The Application of Fuzzy Theory In Solar Activity Prediction*

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Abstract In this paper, the theory and method of fuzzy mathematics are applied to predict the activity of active regions on the solar disk. The index system and standard of fuzzy evaluation, as well as membership function are proposed. A practical software of computer data processing is given. More accurate result of solar activity prediction is obtainable. Accuracy is greater than 90%.

Keywords: Solar activity prediction; fuzzy theory; membership function

1. Introduction

Traditional methods of solar activity prediction are mainly multi-element regression analysis. Due to the lack of necessary mathematical connection between different predicting factors, so far there has been no method accurately forecasting the activity of active regions and solar flares.

According to fuzzy statistics, the relationship between 6800 flares and the characteristic factors (spot class, area, spot count, extent and magnetic class etc.) of 1100 sunspot groups in solar geophysical Data[3] during the period of Jan. 1989-Dec1990 has been analyzed. We have defined the characteristic factors, which can be used for solar activity predication. As the weights of these factors influencing upon the solar activity prediction are different, it is necessary to evaluate comprehensively the influence of the factors upon the solar activity and solar flare.

In this paper, the theory and method of fuzzy mathematics are applied to modeling membership function and to evaluate solar activity thus the accuracy of the solar activity prediction can be effectively increased.

2. Basic Principle and Method

The main factors influencing upon the solar activity include the relative increase u_1 of the radio flux at 10cm wavelength, the spot class u_2, the extent u_3 on the solar disk, the spot area u_4, the spot count u_5, the magnetic class u_6 and the morphology u_7 of the sunspot group.

Hence the set of factors is as follows:

$U=\{u_1\ u_2\ u_3\ u_4\ u_5\ u_6\ u_7\}$

Each factor is divided into four classes according to its intensity, comparable to the four classes of solar activity :

Weak v_1 , Normal v_2, Strong v_3,Very Strong v_4

Hence the set of evaluation is as follows:

$V=\{v_1\ v_2\ v_3\ v_4\}$

From our statistics result of large amount of calculation, the weak indicates that the active regions produce no flares or only few S-class flares. The general indicates that the active regions produce a few S-class flares or one or two class 1 flares. The strong indicates the active regions produce more than two class 1 flares or one or two class 2 flares. The very strong indicates that

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the active regions produce more than two class 2 flares or one or several class 3 flares and class 4 flares.

Based on the previous research results, related data and large amount of calculation, the weight of every factor influencing upon solar activity is given, respectively as

$A=\{a_1(0.15), a_2(0.20), a_3(0.10). a_4(0.10), a_5(0.10), a_6(0.10), a_7(0.25)\}$

With the use of the data in Preliminary Report and Forecast of Solar Geophysical Data,[4,5] Figure 1 indicates the correlation between mean producing rates of the ≥1 class flares and characteristic factors of sunspot groups. Figure 2 indicates the correlation between hold rates of active regions of ≥1 class flares and characteristic factors of the sunspot groups.

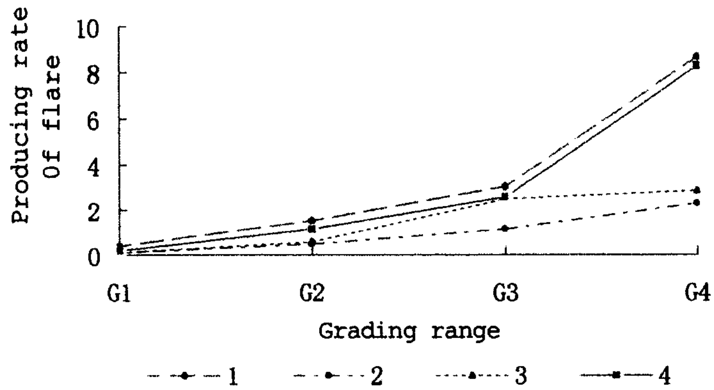


Figure 1 The correlation between producing rates of the class ≥1 flares and the characteristic factors of sunspot groups. The 1, 2, 3 and 4 marks the spot area, spot count, extent and magnetic class, respectively.

