"Polynomials are Fuzzy Numbers"

By
Luisa Nicosia McAllister,professor emeritus of mathematics
Moravian College
1200Main Street
Bethlehem,PA18018
e-mail :luisamca@enter.net

Abstract : *In this paper we prove that polynomials offer perfectly acceptable fuzzy numbers although at first unfeasible because of their sketches often showing wild oscillations. However we show here that given any closed interval, we can find a subinterval on which we have a convex fuzzy set, or a fuzzy number. .It will then follow that their use in networks and graphs will facilitate their modeling because polynomials functions enjoy many properties that make their computational use very desirable.*

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Section1.A Brief Review. It is proper to recall that a fuzzy set is a set without well defined boundaries.

There are many instances in which the verification of the property defining a set cannot be completed without some lingering ambiguities.

Perhaps a classical example is to ask Mr. Who what is his party affiliations.

Normally, we can think of three possible party affiliations, and then start to figure the probability of each of such affiliations.

However, we might be not that interested in such probabilities, rather we wish to emphasize the level of commitment of Mr.Who to his party. In which case, we no longer can use probabilities; rather we discuss levels of commitment, namely the degree of membership, to that party of Mr. .Who.

Thus we create a fuzzy set, by associating to each member of a universe a degree of membership that has nothing to do with a probability value.The part of the universe, from which we pick our elements will be called the **Support** set of the fuzzy set.

Next it helps to review some terminolgy. We shall often use the term *fuzzified* to denote a concept that has a similar one in the ordinary set theory case.

Definition1. A fuzzy set is said to **convex on some interval [p,q]** (in the fuzzified version) if it satisfies the following condition :

For all elements s in some interval [p,q[we have that

m (s)≤ minimum [m(p),m(q)]..(1)

Let m and M denote respectively the minimum and maximum of the polynomial membership function m(x) on a given interval [a,b].

Theorem 1. For any given polynomial P(x) and any closed interval[a,b], there exists at least an interval[p, q] where condition (1) above holds.

Proof :

A polynomial is a continuous function and every continuous function has a maximum and a minimum on a closed interval, as we recall from elementary calculus. Thus, given any polynomial, we know that it has a minimum value, say m, on any closed interval [a, b]. Assume that the membership function m (x) equals m at some point x=p and, likewise, that it equals the maximum M at some point, say x=p, then the inequality (1) holds on [p, q]. **q.e.d**

As it happens, polynomials lend themselves easily to modeling because they represent a cumulative contribution of several factors that are proportional to increasing powers of time when the independent variable x is time. A first consequence is that we have:

Given a graph with n vertices that is complete, namely each vertex is connected to another, then the adjacency matrix of the graph has entries

$$\{P_{Ij}\}$$

Where P_{Ij} is the polynomial joining vertex I to vertex j.

If its coefficients are:

$$a_0, a_1, a_2, \ldots, a_n \quad \ldots\ldots$$

And so on, then each entry equals the product of the row vector of the coefficients with the column vector of the consecutive powers of the variable x beginning from 1, etc, up to x^n , when each polynomial is of the n-th degree. Thus , the entire adjacency matrix equals the matrix product of a matrix whose entries are the coefficients of each polynomial with the matrix of the column vectors of the consecutive powers of x, [1], [2].

Computationally, is this approach feasible?

To conclude, nowadays, we have mathematical software that readily provides the location of extrema, we thus we can apply the above theorem to find the interval on which a polynomial yields a fuzzy number.

Then, , provided that we use consistency, the polynomials provide us with a simple and useful tool on the interval that is detemined.

References

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[2] McAllister L.N. "*Time Representation : An Example*" SPRINGER VERLAG,Lecture Notes in Computer Science, pp.226-230, 1988;