(μ,ν)-Implication of IOFL

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Abstract

In paper [1],the method of (μ, ν)-resolution in intuitionistic operator fuzzy logic (IOFL) is discussed . But it isn't perfectibility . In this paper two different concepts of the implication of IOFL is presented, namely, (μ, ν)-weak implication and (μ, ν)-strong implication . Then the property of these two different implication and the perfectibility of (μ, ν)-resolution is discussed .

Keywords Intuitionistic Operator Fuzzy Logic , (μ, ν)-resolution, (μ, ν)-weak implication

1. Introduction

An intuitionistic fuzzy proposition can be described by two real number on the closed interval [0,1], which represent its truth degree and its false degree .In paper [1] the intuitionistic fuzzy degree can be expressed by operator which lies on the left of the proposition atom . Thus intuitionistic operator fuzzy logic (IOFL) is discussed on the operator lattice $L=\{(\mu,\nu)|\mu,\nu\in[0,1],\ \mu+\nu\leqslant 1\}$. Furthermore, the (μ,ν)-resolution method is presented. For a whole (μ,ν)-resolution principle ,two concepts of the implication in IOFL is defined as follows .

Definition 1.1 Let G is a formula of IOFL, $\mu,\nu\in L$, assume $V_I(G)=(\mu_G,\nu_G)$,the formula G is called (μ,ν)-true if $\mu_G\geqslant\mu$ and $\nu_G\leqslant\nu$ for an arbitrary interpretation I. Whereas, the formula G is called (μ,ν)-false if $\mu_G\leqslant\mu$ and $\nu_G\geqslant\nu$.

2. (μ,ν)-weak implication and (μ,ν)-strong implication

Definiton 2.1 Let G an H are formulas of IOFL, $\mu,\nu\in$L, G is called (μ,ν)-weak implicate H (or H is a weak –logical result of G) if $(G\rightarrow H)$ is (μ,ν)-true, denoted by $G\Rightarrow H$.

Theorem 2.1 Assume $V_I(G)=(\mu_G,\nu_G)$ and $V_I(H)=(\mu_H,\nu_H)$, $(G\rightarrow H)$ is (μ,ν)-true iff if $\mu_G>\nu$ and $\nu_G<\mu$ then $\mu_H\geqslant\mu$ and $\nu_H\leqslant\nu$ for arbitrary interpretation I.

Proof (<=) $\mu_{(G\to H)}=\mu_{(\sim G\vee H)}=\max\{\mu_{\sim G},\mu_H\}=\max\{\nu_G,\mu_H\}=\mu_H\geqslant\mu$ and $\nu_{(G\to H)}=\nu_{(\sim G\vee H)}=\min\{\nu_{\sim G},\nu_H\}=\nu_H\leqslant\nu$, hence $(G\rightarrow H)$ is (μ,ν)-true.

(=>) For an arbitrary interpretation I, $\mu_{(G\to H)}\geqslant\mu$ and $\nu_{(G\to H)}\leqslant\nu$, if $\mu_G>\nu$ and $\nu_G<\mu$, then

$$\mu_{(G\to H)}=\mu_{(\sim G\vee H)}=\max\{\mu_{\sim G},\mu_H\}=\max\{v_G,\mu_H\}\geqslant\mu$$

since $v_G<\mu$, we have $\mu_H\geqslant\mu$,

$$v_{(G\to H)}=v_{(\sim G\vee H)}=\min\{v_{\sim G},v_H\}=\min\{\mu_G,v_H\}\leqslant v$$

and $\mu_G>v$, therefore $v_H\leqslant v$.

Definition 2.2 Assume S is a set of clause, $S_{PR}^{(\mu,v)}$ is called (μ,v)-primary reduced set of S,$(\mu,v)\in L$,,$S_{PR}^{(\mu,v)}$ is obtained by the method as follows : for arbitrary $(\mu^*,v^*)P\in S$,

(1)when $\mu\geqslant0.5$, $v\leqslant0.5$, if $v\leqslant\mu^*\leqslant\mu$ or $v\leqslant v^*\leqslant\mu$, delete $(\mu^*,v^*)P$ from S.

(2)when $\mu<0.5$, $v>0.5$, if $\mu\leqslant\mu^*\leqslant v$ or $\mu\leqslant v^*\leqslant v$, delete $(\mu^*,v^*)P$ from S.

Theorem 2.1 Let C_1 and C_2 are two clauses , $\mu\geqslant0.5$, $v\leqslant0.5$, $C_{1PR}^{(\mu,v)}$ and $C_{2PR}^{(\mu,v)}$ are (μ,v)-primary reduced clause of C_1, C_2, and then

$$(C_{1PR}^{(\mu,v)}\wedge C_{2PR}^{(\mu,v)})\Rightarrow R_{(\mu,v)}(C_{1PR}^{(\mu,v)},C_{2PR}^{(\mu,v)})$$

Proof We can obtain it from definition 2.1 ,definition 2.2 and theorem 2.1.(omitted)

Theorem 2.3 Let C_1 and C_2 are two clauses , assume $(\mu,v)=(0.5,0.5)$, and then $C_1\wedge C_2\Rightarrow R_{(\mu,v)}(C_1,C_2)$.

