On fuzzy weakly semi-precontinuous multifunctions

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Abstract: This paper is devoted to the introduction and study of fuzzy lower and upper weakly semi-precontinuous multifunctions between a general topological space (X,T) and a fuzzy topological space (Y,T_1) .Using the mapping F_* and F^* several equivelent conditions of a lower and upper fuzzy weakly semi-precontinuous multifunction.

Key words: Fuzzy semi-preopen (semi-preclosed) sets; Fuzzy lower and upper semi-precontinuous multifunctions; Fuzzy lower and upper weakly semi-precontinuous multifunctions.

1. Preliminaries

Throughout the paper , by (X,T) or simply by X we will mean a topological space in classical sense, and (Y,T_1) or simply by Y will stand for a fuzzy topological space (fts, for short) as defined by Chang's [1]. Fuzzy sets in Y will be denoted A,B,U,V etc. and interior(semi-interior) and closure (semi-closure) and complement of a fuzzy set A in an fts Y will be denoted by $Int\,A$ $(SIntA)$ $Cl\,A\,(SClA)$ and $1-A$ respectively. A fuzzy set A of fts X is called fuzzy preopen set (fuzzy preclosed set) if $A \le IntClA(A \ge IntClA)$; A fuzzy set A of fts X is called semi-preopen set (fuzzy semi-pre closed set) if there a fuzzy preopen set U such that $U \le A \le ClU$ ($IntU \le A \le U$). Let A be any fuzzy set a fts X ,then fuzzy semi-preclosure ($SPcl\,A$, for short) and fuzzy semi-preinterior ($SP\text{int}\,A$, for short) of A are defined as follows : $SPcl\,A = \cap\{B \mid B$ is fuzzy semi-pre interior and $A \le B\}$ $SP\text{int}\,A = \Upsilon\{B \mid B$ is fuzzy semi-preopen and $B \le A\}$. when a fuzzy set A is quasi-coincident (q-coincident, for short) with a fuzzy set B in (Y,T_1), we shall write $A\,\text{q}\,B$, if A and B are not q-coincident, denoted by $A\,\overline{\text{q}}\,B$. The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd', and 'fts' respectively.

Definition 1.1. Let (X,T) be a topological space in the classical sense and

(Y,T_1) be a fuzzy topological space. $F:X\to Y$ is called a fuzzy multifunction iff for each $x\in X$, $F(x)$ is a fuzzy set in Y.

Definition 1.2. For a fuzzy multifunction $F:X\to Y$, the $F_*(A)$ and $F^*(A)$ of a fuzzy set A in Y are defined as follows:

$$F_*(A)=\{x\in X:\ F(x)\mathrm{q}A\},$$
$$F^*(A)=\{x\in X:\ F(x)\leq A\}$$

Theorem 1.3.[4] For a fuzzy multifunction $F:X\to Y$ we have $F_*(1-A)=X-F^*(A)$, for any fuzzy set A in Y.

Definition 1.4. For a fuzzy multifunction $F:X\to Y$ is called :

(a) fuzzy lower semi-precontinuous (f.l.s.p.c., in short) at a point $x_0\in X$ iff for every fuzzy open set V in Y will $x_0\in F_*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U\subset F_*(V)$,

(b) fuzzy upper semi-precontinuous (f.w.s.p.c., in short) at a point $x_0\in X$ iff for every fuzzy open set V in Y will $x_0\in F^*(V)$), there exists a semi-preopen nbd U of x_0 in X such that $U\subset F^*(V)$,

(c) f.l.s.p.c. (f.u.s.p.c.) on X iff it is respectivety so at each $x_0\in X$,

(d) f.s.p.c. on X iff it is f.l.s.p.c. and f.u.s.p.c. .

Definition 1.5. For a fuzzy multifunction $F:X\to Y$ is called :

(a) fuzzy lower weakly semi-continuous (f.l.w.s.c., in short) at a point $x_0\in X$ iff for every fuzzy open set V in Y will $x_0\in F_*(V)$, there exists a semi-open nbd U of x_0 in X such that $U\subset F_*(SClV)$,

(b) fuzzy upper weakly semi-continuous (f.u.w.s.c., in short) at a point $x_0\in X$ iff for every fuzzy open set V in Y will $x_0\in F^*(V)$, there exists a semi-open nbd U of x_0 in X such that $U\subset F^*(SClV)$,

(c) f.l.w.s.c. (f.u.w.s.c.) on X iff it is respectivety so at each $x_0\in X$,

(d) f.w.s.c. on X iff it is f.l.w.s.c. and f.u.w.s.c. .

