The Properties of the Fuzzy Lower Cut Sets

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Abstract: In this paper, we present a new concept of fuzzy cut sets that is called fuzzy lower cut sets, several properties of them are discussed.

Keywords: Fuzzy set; fuzzy lower cut set; membership function

1. Introduction

When we want to exhibit an element $x \in X$ that typically belong to a fuzzy set A ,we may demand its membership value to be greater than some threshold $\alpha \in]0,1]$ .The set of such elements is the $\alpha - cut$ A_α of A, A_α $= \{x \in X, \mu_A(x) \geq \alpha\}$. One also defines the strong α – cut, $A_{\underline{\alpha}} = \{x \in X, \mu_A(x) > \alpha\}$.

Radecki (1977) has defined level fuzzy sets of a fuzzy set A as the fuzzy set A as the fuzzy sets $\overline{A}_\alpha, \alpha \in]0,1]$, such that

$$\overline{A_\alpha} = \{(x, \mu_A(x)), x \in A_\alpha\}.$$

In this paper ,we propose a new fuzzy cuts which has a remarkable property and it is called fuuzy lower cut sets.

2. Notions

In this section, some basic definitions of fuzzy sets and their operations will be described.

Definition 1. Let A be a fuzzy set, for any $\lambda \in [0,1]$, we define

$$A_{\bar{\alpha}} = \{x \mid 0 < \mu_A(x) \leq \alpha\} \quad (1)$$

$$A_{\bar{\underline{\alpha}}} = \{x \mid 0 < \mu_A(x) < \alpha\} \quad (2)$$

$$A_{[\alpha,\beta]} = A_{\bar{\alpha}} - A_{\bar{\beta}} = A_\alpha \cap A_{\bar{\beta}} \quad (3)$$

where (1)、(2)、(3) are called α – lower cut sets 、α – strong lower cut sets 、interval cut sets, respectively.

As illustrated in Figures 1.

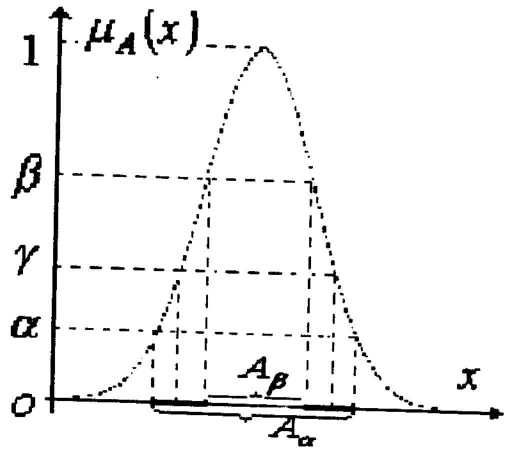

Fig.1

▬ describe $A_{[\alpha,\beta]}$.

Example 1 Let $A = 1/a + 0.75/b + 0.5/c + 0.25/d + 0/e$, we have

$$A_{\bar{1}} = \{a,b,c,d,e\} = X, \qquad A_{\overline{0.7}} = \{c,d\}, \qquad A_{\overline{0.5}} = \{c,d\}$$

$$A_{\bar{1}} = \{b,c,d\}, \qquad A_{\overline{0.75}} = \{c,d\}, \qquad A_{\bar{0}} = \Phi.$$

$$A_{[0.2,0.8]} = \{b,c,d\}, \qquad A_{[0.2,1]} = \{a,b,c,d\}.$$

Example 2 Let A be a fuzzy set with Trapezoidal distribution function, if its membership function $\mu_A(x): R \to [0,1]$ is

$$\mu_A(x) = \begin{cases} 0, x \le a - a_2, or a_2 + a < x \\ \dfrac{a_2 + x - a}{a_2 - a_1}, a - a_2 < x \le a - a_1 \\ 1, a - a_1 < x \le a + a_1 \\ \dfrac{a_2 - x + a}{a_2 - a_1}, a + a_1 < x \le a + a_2 \end{cases}$$

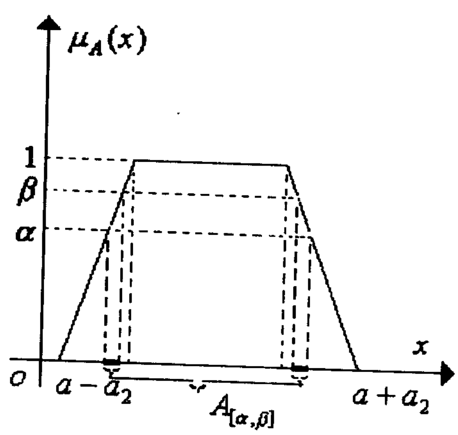

Fig.2

Fig.2 show the graph of the set $A_{[\alpha,\beta]}$ of such an fuzzy set .Then

$$A_{[\alpha,\beta]}=\{x\mid \alpha\le \frac{a_2+x-a}{a_2-a_1}\le\beta\}\cup\{x\mid\alpha\le\frac{a_2-x+a}{a_2-a_1}\le\beta\}$$

3.Properties of the fuzzy lower cut sets

Propsition 3.1. Let A be a fuzzy set, then for any $\alpha,\beta,\gamma\in[0,1]$, we have

1) if $\alpha<\gamma<\beta$, then $A_{[\alpha,\gamma]}\subseteq A_{[\alpha,\beta]}$ and $A_{[\gamma,\beta]}\subseteq A_{[\alpha,\beta]}$;

2) if $\alpha<\gamma<\beta$, then $A_{[\alpha,\beta]}=A_{[\alpha,\gamma]}\cup A_{[\gamma,\beta]}$.

