Modifying operations on intuitionstic fuzzy sets

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Abstract: Combining the notions of intuitionstic fuzzy sets and under the basic framework of classic fuzzy sets theory, this paper proposed modifying operations of intuitionstic fuzzy sets. In the same time, this paper made a study of the properties of these operations. These results extended Zadeh's consideration of PRUF.

Keywords: Fuzzy set, Intuitionstic fuzzy set, Concentration, Dilatation, Contrast intensification.

1. Introduction

One of the major applications of fuzzy set theory may be found in Zadeh's creation of PRUF, an acronym for Probabilistic Relational Universal Fuzzy. PRUF constitutes a first attempt to give a formal description of the meaning of sentences in a natural language. One issue in this description concerns the observation that a lot of appearing predicates resembles the same structure. A concept such as age of a person for example is generated from some primary terms such as the antonyms "young" and "old", the remaining values are built from these primary ones through the use of so-called linguistic modifiers or hedges such as "very", "more-or-less" and the logical connectives "not", "or", "and". In this way one obtains values for the predicate "age" as: not very young, not very old, more-or-less young, not very old and not very young. In this paper, Zadeh's consideration of PRUF will be extended to intuitionstic fuzzy set.

2. Remarks about intuitionstic fuzzy set

Let us introduce the following definitions:

Definition 1[1] . Let X be a non-void classic set ,a three recombination $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in X\}$ is called a intuitionstic fuzzy set on X. $\mu_A: X\to[0,1]$ and $\nu_A: X\to[0,1]$ means respectively the degree of membership and the degree of non-membership of x that belong to A in X , and meet $0\le\mu_A(x)+\nu_A(x)\le 1$, $\forall x\in X$. In fact, where μ_A,ν_A are the membership function of ordinary fuzzy sets .The class of all intuitionstic fuzzy sets on a universe X will be denoted IFS [X] .

Definition 2[1] .Let X be a non-void classic set, $A,B\in \mathrm{IFS}[X]$ .Then, A,B are given as:

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in X\}$$
$$B=\{\langle x,\mu_B(x),\nu_B(x)\rangle | x\in X\}$$

where order and operation are given as:

(1). $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x),\forall x\in X$;

(2). $A=B$ iff $\mu_A(x)=\mu_B(x)$ and $\nu_A(x)=\nu_B(x),\forall x\in X$;

(3). $A\cap B=\{\langle x,\min\{\mu_A(x),\mu_B(x)\}, \max\{\nu_A(x),\nu_B(x)\}\rangle | x\in X\}$;

(4). $A\cup B=\{\langle x,\max\{\mu_A(x),\mu_B(x)\}, \min\{\nu_A(x),\nu_B(x)\}\rangle | x\in X\}$;

(5). $A\cdot B=\{\langle x,\mu_A(x)\cdot\mu_B(x),$

$\nu_A(x)+\nu_B(x)-\nu_A(x)\cdot\nu_B(x)\rangle$
$|x\in X\}$;

(6). $A \hat{+} B=\{\langle x,\mu_A(x)+\mu_B(x)$
$-\mu_A(x)\cdot\mu_B(x),$
$\nu_A(x)\cdot\nu_B(x)\rangle\ |x\in X\}$;

(7). $A\otimes B=$
$\{\langle x,\max\{0,\mu_A(x)+\mu_B(x)-1\},$
$\min\{1,\nu_A(x)+\nu_B(x)\}\rangle|x\in X\}$;

(8). $A\oplus B=$
$\{\langle x,\min\{1,\mu_A(x)+\mu_B(x)\},$
$\max\{0,\nu_A(x)+\nu_B(x)-1\}\rangle|x\in X\}$;

(9). $h(A)=\{\langle x,\nu_A(x),\mu_A(x)\rangle|x\in X\}$

3. Concentration

Concentration is defined as a unary operation on IFS $[X]$:

Con: IFS $[X]$ → IFS $[X]$

$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\}\rightarrow$
$con(A)=\{\langle x,\mu_A^2(x),1-(1-\nu_A(x))^2\rangle|x\in X\}$.

