An algorithmic Framework of Collaborative Clustering

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Abstract *In this study, we develop models of collaborative clustering realized over a collection of databases. The essence of a search for data structures carried out in this environment deals with a determination of crucial common relationships in databases. Depending upon a way in which databases are accessible and can collaborate, we distinguish between a vertical and horizontal collaboration. In the first case, the databases deal with objects defined in the same attribute (feature) space. The horizontal collaboration takes place when we deal with the same objects being defined in different attribute spaces and therefore giving rise to separate databases. We develop a new clustering architecture supporting the mechanisms of collaboration. It is based on a standard FCM (Fuzzy C – Means) method. When it comes to the horizontal collaboration, the clustering algorithms interact by exchanging information about "local" partition matrices. In this sense, the required communication links are established at the level of information granules (more specifically, fuzzy sets or fuzzy relations forming the partition matrices) rather than patterns (data points) that are directly available in the databases. We discuss how this form of collaboration helps meet requirements of data confidentiality. In case of the horizontal collaboration, the method operates at the level of the prototypes formed for each individual database or the induced partition matrices.*

1. Introduction

Undoubtedly, a distributed nature of data is inherent to most information systems. Intelligent agents and their collaboration over the Internet is an excellent testimony to such claim [1][9][13][14][18]. In many areas of everyday activity various databases are constructed, used and maintained independent from each other. In each local environment, one tries to make sense of data by engaging in various activities of data mining and data analysis. The obtained results can be useful to such local community yet they could be of significant interest to the others. This triggers interest in a collaborative effort where the data mining activities could exploit several databases and the ensuing results benefit a larger circle of users. While it sounds appealing, one has to remember that sharing data, especially those of more confidential nature, is a genuine obstacle. This matter has to be taken seriously when moving along any collaborative pursuit in data analysis.

This collaboration-driven task of data mining calls for an orchestrated effort and implies a highly collaborative nature of search for dependencies in data so that that such findings are common and relevant to all databases (as such discoveries of global character are of genuine interest). To shed light on the spectrum of the processing problems, we identify possible scenarios along with existing drawbacks and envision potential mechanisms of collaboration

- Search for a common structure in databases Within a given organizational structure (company, network of sales offices, etc.), there are several local databases of customers (e.g., each supermarket generates its own database or a sales office maintains a local database of its customers). Generally, we can assume that all databases have the same attributes (features) while each database consists of different objects (patterns). To derive some global relationships that are common to all these databases, we should allow the databases to collaborate at the level of the patterns. Quite commonly, we may not be permitted to have access to all databases but eventually could be provided with some general aggregates (say, some synthetic indexes describing data; a mean value or median are a good example in this case). Refer to Figure 1 that illustrates the underlying concept. Bearing this in mind, we can talk about *vertical* (data based) collaboration in the process of knowledge elicitation (that is revealing a common structure in the data).

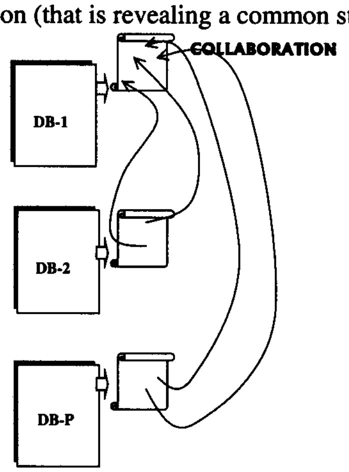

Figure 1. Vertical collaboration between databases at a local level; in each database objects are located in the same data space but deal with the different patterns

- Security issues and discovery of .data structures across different datasets. Consider now that information about the same group of clients is collected in different databases where an individual company (bank, store, etc.) builds its own database. Because of confidentiality and security requirements, the companies cannot share information about clients in a direct manner. However all of them are vitally interested in deriving some associations that help them learn about clients (namely, identifying their profiles and needs). As they are concerned with the same population of clients, we may anticipate that the basic structure of the population of such patterns, in spite of possible minor differences, should hold across all databases. The approach taken in this case would be to build clusters in each database and exchange information at the level of the clusters treated here as information granules. Subsequently, we allow all collaboration processes to be realized at this particular level. In this manner, the security issues are not compromised while a sound mechanism of collaboration/ interaction between the databases becomes established. Graphically, we can envision the situation of such collaboration as the one portrayed in Figure 2. Evidently, in this case we are concerned with a *horizontal* (that is feature-based) collaboration in the search for the data structure.