Proof. Follows easily by applying definition 1.

Remark 1. For any $\alpha,\beta\in[0,1]$,the following Eq.(4) holds

$$A_{\bar\beta}-A_{\bar\alpha}=(A_0-A_\beta)-(A_0-A_\alpha)=A_\alpha-A_\beta. \tag{4}$$

Propsition 3.2 Let A, B are fuzzy sets, for any $\alpha\in[0,1]$, we have the following properties hold:

3) $(A\cup B)_{\bar\alpha}=A_{\bar\alpha}\cap B_{\bar\alpha}$; (5)

4) $(A\cap B)_{\bar\alpha}=A_{\bar\alpha}\cup B_{\bar\alpha}$. (6)

Proof. 3) Since $\forall u\in(A\cup B)_{\bar\alpha}\Leftrightarrow \mu_{(A\cup B)}(u)\le\alpha\Leftrightarrow\mu_A(u)\vee\mu_B(u)\le\alpha$

$\Leftrightarrow\mu_A(u)\le\alpha$ and $\mu_B(u)\le\alpha$

$\Leftrightarrow u\in A_{\bar\alpha}$ and $u\in B_{\bar\alpha}$

$\Leftrightarrow u\in A_{\bar\alpha}\cap B_{\bar\alpha}$.

hence, Eq.(5) is right. In a similar, we can easily verify that (6) holds.

Proposition 3.3 Let T be a index set , $\forall\lambda\in[0,1]$, then the following relations hold:

5) $(\bigcup_{t\in T}A^{(t)})_{\bar\lambda}\subset\bigcap_{t\in T}A^{(t)}_{\bar\lambda}$; (7)

6) $(\bigcup_{t\in T}A^{(t)})_{\bar\lambda}\subset\bigcap_{t\in T}A^{(t)}_{\bar\lambda}$; (8)

7) $(\bigcap_{t\in T}A^{(t)})_{\bar\lambda}\subset\bigcap_{t\in T}A^{(t)}_{\bar\lambda}$; (9)

8) $(\bigcap_{t\in T} A^{(t)})_{\bar{\lambda}} \subset \bigcap_{t\in T} A^{(t)}_{\bar{\lambda}}$; (10)

9) $\bigcap_{t\in T} A_{\bar{\lambda}_t} \subset A_{\overline{(\vee_{t\in T} \lambda_t)}}$; (11)

10) $(A^c)_\lambda = A_{\overline{1-\lambda}}$. (12)

Proof. 5) For any $u \in (\bigcup_{t\in T} A^{(t)})_{\bar{\lambda}} \Rightarrow \mu_{\bigcup_{t\in T} A^{(t)}}(u) \le \lambda \Rightarrow \bigvee_{t\in T} \mu_{A^{(t)}}(u) \le \lambda$

$$\Rightarrow \forall t_0 \in T, \mu_{A^{(t_0)}}(u) \le \lambda \Rightarrow u \in \bigcap_{t\in T} A^{(t)}_{\bar{\lambda}}.$$

That is 5) holds. Similar, we can easily verily that 6) holds.

7) $\forall u \in (\bigcap_{t\in T} A^{(t)})_{\bar{\lambda}} \Leftrightarrow \bigwedge_{t\in T} A^{(t)}(u) \le \lambda \Leftrightarrow \exists t_0, s.t. \mu_{A^{(t_0)}}(u) \le \lambda$

$$\Rightarrow u \in \bigcap_{t\in T} A^{(t)}_{\bar{\lambda}}.$$

Hence, 7) is proven. In a similar way, using the definition 1,we can see that the relations 8) and 9) also hold for any λ.

Finally, since $\forall u \in (A^c)_\lambda \Leftrightarrow \mu_{A^c}(u) \ge \lambda \Leftrightarrow 1-\mu_A(u) \ge \lambda$

$$\Leftrightarrow \mu_A(u) \le 1-\lambda \Leftrightarrow u \in A_{\overline{1-\lambda}}.$$

Therefore, 10) is proven.

Remarks: ① $\forall \lambda \in [0,1], A_{\bar{\lambda}} \subset A_{\bar{\lambda}}$,

② $\lambda \le \eta \Rightarrow A_{\bar{\lambda}} \subseteq A_{\bar{\eta}}, A_{\bar{\lambda}} \subset A_{\bar{\eta}}$,

③ $A_{\bar{1}} = A_0$.

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