(μ,ν)-Resolution Method in Intuitionistic Operator Fuzzy Logic

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Abstract. A primary interpretation for intuitionistic fuzzy operator logic is given. Then the concept of (μ,ν)-complementary literal and (μ,ν)-similar literal about complex literals is presented. In this paper the property of (μ,ν)-false and the (μ,ν)-resolution method of the complex literals are discussed.

Key words: Intuitionistic Operator Fuzzy Logic, (μ,ν)-Resolution, (μ,ν)-False

1 Introduction

From the view of intuitionistic fuzzy logic introduced by K.Atanassov, the true value of a fuzzy proposition can be described by two real number (μ,ν) on the closed interval [0,1], which represents its truth degree and its false degree[1]. In paper [2] the intuitionistic fuzzy degree can be denoted by the operator which lies on the left of the proposition atom. Thus the intuitionistic operator fuzzy logic is discussed on the operator lattice[3] $L=\{(\mu,\nu)|\mu,\nu\in[0,1],\ \mu+\nu\leqq 1\}$.

Definition 1.1[2] Assume $(\mu,\nu)P$ is an atom of IOFL, appointed

$$V_I(\mu,\nu)P=\begin{cases}\mu, & \text{when } P \text{ is appointed } T \text{ by } I\\ \nu, & \text{when } P \text{ is appointed } F \text{ by } I\end{cases}$$

The interpretation of P in this definition is two kinds: true or false. The world described in this system is: any proposition P is certain, crisply, true or false. But owing to different understanding degree or different person generates different intuitionistic fuzzy proposition. This degree denoted by operator (μ,ν). We can interpret the operator (μ,ν): the certainty and uncertainty of P, or the obverse demonstration and inverse demonstration of P and so on.

Definition 1.2[2] Let $L=\{(\mu,\nu)|\mu,\nu\in[0,1],\ \mu+\nu\leqq 1\}$, the operation "$\circ$"of the operator on $(L, *, \oplus, ')$ is defined as follows: for (μ_1,ν_1), $(\mu_2,\nu_2)\in L$, where

$$(\mu_1,\nu_1)\circ(\mu_1,\nu_2)=((\mu_1+\mu_2)/2,(\nu_1+\nu_2)/2)$$
$$(\mu_1,\nu_1)*(\mu_1,\nu_2)=(\min(\mu_1,\mu_2),\max(\nu_1,\nu_2))$$
$$(\mu_1,\nu_1)\oplus(\mu_1,\nu_2)=(\max(\mu_1,\mu_2),\min(\nu_1,\nu_2))$$
$$(\mu_1,\nu_1)'=(\nu_1,\mu_1)$$

So L is an operator lattice.

The operator "$\circ$" can be regarded the evidence of existing P. The different operation can get different interpretation. Therefore we can define and take the operator based on our need.

Definition 1.3[2] Assume S is a set of clause in IOFL, $S_{PR}^{(\mu,\nu)}$ is called the

(μ, ν)-primary reduced set, where $(\mu, \nu)\in L$, $S_{PR}^{(\mu,\nu)}$ is obtained by the method as follows: for any $(\mu^*, \nu^*)P\in S$,

(1)When $\mu\geqq 0.5$ and $\nu\leqq 0.5$,if $\nu\leqq\mu^*\leqq\mu$ or $\nu\leqq\nu^*\leqq\mu$, delete $(\mu^*, \nu^*)P$ from S;

(2)When $\mu<0.5$ and $\nu>0.5$, if $\mu\leqq\mu^*\leqq\nu$ or $\mu\leqq\nu^*\leqq\nu$, delete $(\mu^*, \nu^*)P$ from S.

2 Some results of (μ,ν)-false

Definition 2.1 Let G, H are two formulas of IOFL, $(\mu, \nu)\in L$. For arbitrary interpretation I if $\mu_G\geqq\mu$ and $\nu_G\leqq\nu$ then $\mu_H\geqq\mu$ and $\nu_H\leqq\nu$, then G is called (μ,ν)-strong implicate H, or H is called the logic result of G, denoted by $G\equiv>H$.

Theorem 2.1 Let $\mu\geqq 0.5$ and $\nu\leqq 0.5$, if the (μ,ν)-resolution deduction which can deduce (μ,ν)-□ from the clause set S, then S is (μ, ν)-false.

Theorem 2.2 If the (μ,ν)-resolution deduction which can deduce (μ,ν)-□ from the clause set S, then S is both (μ, ν)-false and (ν,μ)-false.

Proof: If (μ,ν)=(0.5, 0.5), it can prove easily.

If $\mu\geqq 0.5$ and $\nu\leqq 0.5$, the null clause can be obtained by (μ,ν)-resolution, then S is (μ,ν)-false. At last we can obtained the (μ,ν)-resolution formula of two (μ,ν)-complementary literals.

Assume (μ,ν)-complementary literals are $(\mu_1,\nu_1)P_1$ and $(\mu_2,\nu_2)P_2$, $\mu_1>\mu$ and $\nu_1<\nu$, $\mu_2<\nu$ and $\nu_2>\mu$, therefore $S\equiv>(\mu_1, \nu_1)P_1^{\sigma}$, $S\equiv>(\mu_2,\nu_2)P_2^{\sigma}$.