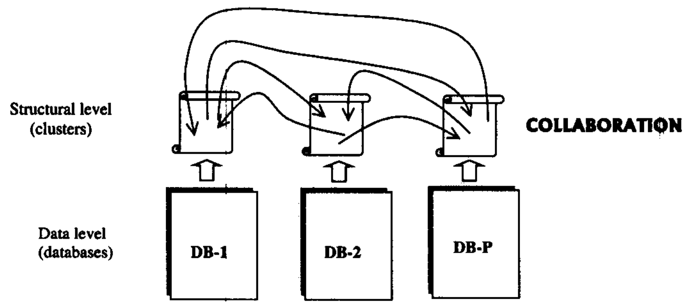

Figure 2. Collaboration between databases at the level of "local" structures (clusters) discovered there; note that no direct collaboration at the data level is allowed

As data structure elicitation is inherently user-oriented and user-friendly, we are interested in the collaborative clustering as its results are information granules. In the sequel, this gives rise to a certain type of collaboration as indicated before, namely a vertical collaborative clustering that involves databases involving various objects and horizontal clustering where we are faced with the same objects but being characterized by various attributes.

As far as the algorithmic issues are concerned, the underlying idea of collaboration dwells on a well-known Fuzzy C-Means (FCM) cf. [3]. The reader may refer to pertinent details as to the generic method that is used as a canvass of the collaborative schemes developed in the study. In general, we can think of clustering [1] [4][7][8][11][12] as a vehicle of forming information granules. It is also worth stressing that fuzzy clustering arose as a fundamental and highly appealing technique in construction of fuzzy models; refer e.g., to [5] [6] [7][15][16][17]. Moreover the collaborative clustering can be cast in the realm of intelligent agents cf. [ ] whose activities may center around discovering and sharing knowledge.

2. The horizontal collaborative clustering

In this section, we introduce all necessary notation, formulate the underlying optimization problem implied by the objective function – based clustering technique and derive the solution in a form of some iterative scheme.

2.1. The notation

In what follows, we consider "p" subsets of data located in different spaces (viz. the patterns there are described by different features). As each subset concerns the same patterns (that is each pattern results as a concatenation of the corresponding subpatterns), the number of elements in each subset is the same and equal to N. We are interested in partitioning the data into "c" fuzzy clusters. The result of clustering completed for each subset of data comes in the form of a partition matrix and a collection of prototypes. We use a bracket notation to identify the specific subset. Hence we use the notation U[ii] and **v**[ii] to denote the partition matrix and the i-th prototype produced by the clustering realized for the ii-th set of data. Similarly, the dimensionality of the patterns (number of their features) in each subset could be different; to underline this we use a pertinent index, say n[ii], ii=1, 2, …,p. The distance function between the i-th prototype and k-th pattern in the same set is denoted by $d_{ik}^2[ii]$, i=1, 2, …,c, k=1, 2, …,N. Again, the index used here underlines the fact that we are dealing with a certain data space pertinent to the ii-th data set (database). Moreover throughout the study, we confine ourselves to the weighted Euclidean distance of the form

$$d_{ik}^2[ii] = \| \mathbf{x}_k - \mathbf{v}_i[ii] \|_{ii} = \sum_{j=1}^{n[ii]} \frac{(x_{kj} - v_{ij}[ii])^2}{\sigma_j^2[ii]}$$

