Homomorphism and Isomorphism of the Intuitionistic Fuzzy Normal Subgroups *

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Abstract In this paper , in the sense of homomorphism and ismorphism between two classical groups , we study the image , the preimage and the inverse mapping of the intuitionistic fuzzy normal subgroups defined by [3]

Keywords Intuitionistic fuzzy normal subgroups , homomorphism, isomorphism.

1 Preliminaries

Definition 1.1[1] Let X be a nonempty classical set. The triad formed as $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ on X is called an intuitionistic fuzzy set on X . Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively , and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$.

All of the intuitionistic fuzzy sets on X are written as $IFS[X]$ for short.

Definition 1.2[2] Let X and Y be nonempty classical sets , $f : X \to Y$ be a mapping, $B = \{< y, \mu_B(y), \nu_B(y) > | y \in Y\}$ be an intuitionistic fuzzy set on Y , i.e., $B \in IFS[Y]$. $F_f^{-1} : IFS[Y] \to IFS[X]$ is the inverse mapping induced by f , the preimage $F_f^{-1}(B)$ of B is an intuitionistic fuzzy set on X . Here , we define

$$F_f^{-1}(B) = \{< x, F_f^{-1}(\mu_B)(x), F_f^{-1}(\nu_B)(x) > | x \in X\}$$

Where $F_f^{-1}(\mu_B), F_f^{-1}(\nu_B)$ obey the classical extension principle of L.A. Zadeh.

Definition 1.3[2] Let X and Y be nonempty classical sets , $f : X \to Y$ be a mapping. If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \in IFS[X]$, $F_f : IFS[X] \to IFS[Y]$ is the mapping induced by f , then

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the image $F_f(A)$ of A is and intuitionistic fuzzy set on Y , and define

$$F_f(A) = \{< y, F_f(\mu_A)(y), \hat{F}_f(\nu_A)(y) > \mid y \in Y\}$$

where

$$F_f(\mu_A)(y) = \begin{cases} \sup\{\mu_A(x) \mid f(x) = y, x \in X\}, & f^{-1}(y) \neq \phi \\ 0 & f^{-1}(y) = \phi \end{cases}$$

$$\hat{F}_f(\nu_A)(y) = \begin{cases} \inf\{\nu_A(x) \mid f(x) = y, x \in X\}, & f^{-1}(y) \neq \phi \\ 1 & f^{-1}(y) = \phi \end{cases}$$

The above definitions are together called the extension principle of the intuitionistic fuzzy sets , the extension principle for short.

Definition 1.4[2] Let G be a classical group, then the intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in G\}$ on G is called an intuitionistic fuzzy subgroup on G , if the follouing conditions are satisfied.

(1) $\mu_A(xy) \geqslant \min\{\mu_A(x), \mu_A(y)\}$, $\nu_A(xy) \leqslant \max\{\nu_A(x), \nu_A(y)\}$; for all $x, y \in G$

(2) $\mu_A(x^{-1}) \geqslant \mu_A(x)$, $\nu_A(x^{-1}) \leqslant \nu_A(x)$, for all $x \in G$

All of the intuitionistic fuzzy subgroups on G are denoted as $IFS[G]$ for short.

Definition 1.5[3] Let G be a clasical group, $A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in G\}$ be an intuitionistic fuzzy subgroup on G , then A is called and intuitionistic fuzzy mormal subgroup on G , if the fuzzy normality is satisfied: i.e. , $\mu_A(xyx^{-1}) \geqslant \mu_A(y), \nu_A(xyx^{-1}) \leqslant \nu_A(y)$. for all $x, y \in G$.

All of the intuitionistic fuzzy normal subgroups on G are denoted as $IFNS[G]$ for short.

Theorem 1.1[2] Let G_1 and G_2 be classical groups, $f: G_1 \to G_2$ be a homomorphic mapping. If $A \in IFS[G_1], B \in IFS[G_2]$, then $F_f(A) \in IFS[G_2], F_f^{-1}(B) \in IFS[G_1]$.

Theorem 1.2[2] Let G_1 and G_2 be classical groups , $f: G_1 \to G_2$ be a homomorphic mapping , $A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in G\} \in IFS[G_1]$, define A^{-1} : $\mu_{A^{-1}}(x) = \mu_A(x^{-1}), \nu_{A^{-1}}(x) = \nu_A(x^{-1})$ for arbitrary $x \in G$, then (1) $A^{-1} \in IFS[G_1]$ (2) $F_f(A^{-1}) = (F_f(A))^{-1}$.