From $0\le\mu_A(x),\nu_A(x)\le 1$,
and $0\le\mu_A(x)+\nu_A(x)\le 1$,
we obtain $0\le\mu_A^2(x)\le\mu_A(x)$,
$1\ge 1-(1-\nu_A(x))^2\ge\nu_A(x)$,
and $con(A)\in$ IFS $[X]$, i.e. $con(A)\subseteq A$.
This means that concentration of a intuitionstic fuzzy set leads to a reduction of the degrees of membership.

The operator "con" reveals nice distributivity properties with respect to intuitionstic union and intersection. These and other properties are given in the following section.

(1). $con(A)\subseteq A$
(2). $con(h(A))\subseteq h(conA)$
(3). $con(A\cup B)=conA\cup conB$
(4). $con(A\cap B)=conA\cap conB$
(5). $con(A\cdot B)=conA\cdot conB$
(6). $conA\hat{+}conB\subseteq con(A\hat{+}B)$
(7). $conA\otimes conB\subseteq con(A\otimes B)$
(8). $conA\oplus conB\subseteq con(A\oplus B)$
(9). $A\subseteq B\Rightarrow conA\subseteq conB$

Proof. As an example we prove property 6, i.e.
$\mu_A^2(x)+\mu_B^2(x)-\mu_A^2(x)\cdot\mu_B^2(x)$
$\le(\mu_A(x)+\mu_B(x)-\mu_A(x)\cdot\mu_B(x))^2$,
$(1-(1-\nu_A(x))^2)\cdot(1-(1-\nu_B(x))^2)$
$\ge 1-(1-\nu_A(x)\cdot\nu_B(x))^2$ or, putting
$a=\mu_A(x),b=\mu_B(x)$,
$c=\nu_A(x),d=\nu_B(x):a^2+b^2-a^2b^2$
$\le(a+b-ab)^2$,
$(1-(1-c)^2)\cdot(1-(1-d)^2)\ge 1-(1-cd)^2$,
or equivalently:
$2a^2b^2+2ab-2a^2b-2ab^2\ge 0$,
$(2c-c^2)(2d-d^2)\ge 2cd-c^2d^2$,
$(1-a)(1-b)\ge 0$, $(1-c)(1-d)\ge 0$. The last inequality follows from
$0\le a,b,c,d\le 1$.

4. Dilatation

Dilatation is defined as a unary operation on IFS $[X]$.

dil: IFS $[X]$ → IFS $[X]$

$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\}\rightarrow$
$dil(A)=\{\langle x,\mu_A^{\frac{1}{2}}(x),1-(1-\nu_A(x))^{\frac{1}{2}}\rangle|x\in X\}$

From $0\le\mu_A(x),\nu_A(x)\le 1$,
and $0\le\mu_A(x)+\nu_A(x)\le 1$,
we obtain $0\le\mu_A(x)\le\mu_A^{\frac{1}{2}}(x)$,
$0\le 1-(1-\nu_A(x))^{\frac{1}{2}}\le\nu_A(x)$, and $dil(A)\in$ IFS $[X]$, i.e. $A\subseteq dilA$. This means that dilatation of an intuitionstic fuzzy set leads to an increase of the degrees of membership.

In the following paragraph we list the main properties of dilatation.

(1). $A\subseteq dilA$,

(2). $h(dilA)\subseteq dil(h(A))$,
(3). $dil(A\cup B)=dilA\cup dilB$,
(4). $dil(A\cap B)=dilA\cap dilB$,

(5). $\mathrm{dil}(A \cdot B) = \mathrm{dil}A \cdot \mathrm{dil}B$,

(6). $\mathrm{dil}(A \hat{+} B) \subseteq \mathrm{dil}A \hat{+} \mathrm{dil}B$,

(7). $\mathrm{dil}(A \otimes B) \subseteq \mathrm{dil}A \otimes \mathrm{dil}B$,

(8). $\mathrm{dil}(A \oplus B) \subseteq \mathrm{dil}A \oplus \mathrm{dil}B$,

(9). $\mathrm{con}(\mathrm{dil}A) = A$

(10). $\mathrm{dil}(\mathrm{con}A) = A$,

(11). $A \subseteq B \Rightarrow \mathrm{dil}A \subseteq \mathrm{dil}B$.