Let $T=(\mu_1, \nu_1) P_1^{\sigma} \wedge (\mu_2, \nu_2) P_2^{\sigma}$, in which $P_1^{\sigma} = P_2^{\sigma}$,So $S\equiv>T$.

Hence $\mu_S<\nu$ and $\nu_S>\mu$. Otherwise, if $\mu_S>\nu$, $\nu_S<\mu$ then $\mu_T>\nu,\nu_T<\mu$, but this is impossible . From above S is (ν,μ)-false.

Definition 2.3 Let $(\mu_{11},\nu_{11})\ldots(\mu_{1n},\nu_{1n})P$ and $(\mu_{21},\nu_{21})\ldots(\mu_{2n},\nu_{2n})P$ are two literals, $(\mu,\nu)\in L$. Assume $V_I(\mu_{11}, \nu_{11})\ldots(\mu_{1n}, \nu_{1n})P=(\mu_1, \nu_1)$, $V_I(\mu_{21}, \nu_{21})\ldots(\mu_{2n}, \nu_{2n})P=(\mu_2, \nu_2)$, if when P is appointed T, $\mu_1>\max(\mu,\nu)$, $\nu_1<\min(\mu,\nu)$ and $\mu_2<\min(\mu,\nu)$, $\nu_2>\max(\mu,\nu)$; when P is appointed F, $\mu_1<\min(\mu,\nu)$, $\nu_1>\max(\mu,\nu)$ and $\mu_2>\max(\mu,\nu)$, $\nu_2<\min(\mu, \nu)$, then these two literals are called (μ, ν)-complementary literals each other.

Definition 2.4 Let $(\mu_{11}, \nu_{11})\ldots(\mu_{1n},\nu_{1n})P$ and $(\mu_{21},\nu_{21})\ldots(\mu_{2n},\nu_{2n})P$ are two lilterals, $(\mu,\nu)\in L$, Assume $V_I(\mu_{11},\nu_{11})\ldots(\mu_{1n}, \nu_{1n})P=(\mu_1, \nu_1)$, $V_I(\mu_{21}, \nu_{21})\ldots(\mu_{2n},\nu_{2n})P=(\mu_2,\nu_2)$, if when P is appointed T, $\mu_1>\max(\mu,\nu)$, $\nu_1<\min(\mu,\nu)$ and $\mu_2>\max(\mu,\nu)$, $\nu_2<\min(\mu, \nu)$; when P is appointed F, $\mu_1<\min(\mu,\nu)$, $\nu_1>\max(\mu,\nu)$ and $\mu_2<\min(\mu,\nu)$, $\nu_2>\max(\mu, \nu)$, then these two literals are called (μ, ν)-similar literals.

3 Application

For instance a production rule

if A then B (μ^*, ν^*)

is described by the formula in IOFL as follows:

$$(\mu^*, \nu^*)(A\rightarrow B) \text{ or } (\mu^*, \nu^*)(\sim A\vee B)$$

(μ^*, v^*) is the intuitionistic fuzzy degree of this rule.

A group of production rule A and a group of fact B are known:

$$A\begin{cases} If \quad E_1 \quad then \quad E_2 \quad (0.7,0.2) \\ If \quad E_2 \quad then \quad E_2 \quad (0.9,0.1) \\ If \quad E_4 \quad then \quad E_5 \quad (0.6,0.2) \end{cases}$$

$$B\begin{cases} (1,0)E_4 \\ (0.8,0.1)E_5 \end{cases}$$

We can prove that $(0.8,0.1)H$ will be deduced from A and B.

Using (μ, v)-resolution method, we can prove $A \wedge B \rightarrow (0.8,0.1)H$ is (μ, v)-false.

$A \wedge B \wedge \sim(0.8,0.1)H$ can decompose the set of clause:

(1)$(0.7,0.2)((0,1)E_1 \vee (1,0)E_2)$

(2)$(0.9,0.1)((0,1)E_2 \vee (1,0)H)$

(3)$(0.6,0.2)((0,1)E_4 \vee (0,1)E_5 \vee (1,0)E_1)$

(4)$(1,0)E_4$

(5)$(0.8,0.1)E_5$

(6)$(0.1,0.8)H$

Take (μ, v)=(0.6,0.2), resolute with (0.6,0.2), then

(7)$(0.6,0.2)((1,0)E_1)$	from(3),(4),(5)
(8)$(0.7,0.2)((1,0)E_2)$	from(1),(7)
(9)$(0.9,0.1)((1,0)H)$	from(2),(8)
(10)□	from(6),(9)

Because we can deduce the conclusion $(0.8,0.1)H$ from A and B, this theorem is (0.2,0.6)-true. At last we can deduce □ but not (μ, v)-□, this theorem is also (0.6,0.2)-true. The intuitionistic fuzzy degree of the conclusion H which deduce from A and B is (0.8,0.1) and this deduction isn't credible completely. It is likely 0.6-true, but likely 0.2-false. The intuitionistic fuzzy degree in this deduction is taken (0.6,0.2). We can also choose anther intuitionistic fuzzy degree. How to find an intuitionistic fuzzy degree by a more simple and convenient algorithm is an opening Problem.

Reference

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