On A Kind of Fuzzy Relational Equation

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Abstract:

In this paper,the following fuzzy relational equation

$$\bigvee_{i=1}^{n}(a_{ij}x_i) = \lambda \wedge x_j$$

is discussed. The compatible condition of this equation is suggested. The equal solution theorem and solution structure of this equation are put forward.The soving steps are given by an example.

Keywords:

Fuzzy relational equation,Compatible condition,Solution structure

1. General expression of fuzzy relational equation

Let $A=(a_{ij})$ be a n-order fuzzy matrix,$a_{ij}\in[0,1]$; $X = (x_1,x_2,\cdots,x_n)$ is a fuzzy vector, $x_i\in[0,1]$; λ is a real number, $\lambda\in[0,1]$. We can imply the following equation by using the calculation of fuzzy logic relation

$$A\wedge X=\lambda\wedge X \tag{1}$$

This equation can be expressed in detail as

$$\bigvee_{i=1}^{n}(a_{ij}x_i) = \lambda \wedge x_j \qquad j=1,2,\cdots,n \tag{2}$$

where the symbols " $\vee$ " and " $\wedge$ " mean fuzzy logic "sum" and " product " respectively.

2. Sectional system and compatible condition

In order to give the exist judgment of solution of equation (2) , we introduce a concept of sectional system.

For any number $t\in[0,1]$, the following fuzzy system

$$\begin{cases}\bigvee_{i=1}^{n}(a_{ij}x_i) = t \\ \lambda\wedge x_j = t\end{cases} \qquad j=1,2,\cdots,n \tag{3}$$

is called a sectional system of equation (2) . If for some $t\in[0,1]$, the sectional system (3) exists a solution $(x_1, x_2, \cdots, x_n)$,then the sectional system (3) is called compatible.

It is clearly that, the system (3) is compatible, the number t must satisfy the condition

$$0\leqslant t\leqslant \min\{\lambda, M_j; j=1,2,\cdots,n\} \tag{4}$$

where, $M_j=\bigvee_{i=1}^{n} a_{ij}$

Theorem 1. Fuzzy relational equation (2) is compatible,if and only if,the sectional system (3) is compatible for any t satisfied the condition (4).

3. Equal solution theorem and constructure of solution

According to the fuzzy logic calculation, we directly have the following equal solution theorem.

Theorem 2. Fuzzy relational equation(2) has the same solutions as the fuzzy system

$$\begin{cases} a_{ij} \wedge x_i \le \lambda \wedge x_j, i = 1,2,\cdots,n \\ a_{ij} \wedge x_i = \lambda \wedge x_j, \exists i \end{cases} \tag{5}$$

In the system(5),the solution of inequation

$$a_{ij} \wedge x_i \leqslant \lambda \wedge x_j \tag{6}$$

is denoted as G_{ij}, and the solution of eqution

$$a_{ij} \wedge x_i = \lambda \wedge x_j \tag{7}$$

is denoted as H_{ij},then the solution of j-th equation in equation system(2) is

$$G_j = \bigcup_{k=1}^{n}[(\bigcap_{i=1}^{n} G_{ij}) \cap H_{kj}] \qquad j=1,2,\cdots,n \tag{8}$$

Therefore we have the solution of equation system (2) expressed as

$$G = \bigcap_{j=1}^{n} G_j \tag{9}$$

Theorem 3. The structure of solutions of fuzzy relational equation system (2) is expressed by (8) and (9).

In the formula (8),the calculating of G_{ij} and H_{ij} can be obtained by the analyse of fuzzy logic function.

4. Calculating example

In this section, we give the soving steps of system (2) by a simple example.

Example. Soving the fuzzy relational equation system

$$\begin{cases} (0.5 \wedge x_1) \vee (0.2 \wedge x_2) = 0.4 \wedge x_1 \\ (0.2 \wedge x_1) \vee (0.5 \wedge x_2) = 0.4 \wedge x_2 \end{cases} \tag{10}$$

Solution:

Step1. Calculating the solutions of each simple inequation and equation in (10):

$G_{11} = [0,0.4] \times [0,1]$ $\qquad$ $H_{11} = [0,0.4] \times [0,1]$

$G_{12} = G_{21} = [0 \leqslant x_1 = x_2 \leqslant 0.2] \cup [0.2,1] \times [0.2,1]$ $\qquad$ $H_{12} = H_{21} = [0 \leqslant x_1 = x_2 \leqslant 0.2]$

$H_{22} = [0,1] \times [0,0.4]$

$G_{22} = [0,1] \times [0,0.4]$

Step2. Calculating the solutions of each equation in (10):

$$G_1 = [(G_{11} \cap G_{12}) \cap H_{11}] \cup [(G_{11} \cap G_{12}) \cap H_{12}] = [0 \leqslant x_1 = x_2 \leqslant 0.2] \cup [0.2,0.4] \times [0.2,1]$$

$$G_2 = [(G_{21} \cap G_{22}) \cap H_{21}] \cup [(G_{21} \cap G_{22}) \cap H_{22}] = [0 \leqslant x_1 = x_2 \leqslant 0.2] \cup [0.2,1] \times [0.2,0.4]$$

Step3. Calculating the solution of fuzzy relational equation system(10):

$$G = G_1 \cap G_2 = [0 \leqslant x_1 = x_2 \leqslant 0.2] \cup [0.2,0.4] \times [0.2,0.4]$$

References

[1]Wang Peizhuang, Fuzzy set theory and its application, Science and technology press,Shanghai,1983,109-123.

[2] Liu Fengqi, Fuzzy mathematics and its application, Science press,Bei-jing,1988,130-159.

[3] Liu Jinlu,Fuzzy logic functional equation, J.of Fuzzy Math,3(1996) 787-792.

[4] Liu Jinlu and Shi kaiquan,On the F-quadratic equation, J.of Fuzzy Math.3(1996)793-800.