Countably Strong Lowen's Compactness in L-fts *

Bai Shi-Zhong

Department of Mathematics, Wuyi University
Jiangmen Guangdong 529020, P.R.China

Abstract: *In this paper, the concepts of countably strong Lowen's compactness and strong Lowen's Lindelöf property in L-fuzzy topological spaces are introduced. And these concepts are defined for arbitrary L-fuzzy subsets. Their basic properties are studied, and also the cover forms and finite intersection properties are described.*

Key words: *L-fuzzy topology; Remote-neighborhood; Countably Strong Lowen's compact set; Strong Lowen's Lindelof set*

1 Introduction

R.Lowen has introduced a fuzzy compactness for fuzzy topological spaces in [6], and we'll call it Lowen's compactness. Wang has generalized it to L-fuzzy topological spaces in [8]. We have introduced SR-compactness and strong Lowen's compactness in L-fuzzy topological spaces in [2,4]. The strong Lowen's compactness is a kind of compactness between Lowen's compactness and SR-compactness in L-fuzzy topological spaces[4]. In this paper, we introduce the concepts of countably strong Lowen's compactness and strong Lowen's Lindelöf property in L-fuzzy topological spaces. They are defined for arbitrary L-fuzzy subsets. Their basic properties and characteristic properties are studied.

2 Preliminaries

In this paper, $L = L(\leq, \vee, \wedge, ')$ always denotes a fuzzy lattice, i.e., a completely distributive lattice with an order-reversing involution "$'$". 0 and 1 denote the least

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and the greatest element in L, respectively. L^X denotes the set of all L-fuzzy sets on a nonempty crisp set X. $M(L)$ denotes the set of all nonzero irreducible elements of L. $M^*(L^X)$ denotes the set of all nonzero irreducible elements of L^X. $\beta(\alpha)$ and $\beta^*(\alpha) = \beta(\alpha) \cap M(L)$ respectively denote greatest minimal set and standard minimal set of α $(\alpha \in L)$. $\alpha^*(r) = (\beta^*(r'))'$ denotes maximal set of r (r is a prime element of L and $r < 1$).

For each $\phi \subset L$, we define $\phi' = \{A' : A \in \phi\}$, $\bigvee \phi = \bigvee\{P : P \in \phi\}$, $\bigwedge \phi = \bigwedge\{P : P \in \phi\}$. For $r \in L$, $\varepsilon_r(A) = \{x \in X : A(x) \geq r\}$. We will denote L-fuzzy topological space by L-fts.

Definition 2.1(Bai[1]). Let (L^X, δ) be an L-fts, $A \in L^X$. Then A is called a strongly semiopen set iff there is a $B \in \delta$ such that $B \leq A \leq B^{-o}$, and A is called a strongly semiclosed set iff there is a $B \in \delta'$ such that $B^{o-} \leq A \leq B$, where B^o and B^- are the interior and closure of B, respectively. $SSO(L^X)$ and $SSC(L^X)$ will always denote the family of strongly semiopen sets and family of strongly semiclosed sets of an L-fts (L^X, δ), respectively.

Definition 2.2(Bai[2]). Let (L^X, δ) be an L-fts and $x_\lambda \in M^*(L^X)$.$A \in SSC(L^X)$ is called a strongly semiclosed remoted-neighborhood, or briefly, SSC-RN of x_λ , if $x_\lambda \notin A$. The set of all SSC-RNs of x_λ is denoted by $\xi(x_\lambda)$.

Definition 2.3(Bai[2]). Let (L^X, δ) be an L-fts, $A \in L^X$ and $\alpha \in M(L)$, $\phi \subset SSC(L^X)$ is called an α-SS-remote neighborhood family of A (briefly α-SS-RF of A) if for each x_α in A, there is $P \in \phi$ such that $P \in \xi(x_\alpha)$.

Definition 2.4(Bai[3]). Let (L^X, δ) be an L-fts and $A \in L^X$. A is called strong Lowen's compact, if for each α-net S in $A(\alpha \in M(L))$ and each $r \in \beta^*(\alpha), S$ has an SS-cluster point in A with height r. Specifically, when $A = 1_X$ is strong Lowen's compact, we call (L^X, δ) a strong Lowen's compact space.

Theorem 2.5(Bai[3]). Let (L^X, δ) be an L-fts and $A \in L^X$. A is strong Lowen's compact iff for each $r \in \beta^*(\alpha)$ and each r-SS-RF ϕ of A has a finite subfamily ψ of ϕ such that ψ is an α-SS-RF of $A(\alpha \in M(L))$.