The objective function guiding the formation of the clusters that is completed for each subset assumes a well-known form as being encountered in the standard FCM algorithm

$$\sum_{k=1}^{N}\sum_{i=1}^{c} u_{ik}^2[ii] d_{ik}^2[ii]$$

ii=1,2, …,p. The collaboration between the subsets is established through a matrix of connections (interaction coefficients or interactions, for brief).Each entry of the collaborative matrix states describes an intensity of the interaction. In general, α[ii, kk] assumes nonnegative values. The higher the value of the interaction coefficient, the stronger the collaboration between the corresponding subsets. To accommodate the collaboration effect in the optimization process, the objective function is expanded into the form

$$Q[ii] = \sum_{k=1}^{N}\sum_{i=1}^{c} u_{ik}^2[ii] d_{ik}^2[ii] + \sum_{\substack{jj=1 \\ jj\neq ii}}^{P} \alpha[ii, jj] \sum_{k=1}^{N}\sum_{i=1}^{c} \{u_{ik}[ii] - u_{ik}[jj]\}^2 d_{ik}^2[ii]$$

ii=1, 2, …, p. The role of the second term standing in the above expression is to make the clustering based on the ii-th subset "aware" of the other partitions. It becomes obvious that if the structures in all datasets are similar then the differences between the partition matrices tend to be lower. On the other hand, if we encounter higher differences, we anticipate that the collaboration will be able to address these needs.

As usual, we require that the partition matrix satisfies 'standard" requirements of membership grades summing to 1 for each patterns and the membership grades contained in the unit interval. All in all, the collaborative clustering converts into the following family of "p" optimization problems with membership constraints

Min Q[ii] subject to U[ii] $\in$ $\mathcal{U}$[ii]

where $\mathcal{U}$[ii] is a family of all fuzzy partition matrices, namely

$$\mathcal{U}[ii] = . \{u_{ik}[ii] \in [0,1] \mid \sum_{i=1}^{c} u_{ik}[ii] = 1 \text{ for all k and } 0 < \sum_{k=1}^{N} u_{ik}[ii] < N \text{ for i}\}.$$

The minimization is carried out with respect to the fuzzy partition and the prototypes. This problem and its solution are discussed in detail in the ensuing section.

2.2. Optimization details of the collaborative clustering

The above optimization task splits into two problems, namely a determination of the partition matrix U[ii] and the prototypes $\mathbf{v}_1$[ii], $\mathbf{v}_2$[ii], …, $\mathbf{v}_c$[ii]. These problems are solved separately for each of the collaborating subsets of patterns.To determine the partition matrix, we exploit a technique of Lagrange multipliers so that the constraint occurring in the problem becomes integrated as a part of the objective function considered in the constraint-free optimization. The objective function V[ii] that is considered for each "k" separately comes in the form

$$V[ii] = \sum_{i=1}^{c} u_{ik}^2[ii] d_{ik}^2[ii] + \sum_{\substack{jj=1 \\ jj\neq ii}}^{P} \alpha[ii, jj] \sum_{i=1}^{c} \{u_{ik}[ii] - u_{ik}[jj]\}^2 d_{ik}^2[ii] - \lambda(\sum_{i=1}^{c} u_{ik}[ii] - 1)$$

where λ denotes a Lagrange multiplier. The necessary conditions leading to the local minimum of V[ii] read as follows

$$\frac{\partial V[ii]}{\partial u_{st}[ii]} = 0, \quad \frac{\partial V[ii]}{\partial \lambda} = 0$$

s = 1, 2, …,c, t = 1, 2, …,N. Completing quire lengthy calculations, we arrive at the following formula for the partition matrix

$$u_{st}[ii] = \frac{\varphi_{st}[ii]}{1+\psi[ii]} + \frac{1}{\sum_{j=1}^{c}\frac{d_{st}^2}{d_{jt}^2}}[1-\sum_{j=1}^{c}\frac{\varphi_{jt}[ii]}{1+\psi[ii]}]$$

with the auxiliary notation

$$\varphi_{st}[ii] = \sum_{\substack{jj=1\\ jj\neq ii}}^{P} \alpha[ii,jj]u_{st}[jj]$$

and

$$\psi[ii] = \sum_{\substack{jj=1\\ jj\neq ii}}^{P} \alpha[ii,jj]$$

In the calculations of the prototypes we use explicitly the weighted Euclidean distance between the patterns and the prototypes. The necessary condition for the minimum of the objective function is of the form $\nabla_{v[ii]}Q = 0$. The resulting prototypes are equal to

