social system. Especially the description for some artificial intelligence system is also very suitable. Modeling with fuzzy Petri nets has greater representation ability. The fuzzy Petri nets in this paper can be used to knowledge representation and reasoning and other systems. It is either convenient for analysis or the model for many real problems.

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