3 Countably Strong Lowen's compact sets

Definition 3.1. Let (L^X, δ) be an L-fts and $A \in L^X$. A is called a countably strong Lowen's compact set, if for each $r \in \beta^*(\alpha)$ and each countable r-SS-RF ϕ of A has a finite subfamily ψ of ϕ such that ψ is an α-SS-RF of $A(\alpha \in M(L))$. Specifically, when $A = 1_X$ is countably strong Lowen's compact, we call (L^X, δ) a countably strong Lowen's compact space.

Definition 3.2. Let (L^X, δ) be an L-fts, $A \in L^X, r$ be a prime element of L and $r < 1$. $\mu \subset SSO(L^X)$ is called an r-S-cover of A if for each $x \in \varepsilon_{r'}(A)$, there is $U \in \mu$ such that $U(x) \not\leq r$.

Lemma 3.3. Let (L^X, δ) be an L-fts, $A \in L^X$ and $\mu \subset SSO(L^X)$. Then μ is an r-S-cover of A iff $\mu' \subset SSC(L^X)$ is r'-SS-RF of A, where r is a prime element of L and $r < 1$.

Proof. Let μ be an r-S-cover of A. Then for each $x \in \varepsilon_{r'}(A)$ there is an $U \in \mu$ such that $U(x) \not\leq r$, and so $r' \not\leq U'(x)$. This shows that $U' \in \xi(x_{r'})$, and hence μ' is r'-SS-RF of A.

Conversely, let $\mu' \subset SSC(L^X)$ be r'-SS-RF of A, where r is a prime element of L and $r < 1$. Then for each $x_{r'}$ in A, there is $U' \in \mu'$ such that $U' \in \xi(x_{r'})$, and so $r' \not\leq U'(x)$, then $U(x) \not\leq r$. This shows that μ is an r-S-cover of A.

Theorem 3.4. Let (L^X, δ) be an L-fts and $A \in L^X$. Then A is countably strong Lowen's compact iff for each $r \in (\beta^*(\alpha))'(i.e., r' \in \beta^*(\alpha))(\alpha \in M(L))$ and every countable r-S-cover μ of A, there is a finite subfamily ν of μ such that ν is an α'-S-cover of A.

Proof. This follows directly from Definition 3.1 and Lemma 3.3.

Definition 3.5. Let (L^X, δ) be an L-fts, $\alpha \in L, A \in L^X$ and $\phi \subset L^X.\phi$ is called a family with the α-finite intersection property in A, if for each $\psi \in 2^{(\phi)}$ (where $2^{(\phi)}$ is a set of all the finite subfamily of ϕ) there is an $x \in \varepsilon_\alpha(A)$ such that $(\bigwedge \psi)(x) \geq \alpha$.

Lemma 3.6. Let (L^X, δ) be an L-fts, $A \in L^X, \phi \subset SSC(L^X)$ and $\alpha \in M(L)$. Then ϕ has the α-finite intersection property in A iff ψ' is not an α'-S-cover of A for each $\psi \in 2^{(\phi)}$.

Proof. ϕ has the α-finite intersection property in $A(\alpha \in M(L))$ iff $\forall \psi \in 2^{(\phi)}$, $\exists x \in \varepsilon_\alpha(A), (\bigwedge \psi)(x) \geq \alpha$ iff $\forall \psi \in 2^{(\phi)}, \exists x \in \varepsilon_\alpha(A), (\bigvee \psi')(x) \leq \alpha'$ iff $\forall \psi \in 2^{(\phi)}, \psi'$ is not an α'-S-cover of A.

Theorem 3.7. Let (L^X, δ) be an L-fts and $A \in L^X$. Then A is countably strong Lowen's compact iff for each $\alpha \in M(L)$ and every countable subfamily $\phi \subset SSC(L^X)$ which has the α-finite intersection property in A, there is $x \in \varepsilon_\alpha(A)$ and some $r \in (\beta^*(\alpha))'$ such that $(\bigwedge \phi)(x) \geq r'$.
Proof. This follows directly from Theorem 3.4. and Lemma 3.6.

Theorem 3.8. Let A and B be two countably strong Lowen's compact sets in an L-fts (L^X, δ). Then $A \vee B$ is also countably strong Lowen's compact.
Proof. Suppose for each $r \in \beta^*(\alpha)(\alpha \in M(L))$, $\phi \subset SSC(L^X)$ be a countable r-SS-RF of $A \vee B$. Then ϕ is a countable r-SS-RF of both A and B. Since A and B are both countably strong Lowen's compact sets, there exist finite subfamily ψ_1 and ψ_2 of ϕ such that ψ_1 and ψ_2 are α-SS-RF of A and B, respectively. Put $\psi = \psi_1 \cup \psi_2$. Clearly, ψ is a finite subfamily of ϕ, and also an α-SS-RF of $A \vee B$. Thus, by Definition 3.1 $A \vee B$ is countably strong Lowen's compact.