$$v_{st}[ii] = \frac{A_{st}[ii] + C_{st}[ii]}{B_s[ii] + D_s[ii]}$$

s=1,2, …, c, t=1, 2, …, n[ii], ii=1, 2, …P

The coefficients in the above expression are as follows

$$A_{st}[ii] = \sum_{k=1}^{N} u_{sk}^2[ii]x_{kt}[ii]$$

$$B_s[ii] = \sum_{k=1}^{N} u_{sk}^2[ii]$$

$$C_{st}[ii] = \sum_{\substack{jj=1\\ jj\neq ii}}^{P} \alpha[ii,jj]\sum_{k=1}^{N}(u_{sk}[ii]-u_{sk}[jj])^2 x_{kt}[ii]$$

$$D_s[ii] = \sum_{\substack{jj=1\\ jj\neq ii}}^{P} \alpha[ii,jj]\sum_{k=1}^{N}(u_{sk}[ii]-u_{sk}[jj])^2$$

(note that $\mathbf{x}_k[ii]$ denotes a k-th pattern coming from the ii-th subset of patterns).

2.3. The detailed clustering algorithm

The general clustering scheme consists of two phases:

- generation of clusters without collaboration. This phase involves the use of the FCM algorithm applied individually to each subset of data. Obviously, the number of clusters needs to be the same for all the datasets. During this phase we seek independently a structure in each subset of data
- collaboration of the clusters. Here we start with the already computed partition matrices, set up the collaboration level (through the values of the interaction coefficients arranged in α[ii,jj]) and proceed with a simultaneous optimization of the partition matrices

Moving on to the formal algorithm, the computational details are organized in the following way

<u>Given</u>: subsets of patterns $\mathbf{X}_1$, $\mathbf{X}_2$, …, $\mathbf{X}_P$

Select: distance function, number of clusters (c), termination criterion, and collaboration matrix α[ii,jj].
Initiate randomly all partition matrices U[1] , U[2], …, U[p]
Phase I

For each data
repeat
compute prototypes {v_i[ii] }, i=1, 2, …,c and partition matrices U[ii] for all subsets of patterns
until a termination criterion has been satisfied
Phase II
repeat
For the given matrix of collaborative links α[ii,jj] compute prototypes and partition matrices U[ii] using (4) and(7)
until a termination criterion has been satisfied

The termination criterion relies on the changes to the partition matrices obtained in successive iterations of the clustering method, for instance a Tchebyschev distance could serve as a sound measure of changes in the partition matrices. Subsequently, when this distance is lower than an assumed threshold value (ε > 0), the optimization is terminated.

3. Vertical collaborative clustering

As already discussed, the vertical collaborative clustering is concerned with a collection of databases involving different patterns defined in the same feature space so that the patterns do not repeat across the databases. As the feature space is common throughout the databases we can use prototypes as a means of facilitating the collaboration between the databases. The detailed algorithm discussed in the next section concentrates on this form of collaboration.

3.1. The algorithm

We start with an introduction of the objective function that takes into account the vectors of prototypes specific for each database. With the same notation as before, the objective function is given as

$$Q[ii] = \sum_{i=1}^{c} \sum_{i=1}^{N[ii]} u_{ik}^2[ii] d_{ik}^2 + \sum_{\substack{jj=1 \\ jj \neq ii}}^{P} \beta[ii.jj] \sum_{k=1}^{N[ii]} \sum_{i=1}^{c} u_{ik}^2[ii] \| \mathbf{v}_i[ii] - \mathbf{v}_i[jj] \|^2$$

where β[ii,jj] (> 0) describes a level of collaboration between the datasets and || || denotes a distance function between the prototypes. The optimization of (15) is carried out for the partition matrix U[ii] and the prototypes of the clusters **v**[ii]. This implies two separate optimization problems where the first one involving the partition matrix is subject to constraints. Not including all computational details, the final expression governing computations of the partition matrix reads in the form

$$u_{st} = \frac{1}{\sum_{j=1}^{c} \frac{D_{st}^2}{D_{jt}^2}}$$

t=1, 2,…, N[ii], s=1,2,…,c where D_{st} is computed as follows

$$D_{st}^2 = d_{st}^2 + \sum_{\substack{jj=1 \\ jj \neq ii}}^{P} \beta[ii, jj] \| \mathbf{v}_s[ii] - \mathbf{v}_s[jj] \|^2$$