Proof. As an example we show property 6, i.e.

$$\mu_A^{\frac{1}{2}}(x) + \mu_B^{\frac{1}{2}}(x) - \mu_A^{\frac{1}{2}}(x) \cdot \mu_B^{\frac{1}{2}}(x) \geq (\mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x))^{\frac{1}{2}},$$

$$(1-(1-\nu_A(x))^{\frac{1}{2}}) \cdot (1-(1-\nu_B(x))^{\frac{1}{2}}) \leq 1-(1-\nu_A(x) \cdot \nu_B(x))^{\frac{1}{2}}$$ or, putting

$a = \mu_A(x), b = \mu_B(x)$,

$c = \nu_A(x), d = \nu_B(x) : \sqrt{a} + \sqrt{b} - \sqrt{ab} \geq (a+b-ab)^{\frac{1}{2}}$,

$(1-(1-c)^{\frac{1}{2}}) \cdot (1-(1-d)^{\frac{1}{2}}) \leq 1-(1-cd)^{\frac{1}{2}}$,

or equivalently: $a + b - ab \leq 1, \sqrt{1-cd} \leq 1$.

The last inequality follows from

$0 \leq a, b, c, d \leq 1$

A simple counterexample shows that the reverse inclusion of 6. does not hold: for $A = B = \{\langle x, 1/4, 3/4\rangle | x \in X\}$ we obtain

$\mathrm{dil}(A \hat{+} B) = \{\langle x, \sqrt{7}/4, 1-\sqrt{7}/4\rangle | x \in X$,

$\mathrm{dil}A \hat{+} \mathrm{dil}B = \{\langle x, 3/4, 1/4\rangle | x \in X$.

5. Contrast intensification

Contrast intensification operator is introduced through a combination of concentration and dilatation. Contrast intensification is a unary operation on IFS [X] defined as:

Int: IFS [X] →IFS [X] ,

$A \to \mathrm{int}A, \forall A \in \mathrm{IFS}[X]$. Where:

$\mathrm{int}A = \mathrm{con}A$, if $0 \leq \mu_A(x) < 1/2$

$= \mathrm{con}A$, if $1/2 \leq \mu_A(x) \leq 1$.

In the following paragraph we the main properties of contrast intensification operators:

(1). $\mathrm{int}(A \cup B) = \mathrm{int}A \cup \mathrm{int}B$,

(2). $\mathrm{int}(A \cap B) = \mathrm{int}A \cap \mathrm{int}B$,

(3). $\mathrm{int}(A \cdot B) \neq \mathrm{int}A \cdot \mathrm{int}B$,

(4). $A \subseteq B \Rightarrow \mathrm{int}A \subseteq \mathrm{int}B$.

Proof. The proofs being similar we restrict ourselves to 1. First we consider those points x of X for which $\mu_A(x) < 1/2$ and $\mu_B(x) < 1/2$. Then: $\mathrm{int}A = \mathrm{con}A$ and $\mathrm{int}B = \mathrm{con}B$.

Since $\max(A(x), B(x)) < 1/2$, we have: $\mathrm{int}(A \cup B) = \mathrm{con}(A \cup B)$.

Hence property 1 is an immediate consequence of the distributivity of con with respect to $\cup$. Secondly we consider those points x of X for which $\mu_A(x) \geq 1/2$ and $\mu_B(x) \geq 1/2$. Then: $\mathrm{int}A = \mathrm{dil}A$, $\mathrm{int}B = \mathrm{dil}B$ and $\mathrm{int}(A \cup B) = \mathrm{dil}(A \cup B)$, and again the distributivity of dil ensures 1. Finally, let $\mu_A(x)$ and $\mu_B(x)$ be separated by $1/2$, for example: $\mu_B(x) < 1/2 \leq \mu_A(x)$. In this case we may write:

$\mathrm{int}A = \mathrm{dil}A, \mathrm{int}B = \mathrm{con}B$,

$\mathrm{int}(A \cup B) = \mathrm{dil}(A \cup B)$.

From $\mathrm{con}B \subseteq B \subset A \subseteq \mathrm{dil}A$, we obtain $\mathrm{dil}A \cup \mathrm{con}B = \mathrm{dil}A$. on the other hand we may write $\mathrm{int}(A \cup B) = \mathrm{dil}(A \cup B) = \mathrm{dil}A$.

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