2. Lower and upper weakly semi-precontinuous fuzzy multifunctions

Definition 2.1. For a fuzzy multifunction $F:X\to Y$ is called :

(a) fuzzy lower weakly semi-precontinuous (f.l.w.s.p.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F_*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U \subset F_*(SClV)$,

(b) fuzzy upper weakly semi-precontinuous (f.u.w.s.p.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F^*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U \subset F^*(SClV)$,

(c) f.l.w.s.p.c. (f.u.w.s.p.c.) on X iff it is respectivety so at each $x_0 \in X$,

(d) f.w.s.p.c. on X iff it is f.l.w.s.p.c. and f.u.w.s.p.c. .

Remark 2.2. It is clear from Difinition 2.1 and 1.4,1.5 that every fuzzy semi-continuous (weakly semi-continuous, almost semi-continuous [7], weakly semi-continuous [8], semi-precontinuous [9]) multifunctions is f.w.s.p.c. where as the converse is not true.

Theorem 2.3. Let $F: X \to Y$ is f.l.w.s.p.c.(f.u.w.s.p.c.) iff for every fuzzy open set V in Y, $F_*(V) \subset SP\operatorname{int} F_*(SClV)$ (respectively, $F^*(V) \subset SP\operatorname{int} F^*(SClV)$).

Proof. we prove the theorem for the case of f.l.w.s.c. of F only; there other case is quite similar.

Let F be f.l.w.s.p.c. and V be f.open in Y. If $x \in F_*(V)$, then there exists semi-preopen nbd U of x in X such that $U \subset F_*(SClV)$ and hence $U \subset SP\operatorname{int} F_*(SClV)$.

Conversely, let $x \in X$ and V be fuzzy open set in Y such that $x \in F_*(V)$. Then $x \in F_*(V) \subset SP\operatorname{int} F_*(SClV) = U$ (say). Thus U is a semi-preopen nbd of x such that $U \subset F_*(SClV)$ and consequently, F is f.l.w.s. p.c. .

Theorem 2.4. If $F: X \to Y$ is f.l.w.s.p.c., for every fuzzy pre-semiopen set V in Y, $F_*(V) \subset SP\operatorname{int} F_*(SClV)$.

Proof. Let F be f.l.w.s.p.c. and V be any fuzzy pre-semiopen set in Y. For any $x \in F_*(V)$, then there exists semi-preopen nbd U_x of x in X such that $U_x \subset F_*(SClIntSCl\, V) \subset F_*(SClV)$ and hence $F_*(V) \subset \cup\{U_x : x \in F_*(V)\} \subset F_*(SClV)$, and hence $F^*(V) \subset SP\operatorname{int} F^*(SClV)$.

Theorem 2.5. If $F: X \to Y$ is f.u.w.s.p.c., for every fuzzy pre-semiopen set V in Y, $F^*(V) \subset SP\operatorname{int} F^*(SClV)$.

Proof. Similar to Theorem 2.4. and omitted .

Theorem 2.6. If a fuzzy multifunction $F: X \to Y$ is f.l.w.s.p.c. on X, then $SPcl\ F_*(V) \subset F_*(SClV)$ for any fuzzy open set V in Y.

Proof. Let $x \notin F_*(SClV)$, where V is a fuzzy open set in Y. Then $F(x)$ $\bar{q}\, SPcl\ V$ so $F(x) \leq 1 - SPcl\ V$. By fuzzy lower weakly semi-precontinuity of F, there exists a semi-preopen nbd U of x in X such that $F(z) \leq SPcl(1 - SCl\ V) \leq 1 - V$ for all $z \in U$. Thus $F(z)$ $\bar{q} V$, i.e $z \notin F_*(V)$, for all $z \in U$, hence $U = I\ F_*(V) = \varnothing$. Since U is a semi-preopen nbd of x, it then follows that $x \notin F_*(SClV)$.

Theorem 2.7. If a fuzzy multifunction $F: X \to Y$ is f.u.w.s.p.c. on X, then $SPcl\ F^*(V) \subset F^*(SClV)$ for any fuzzy open set V in Y.

Proof. Similar to Theorem 2.6. and omitted .

Theorem 2.8. Let $F: X \to Y$ is fuzzy multifunctions then following are equivalent:

(a) F is f.l.w.s.p.c.,

(b) for any fuzzy open set V in Y, $F_*(V) \subset SP\operatorname{int} F_*(SClV)$ and $F^*(V) \subset F^*(SClV)$.

Proof. Evident.

Theorem 2.9. Let $F: X \to Y$ is fuzzy lower weakly semi-precontinuity if for each fuzzy open set V, $F_*(ClV)$ is fuzzy semi-preopen set.

Proof. Straightforward.

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