Proof When $(\mu,v)=(0.5,0.5)$, there is $C_1=C_{1PR}^{(\mu,v)}$ and $C_2=C_{2PR}^{(\mu,v)}$

From theorem 2.1 we can get $C_1\wedge C_2\Rightarrow R_{(\mu,v)}(C_1,C_2)$.

Definition 2.3 Assume G and H are two formulas of IOFL, $(\mu,v)\in L$, for arbitrary interpretation I, if $\mu_G\geqslant\mu$ and $v_G\leqslant v$ there must be $\mu_H\geqslant\mu$ and $v_H\leqslant v$, G is called (μ,v)-implication H or H is a logical result of G, denoted $G\equiv>H$.

The following propositions are obviously.

Proposition 2.1 When $\mu>0.5$ and $v<0.5$, if $G=>H$ then $G\equiv>H$; When $\mu=0.5$ and $v=0.5$, if $G\equiv>H$ then $G=>H$.

Proposition 2.2 Let G is a formula ,

(1)When $\mu\leqslant0.5$ and $v\geqslant0.5$, $A=>A$;

(2)$A\equiv>A$.

Proposition 2.3 Let A, B, C are the formulas of IOFL respectively,

(1)When $\mu>0.5$ and $v<0.5$, if $A=>B$, $B=>C$ then $A=>C$;

(2)If $A\equiv>B$ and $B\equiv>C$ then $A\equiv>C$.

Proposition 2.4 Let A, B, C are formulas of IOFL

(1) If $A=>B$ and $A=>C$ then $A=>(B\wedge C)$;

(2) If $A\equiv>B$ and $A\equiv>C$ then $A\equiv>(B\wedge C)$.

Theorem 2.4 Let C_1 and C_2 are two clauses, $\mu\geqslant0.5$ and $v\leqslant0.5$, and then

$$C_1\wedge C_2\equiv>R_{(\mu,v)}(C_1,C_2).$$

Corollary Let C_1 and C_2 are two clauses, $\mu\geqslant0.5$ and $v\leqslant0.5$, for arbitrary interpretation I, if

$$\mu_{(C1\wedge C2)}>\mu,\ v_{(C1\wedge C2)}<v$$

then $\mu_{R_{(\mu,v)}(C_1,C_2)} > \mu, v_{R_{(\mu,v)}(C_1,C_2)} < v$

3.the perfectibility of (μ,ν)-resolution princleple

Definition 3.1 For $(\mu,v) \in L$, (μ^*, v^*)P is an arbitrary word of a clause which satisfied with

$$v \leqslant \mu^* \leqslant \mu \text{ or } v \leqslant v^* \leqslant \mu$$

This clause is called (μ,v)-null clause , denoted by (μ,v)-□.

Theorem 3.1 Let $\mu \geqslant 0.5$ and $v \leqslant 0.5$ if a deduction that (μ,v)-□ can be deduced from S with (μ,v)-resolution method exists ,then S is (μ,v)-false.

Proof If otherwise , there will be an interpretation I, cause $\mu_S > \mu$ and $v_S < v$ from theorem 2.4 there is $C_1 \wedge C_2 \Rightarrow R_{(\mu,v)}(C_1,C_2)$
from proposition 2.3 and the corollary of theorem 2.4 there is

$$\mu_{(\mu,v)\text{-}\square} > \mu,\ v_{(\mu,v)\text{-}\square} < v,$$

It is a contradiction for definition 3.1.

Theorem 3.2[2] For $(\mu, v) \in L$, if the clause set S is (μ, v)-false, there must be a (μ,v)- resolution deduction which can deduce (μ,v)-□ from S.

From theoerm 5 and theorem 6 can obtained follow

Theorem 3.2 (Perfectibility Theorem) Assume $\mu \geqslant 0.5$ and $v \leqslant 0.5$, S is a clause set, then S is (μ,v)-false iff there is a (μ,v)-resolution deduction which can deduce (μ,v)-□ from S.

From above, in order to keep the intuitionistic property of two clauses, $(\mu, v)=(0.5,0.5)$ should be taken in (μ, v)-weak implication; While $\mu \geqslant 0.5$ and $v \leqslant 0.5$ should be taken in (μ,v)-strong implication ,that can make the (μ,v)-resolution formula of two clause is logical result of their parent clause.

When $\mu+v=1$, it can be obtained λ-weak implication and λ-strong implication which defined in paper[3].

References

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