The prototypes are given by the expression

$$v_{st}[ii] = \frac{F_{st}[ii] - A_{st}[ii]}{C_{st}[ii] - B_{st}[ii]}$$

s=1, 2, …, c, t=1,2,…,n with the following concise notation

$$A_{st}[ii] = \sum_{k=1}^{N[ii]} u_{sk}^2[ii] x_{kt}[ii]$$

$$B_{st}[ii] = \sum_{k=1}^{N[ii]} u_{sk}^2[ii]$$

$$C_{st}[ii] = \sum_{\substack{jj=1 \\ jj \neq ii}}^{P} \beta[ii, jj] \sum_{k=1}^{N[ii]} u_{sk}^2[ii]$$

$$F_{st}[ii] = \sum_{\substack{jj=1 \\ jj \neq ii}}^{P} \beta[ii, jj] \sum_{k=1}^{N[ii]} u_{sk}^2[ii] v_{st}[jj]$$

The overall computing scheme can be presented in the following fashion

Given: subsets of patterns $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_P$ located in the same feature space
Select: distance function, number of clusters (c), termination criterion, and collaboration matrix β[ii,jj].
Initiate randomly all partition matrices U[1] , U[2], …, U[p]
Phase I

For each data
repeat
compute prototypes $\{\mathbf{v}_i[ii]\}$, i=1, 2, …,c and partition matrices U[ii] for all subsets of patterns
until a termination criterion has been satisfied

Phase II
repeat
For the given matrix of collaborative links β[ii,jj] compute prototypes and partition matrices U[ii] using (19) and (16)
until a termination criterion has been satisfied

The vertical collaboration could be realized not only by the prototypes as discussed above but we may establish another vehicle of communication coming in the form of so-called induced partition matrices. The crux of this collaboration is as follows. Consider the prototypes of the clusters located in the data space of the jj-th database, say $\mathbf{v}_s[jj]$, s=1, 2, …,c. Now let us position these prototypes in the data space of the ii-th database. For any element $\mathbf{x}_t$ in this data space (t=1, 2,…N[ii]), we can compute *induced* membership grades (viz the grades being induced by the prototypes from the different space) in the form

$$u_{st}^{\sim}[ii][jj] = \frac{1}{\sum_{j=1}^{c} \frac{d_{st}^{\sim}[ii][jj]}{d_{jt}^{\sim}[ii][jj]}}$$

where the distance $\|.\|_{ii}$ is computed in the ii-th space (and this fact is clearly identified by the corresponding subscript), namely

$$d_{st}^{\sim}[ii][jj] = \| \mathbf{x}_t - \mathbf{v}_s[jj] \|_{ii}^2$$

Now the objective function for the ii-th dataset can be written down in the form

$$Q = \sum_{i=1}^{c} \sum_{k=1}^{N[ii]} u_{ik}^2[ii] d_{ik}^2[ii] + \sum_{\substack{jj \\ jj \neq ii}}^{P} \alpha[ii, jj] \sum_{i=1}^{c} \sum_{k=1}^{N[ii]} (u_{ik}[ii] - u_{ik}^{\sim}[ii][jj])^2 d_{ik}^2[ii]$$

with the notation introduced above.

The standard optimization requires two steps, that is the calculations of the partition matrix and the prototypes. Let us start with the partition matrix. Recalling that this implies a constrained optimization, we use Lagrange multipliers that place the standard identity constraint as a part of the objective function, that is

$$V=\sum_{i=1}^{c} u_{ik}^{2}[ii]d_{ik}^{2}[ii]+\sum_{\substack{jj\\ jj\neq ii}}^{P}\alpha[ii,jj]\sum_{i=1}^{c}(u_{ik}[ii]-\tilde{u}_{ik}[ii][jj])^{2}d_{ik}^{2}[ii]+\lambda(1-\sum_{i=1}^{c}u_{ik}[ii])$$

for all k=1,2,..., N[ii]. The partition matrix is then equal to

$$u_{st}[ii]=\frac{1-\sum_{i=1}^{c}\frac{F_{it}[ii]}{D_{it}[ii]}}{\sum_{i=1}^{c}\frac{D_{st}[ii]}{D_{it}[ii]}}+\frac{F_{st}[ii]}{D_{st}[ii]}$$