2 Main results

Theorem 2.1 Let G_1 and G_2 be classical groups, $f: G_1 \to G_2$ be a surjective homomorphism mapping. If $A \in IFNS[G_1]$, then $F_f(A) \in IFNS[G_2]$.

Proof By theorem 1.1[2], clearly, we have $F_f(A) \in IFS[G_2]$. So we need only prove the fuzzy normality.

On the hand, for arbitrary $y_1, y_2 \in G_2$, by the extension principle, $f: G_1 \to G_2$ is a surjective homomorphism mapping, i.e., $f(G_1) = G_2$. then, for all $y_1, y_2 \in G_2$, $f^{-1}(y_1) \neq \phi, f^{-1}(y_2) \neq \phi, f^{-1}(y_1 y_2 y_1^{-1}) \neq \phi$, and we have

$$F_f(\mu_A)(y_1 y_2 y_1^{-1}) = \sup_{z \in f^{-1}(y_1 y_2 y_1^{-1})} \mu_A(z)$$

$$F_f(\mu_A)(y_2) = \sup_{z \in f^{-1}(y_2)} \mu_A(z)$$

For $\forall x_2 \in f^{-1}(y_2), \forall x_1 \in f^{-1}(y_1)$, then $x_1^{-1} \in f^{-1}(y_1^{-1})$, since $A \in IFNS[G_1]$.

We get $\mu_A(x_1 x_2 x_1^{-1}) \geqslant \mu_A(x_2)$

As f is a homomorphic mapping

Thus $f(x_1 x_2 x_1^{-1}) = f(x_1)f(x_2)f(x_1^{-1}) = f(x_1)f(x_2)(f(x_1))^{-1} = y_1 y_2 y_1^{-1}$

Consequently $x_1 x_2 x_1^{-1} \in f^{-1}(y_1 y_2 y_1^{-1})$

Therefore $\sup_{z \in f^{-1}(y_1 y_2 y_1^{-1})} \mu_A(z) \geqslant \sup_{x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)} \mu_A(x_1 x_2 x_1^{-1}) \geqslant \sup_{x_2 \in f^{-1}(y_2)} \mu_A(x_2)$

i.e., $F_f(\mu_A)(y_1 y_2 y_1^{-1}) \geqslant F_f(\mu_A)(y_2)$, for all $y_1, y_2 \in G_2$

On the other hand, similarly, $y_1, y_2 \in G_2$, $f^{-1}(y_1) \neq \phi, f^{-1}(y_2) \neq \phi, f^{-1}(y_1 y_2 y_1^{-1}) \neq \phi$ for arbitrary $y_1, y_2 \in G_2$, and $x_2 \in f^{-1}(y_2), x_1 \in f^{-1}(y_1)$, then $x_1^{-1} \in f_1^{-1}(y_1^{-1})$, and $\nu_A(x_1 x_2 x_1^{-1}) \leqslant \nu_A(x_2)$

Thus $\inf_{z \in f^{-1}(y_1 y_2 y_1^{-1})} \nu_A(z) \leqslant \inf_{x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)} \nu_A(x_1 x_2 x_1^{-1}) \leqslant \inf_{x_2 \in f^{-1}(y_2)} \nu_A(x_2)$

i.e., $\hat{F}_f(\nu_A)(y_1 y_2 y_1^{-1}) \leqslant \hat{F}_f(\nu_A)(y_2)$ for all $y_1, y_2 \in G_2$

Hence $F_f(A) \in IFNS[G_2]$

Theorem 2.2 Let G_1 and G_2 be classical groups, $f: G_1 \to G_2$ be a homomorphic mapping. If $B \in IFNS[G_2]$, then $F_f^{-1}(B) \in IFNS[G_1]$.

Proof By theorem 1.1[2], Evidently, $F_f^{-1}(B) \in IFS[G_1]$, thus, we need only prove the fuzzy normality.