Theorem 3.9. Let A be a countably strong Lowen's compact set in L-fts (L^X, δ). Then for each $B \in SSC(L^X)$, $A \wedge B$ is countably strong Lowen's compact.
Proof. It is similar to the proof of Theorem 2.1 in [3].

Corollary 3.10. Let (L^X, δ) be a countably strong Lowen's compact space and $B \in SSC(L^X)$. Then B is countably strong Lowen's compact.

Definition 3.11(Wang[8]). Let $(L^X, \delta), (L^Y, \tau)$ be two L-ftses and $f : L^X \to L^Y$ be a mapping induced by a crisp mapping $f : X \to Y$. We define the $f : L^X \to L^Y$ and its inverse mapping $f^{-1} : L^Y \to L^X$ as the following:

$\forall A \in L^X, y \in Y, f(A)(y) = \bigvee\{A(x) : x \in X, f(x) = y\}$,

$\forall B \in L^Y, x \in X, f^{-1}(B)(x) = B(f(x))$.

Then $f : L^X \to L^Y$ is called an L-fuzzy mapping.

Definition 3.12(Bai[2]). Let (L^X, δ) and (L^Y, τ) be two L-ftses and $f : (L^X, \delta) \to (L^Y, \tau)$ an L-fuzzy mapping. f is called S-irresolute mapping if $f^{-1}(B) \in SSO(L^X)$ for each $B \in SSO(L^Y)$.

Theorem 3.13. Let A be a countably strong Lowen's compact set in (L^X, δ), and $f : (L^X, \delta) \to (L^Y, \tau)$ an S-irresolute mapping. Then $f(A)$ is countably strong Lowen's compact in (L^Y, τ).
Proof. It is similar to the proof of Theorem 2.2 in [3].

Corollary 3.14. Let (L^X, δ) be a countably strong Lowen's compact space and $f : (L^X, \delta) \to (L^Y, \tau)$ an onto S-irresolute mapping. Then (L^Y, τ) is a countably strong Lowen's compact space.

4 Strong Lowen's Lindelöf sets

Definition 4.1. Let (L^X, δ) be an L-fts and $A \in L^X$. A is called a strong Lowen's Lindelöf set, if for each $r \in \beta^*(\alpha)$ and each r-SS-RF ϕ of A has a countable subfamily ψ of ϕ such that ψ is an α-SS-RF of $A(\alpha \in M(L))$. Specifically, when $A = 1_X$ is strong Lowen's Lindelöf, we call (L^X, δ) a strong Lowen's Lindelöf space.

Theorem 4.2. Let (L^X, δ) be an L-fts and $A \in L^X$. Then A is a strong Lowen's Lindelöf set iff for each $r \in (\beta^*(\alpha))'(\alpha \in M(L))$ and every r-S-cover μ of A, there is a countable finite subfamily ν of μ such that ν is an α'-S-cover of A.

Theorem 4.3. Let A and B be two strong Lowen's Lindelöf sets in an L-fts (L^X, δ). Then $A \vee B$ is also a strong Lowen's Lindelöf set.

Theorem 4.4. Let A be a strong Lowen's Lindelöf set in L-fts (L^X, δ). Then for each $B \in SSC(L^X)$, $A \wedge B$ is a strong Lowen's Lindelöf set.

Corollary 4.5. Let (L^X, δ) be a strong Lowen's Lindelöf space and $B \in SSC(L^X)$. Then B is a strong Lowen's Lindelöf set.

Theorem 4.6. Let A be a strong Lowen's Lindelöf set in (L^X, δ), and $f : (L^X, \delta) \to (L^Y, \tau)$ an S-irresolute mapping. Then $f(A)$ is a strong Lowen's Lindelöf set in (L^Y, τ).

Corollary 4.7. Let (L^X, δ) be a strong Lowen's Lindelöf space and $f : (L^X, \delta) \to (L^Y, \tau)$ an onto S-irresolute mapping. Then (L^Y, τ) is a strong Lowen's Lindelöf space.

Theorem 4.8. Let (L^X, δ) be an L-fts and $A \in L^X$. Then A is a strong Lowen's compact set iff A is a countably strong Lowen's compact and strong Lowen's Lindelöf set.

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