Pseudo-fuzzy Linear Inequation and Equation

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Abstract:

In this paper, an new concept of pseudo-fuzzy linear function is suggested. The elementary properties of this kind functions are discussed. Inequation and equation of pseudo- fuzzy linear function are considered in detail.

Keywords:

PFL-function, PFL-inequation, PFL-equation

1. Pseudo-fuzzy linear function and its properties

In the analysis of fuzzy logic electric circuit which include amplifiers, the function has the following characteristics:

(a) The independent variables are fuzzy value variables, produced by a constant number, then make logic calculating.

(b) The value domain is any internal other than [0,1]. This kind of functions first meet in the reference [1]. As the independent variables take value in {0,1}, then this function just is the so-called pseudo-Boole function[2].

Definition 1. Let $F:[0,1]^n \rightarrow R$ be a mapping with fuzzy variables, it expresses as

$$F(x_1,x_2,\cdots,x_n)=\bigvee_{i=1}^{n}(a_i x_i) \tag{1}$$

where x_i is a fuzzy variable, and a_i is a constant positive number, then $F(x_1,x_2,\cdots,x_n)$ is called a pseudo- fuzzy linear function with n variables, denoted as PFL-function.

It is obviously that, F has the following elementary properties.

Theorem 1. PFL-function $F(x_1,x_2,\cdots,x_n)$ is consistent increasing with fuzzy vector $X=(x_1,x_2,\cdots,x_n)$,That is, if $X=(x_1,x_2,\cdots,x_n)$,$Y=(y_1,y_2,\cdots, y_n)$,and $X\leqslant Y$, then

$$F(x_1,x_2,\cdots,x_n)\leqslant F(y_1,y_2,\cdots,y_n) \tag{2}$$

Theoerm 2. PFL-function $F(x_1, x_2, \cdots, x_n)$ is a boundary function, and satisfies

$$0\leqslant F(x_1,x_2,\cdots,x_n)\leqslant M = \bigvee_{i=1}^{n} a_i \tag{3}$$

Theorem 3. PFL-function $F(x_1,x_2,\cdots,x_n)$ is a continuous function.

The proofs of the above three theorems are obviously, hence omit.

2. Pseudo-fuzzy linear inequation

Definition 2. Let $F(x_1,x_2,\cdots,x_n)$ be a PFL-function, λ is a real number, the following inequation

$$F(x_1,x_2,\cdots,x_n)\leqslant \lambda \tag{4}$$

is called a pseudo-fuzzy linear inequation,denoted as PFL-inequation. If $X=(x_1,x_2,\cdots,x_n)$ satisfies(4), then X is called a solution of PFL-inequation (4).

We can easily imply the below judgment theorem by using the properties of PFL-function.

Theorem 4. The solutions of PFL-inequation(4) satisfy

(i) If $\lambda \geqslant M$, then $X=[0,1]^n$. it is called full-solution;

(ii) If $0 \leqslant \lambda \leqslant M$, then PFL-inequation(4) has solutions;

(iii) If $\lambda < 0$,then $X=\Phi$ .it is called no-solution.

where $M = \bigvee_{i=1}^{n} a_i$

From the definition of generalized logic calculation we have the following equal solution theorem.

Theorem 5. PFL-inequation(4) has the same solution as the following inequation system:

$$a_i x_i \leqslant \lambda \qquad i=1,2,\cdots,n \tag{5}$$

Theorem 5 gives the solution structure of PFL-inequation(4).

From (5) we know the solutions

$$x_i = \begin{cases} [0, \lambda / a_i], & a_i > \lambda \\ [0,1], & a_i \le \lambda \end{cases} \tag{6}$$

Example 1. Solving PFL-inequation

$$5x_1 \vee 8x_2 \vee 6x_3 \leqslant 7$$

Solution. From theorem 5 we directly have

$$X=[0,1] \times [0, 7/8] \times [0,1]$$

3. Pseudo-fuzzy linear equation

Definition 3. Let $F(x_1,x_2,\cdots,x_n)$ be a PFL-function, λ is a real number, then we call the following equation

$$F(x_1,x_2,\cdots,x_n) = \lambda \tag{7}$$

a pseudo-fuzzy linear equation, denoted as PFL-equation, If $X \in [0,1]^n$ satisfies the equation (7), then X is called a solution of PFL-equation(7).

From the properties of generalized fuzzy logic calculation we have the following equal solution theorem.

Theorem 6. PFL-equation (7) has the same solutions as the following simple system $a_i x_i \le \lambda$, $i = 1,2,\cdots,n$

$$\begin{cases} a_i x_i \le \lambda \ , \ i = 1,2,\cdots,n \\ a_i x_i = \lambda \ , \ for\ some\ j \end{cases} \tag{8}$$

Theorem 6 implies the structure of solutions of PFL-equation (7) , that is the following solution structure theorem.

Theorem 7. The solutions of PFL-equation (7) satisfy the following conclusions:

(i) If $\lambda < 0$ or $\lambda > M$,then $X=\Phi$;

(ii) If $0 \leqslant \lambda \leqslant M$, and there are a_{jk} satisfy $a_{jk} > \lambda$, then jk-th branch solution is

$$X_{jk} = [0,1]^{jk-1} \times [0, \lambda / a_{jk}] \times [0,1]^{n-jk} \qquad j,k=1,2,\cdots,m \tag{9}$$

then the solution of PFL-equation(7) is

$$X = \bigcup_{j,k=1}^{m} X_{jk} \tag{10}$$

where $M = \bigvee_{i=1}^{n} a_i$, where X_{jk} is called jk-th branch solution of PFL-equation(7).

Example 2. Soving the following PFL-equation

$$5x_1 \vee 8x_2 \vee 9x_3 \vee 3x_4 = 6 \tag{11}$$

Solution. From theorem 7 we know that, there are two branch solutions of PFL-equation(10), using the formal(8) we calculate the branch-solutions.

$$X_1 = [0,1] \times 3/4 \times [0,1]^2 \qquad X_2 = [0,1]^2 \times 2/3 \times [0,1]$$

Therefore the solution of PFL-equation(10) is

$$X = X_1 \cup X_2 = [0,1] \times 3/4 \times [0,1]^2 \cup [0,1]^2 \times 2/3 \times [0,1]$$

References

[1] Liu Feng-qi, Fuzzy mathematics and its applications,Science press,Beijing,1988,120-149.

[2] Liu Ying-cai, Theory of Boolean methods, Scientific and technological literature press,Shanghai,1993,130-159.

[3] Wang Peizhuang, Theory of fuzzy set and its application, Scienceand techonology press,Shanghai,1983,109-123.