Since $B \in IFNS[G_2]$, for arbitrary $x, y \in G_1$, form the extension principle, we obtain

$$F_f^{-1}(\mu_B)(xyx^{-1}) = \mu_B(f(xyx^{-1})) = \mu_B(f(x)\cdot f(y)\cdot f(x^{-1}))$$
$$= \mu_B(f(x)\cdot f(y)\cdot (f(x))^{-1}) \geqslant \mu_B(f(y)) = F_f(\mu_B)(y)$$

Similary , we get

$$F_f^{-1}(\nu_B)(xyx^{-1}) = \nu_B(f(xyx^{-1})) = \nu_B(f(x)\cdot f(y)\cdot f(x^{-1}))$$
$$= \nu_B(f(x)\cdot f(y)\cdot (f(x))^{-1}) \leqslant \nu_B(f(y)) = F_f(\mu_B)(y)$$

Therefore $F_f^{-1}(B) \in IFNS[G_1]$

Theorem 2.3 Let G_1 and G_2 be classical groups , $f: G_1 \to G_2$ be a homomorphic mapping . If $A \in IFNS[G_1]$,then $A^{-1} \in IFNS[G_1]$ and $F_f(A^{-1}) = (F_f(A))^{-1}$.

Proof Let $A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in G_1\}$, then $A^{-1} = \{< x, \mu_{A^{-1}}(x), \nu_{A^{-1}}(x) > \mid x \in G_1\}$, where $\mu_{A^{-1}}(x) = \mu_A(x^{-1}), \nu_{A^{-1}}(x) = \nu_A(x^{-1})$

By $A \in IFNS[G_1]$ and theorem 1.2[2], we know $A^{-1} \in IFS[G_1]$

For arbitrary $x, y \in G_1$,we have

$$\mu_{A^{-1}}(xyx^{-1}) = \mu_A((xyx^{-1})^{-1}) \geqslant \mu_A(xyx^{-1}) \geqslant \mu_A(y) = \mu_{A^{-1}}(y^{-1}) \geqslant \mu_{A^{-1}}(y)$$
$$\nu_{A^{-1}}(xyx^{-1}) = \nu_A((xyx^{-1})^{-1}) \leqslant \nu_A(xyx^{-1}) \leqslant \nu_A(y) = \nu_{A^{-1}}(y^{-1}) \leqslant \nu_{A^{-1}}(y)$$

i.e., the fuzzy normality holds.

Consepuently , we get $A^{-1} \in IFNS[G_1]$

From theorem 2.1, we have $F_f(A) \in IFNS[G_2]$, thus $F_f(A^{-1}) \in IFNS[G_2]$, of course, $F_f(A) \in IFS[G_2]$, $F_f(A^{-1}) \in IFS[G_2]$, utilizimg theorem 1.2[2],we can infer that $F_f(A^{-1}) = (F_f(A))^{-1}$.

Corollary 2.4 Let G_1 and G_2 be two classical groups , $f: G_1 \to G_2$ be a homomorphic mapping . If $B \in IFNS[G_2]$,then $(F_f^{-1}(B))^{-1} = F_f^{-1}(B^{-1})$.

Theorem 2.5 Let G_1 and G_2 be two classical groups , $f: G_1 \to G_2$ be an isomorphic mapping . If $A \in IFNS[G_1]$,then $F_f^{-1}(F_f(A)) = A$.

Proof For arbitrary $x \in G_1$, *Let* $f(x) = y$, as f is an isomorphic mapping. $f^{-1}(y) = \{x\}$. Applying the extension principle ,we obtain

$$F_f^{-1}(F_f(\mu_A))(x) = F_f(\mu_A)(f(x)) = F_f(\mu_A)(y)$$

$$= \sup_{x \in f^{-1}(y)} \mu_A(x) = \mu_A(x)$$

$$\hat{F}_f^{-1}(F_f(\nu_A))(x) = \hat{F}_f(\nu_A)(f(x)) = \hat{F}_f(\nu_A)(y)$$

$$= \inf_{x \in f^{-1}(y)} \nu_A(x) = \nu_A(x)$$

Hence, we have $F_f^{-1}(F_f(A)) = A$.

Corollary 2.6 Let G_1 and G_2 be two classical groups , $f : G_1 \to G_2$ be an isomorphic mapping . If $B \in IFNS[G_2]$,then $F_f(F_f^{-1}(B)) = B$.

Corollary 2.7 Let G and be a classical group , $f : G \to G$ be an automorphic mapping . If $A \in IFNS[G]$,then $F_f(A) = A$ iff$F_f^{-1}(A) = A$.

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