s=1,2, ..., c, t=1, 2, ..., N[ii] with the following notation

$$D_{st}[ii]=2d_{st}^{2}[ii](1+\sum_{\substack{jj=1\\ jj\neq ii}}^{P}\alpha[ii,jj])$$

$$F_{st}[ii]=2d_{st}^{2}\sum_{\substack{jj=1\\ jj\neq ii}}^{P}\alpha[ii,jj]\tilde{u}_{st}[ii][jj])d_{st}^{2}$$

Proceeding with the computations of the prototypes, let us introduce the notation

$$A=\sum_{k=1}^{N[ii]}u_{sk}^{2}[ii]x_{kt}[ii]$$

$$B=\sum_{k=1}^{N[ii]}u_{sk}^{2}[ii]$$

$$C=\sum_{\substack{jj=1\\ jj\neq ii}}^{P}\alpha[ii,jj]\sum_{k=1}^{N[ii]}(u_{sk}[ii]-\tilde{u}_{sk}[ii][jj])^{2}x_{kt}[ii]$$

$$D=\sum_{\substack{jj=1\\ jj\neq ii}}^{P}\alpha[ii,jj]\sum_{k=1}^{N[ii]}(u_{sk}[ii]-\tilde{u}_{sk}[ii][jj])^{2}$$

and finally

$$v_{st}[ii]=\frac{A+C}{B+D}$$

Let us underline that this type of vertical collaboration occurs in the more abstract space of the information granules (partition matrices) than the previous variant of the collaboration.

4. Concluding remarks

We have introduced an idea of collaborative processing, in general and collaborative clustering, in particular. It has been shown that a communication and collaboration between separate datasets can be effectively realized at the more abstract level of membership grades (partition matrices) and prototypes. Two types of collaboration (vertical and horizontal) were studied in detail. We provided a complete clustering algorithm by dwelling the method on the standard FCM method. The quantification of the collaboration effect can be realized either at the level of the prototypes or the partition matrices. An interesting expansion of the method discussed here involves a partial (limited) collaboration where not all patterns are available to form a collaborative link. This simply calls for an extra Boolean vector $\mathbf{b} = [b_1\ b_2 \ldots b_N]$ modifying the objective function in the form

$$Q[ii] = \sum_{k=1}^{N} \sum_{i=1}^{c} u_{ik}^2[ii] d_{ik}^2[ii] + \sum_{\substack{jj=1 \\ jj \neq ii}}^{P} \alpha[ii, jj] \sum_{k=1}^{N} \sum_{i=1}^{c} \{u_{ik}[ii] - u_{ik}[jj]\}^2 b_k d_{ik}^2[ii]$$

where b_k assumes 1 when the k-th pattern is available for collaboration (otherwise b_k is set to 0).

In general, we can envision the collaboration mechanism to take place both at the vertical (data) as well as horizontal (feature) level, see Figure 15. In terms of the objective function, this approach merges the two methods introduced before.

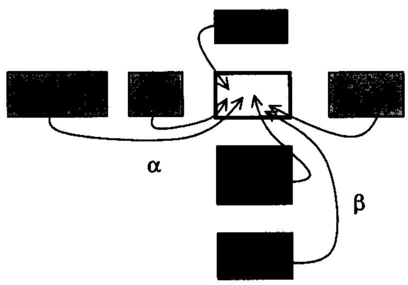

Figure 15. Vertical and horizontal mode of collaboration between databases

As a matter of fact, we can put down the following expression to emphasize the collaboration mechanism being in place

$$U[ii] = F(\mathbf{U}[jj], \mathbf{v}[jj])$$

where $\mathbf{U}$ and $\mathbf{v}$ are used to here denote the information feedback of the other part of the system (both vertical and horizontal)

The approach presented here could be easily generalized to support more specific ideas such as rule-based systems. In this case, we are concerned with the reconciliation of rules in each subset of data. Obviously, the optimization details need to be refined, as the specificity of the problem requires further in-depth investigations of a number of issues related to rules such as their specificity, consistency and completeness.

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