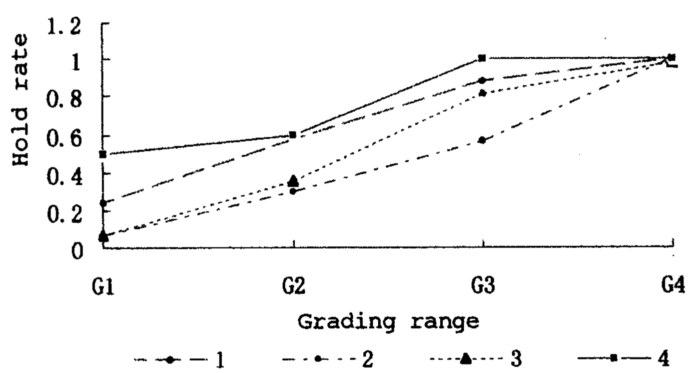


Figure 2 The correlation between hold rates of the active regions of class ≥1 flares and the characteristic factors of sunspot groups. The 1, 2, 3 and 4 marks the spot area, spot count, extent and magnetic class, respectively.

Four grading ranges and the weight of the influencing factors are listed in Table 1

Grading standard	u_1 (%)	u_2	u_3 (helio°)	u_4 (10^{-6}hemi)	u_5 (°)	U_6	u_7
Weak(G_1)	<5	1-10	0-5	1-250	1-5	A=1	0
General(G_2)	5-10	11-20	6-10	251-750	6-10	B=2	0
Stronger(G_3)	11-15	21-30	11-15	751-1250	11-35	BD,BG=3 .	0
Very strong(G_4)	>15	31-40	>15	>1250	>35	BGD=4	1 或 0 *
Effecting weights	0.15	0.20	0.1	0.1	0.1	0.1	0.25

● When the sunspot group is a δ-type one with a single multiple structure, it takes as one, otherwise as 0.

Based on the grading ranges in Table 1 and the correlation between. Solar activity and characteristic factors, the membership function in different forms for different influencing factors can be determined by the following. Such as, for u_1, u_2 and u_3, the membership function is described by a trapezoid distribution. For u_4 and u_5, the membership function is taken as the cauchy distribution. For u_6, the membership function is described by a rectangular distribution.[1,2]

Function of trapezoid distribution is as follows:

$$\mu_{weak} = \begin{cases} 1 & 0 \le X \le D_1 \\ \dfrac{D_2 - X}{D_2 - D_1} & D_1 < X_2 \le D_2 \\ 0 & X > D_2 \end{cases} \qquad \mu_{normal} = \begin{cases} \dfrac{X}{D_1} & 0 < X \le D_1 \\ 1 & D_1 < X \le D_2 \\ \dfrac{D_3 - X}{D_3 - D_2} & D_2 < X \le D_3 \\ 0 & X > D_3 \end{cases}$$

$$\mu_{strong} = \begin{cases} 0 & X \le D_1 \\ \dfrac{X - D_1}{D_2 - D_1} & D_1 < X \le D_2 \\ 1 & D_2 < X \le D_3 \\ \dfrac{D_4 - X}{D_4 - D_3} & X > D_3 \end{cases} \qquad \mu_{Very\ Strong} = \begin{cases} 0 & X \le D_2 \\ \dfrac{X - D_2}{D_3 - D_2} & D_2 < X \le D_3 \\ 1 & X > D_3 \end{cases}$$

Function of Cauchy distribution is as follows:

$$\mu_{weak} = \begin{cases} 1 & 0 \le X \le D_1 \\ \dfrac{1}{1 + [(X - D_1)/5]^2} & D_1 < X \le D_2 \\ 0 & X > D_2 \end{cases} \qquad \mu_{normal} = \begin{cases} \dfrac{1}{1 + (X/5)^{-2}} & 0 < X \le D_1 \\ 1 & D_1 < X \le D_2 \\ \dfrac{1}{1 + [(X - D_2)/5]^2} & D_2 < X \le D_3 \\ 0 & X > D_3 \end{cases}$$

$$\mu_{strong} = \begin{cases} 0 & X \le D_1 \\ \dfrac{1}{1 + [(X - D_1)/5]^{-2}} & D_1 < X \le D_2 \\ 1 & D_2 < X \le D_3 \\ \dfrac{1}{1 + [(X - D_3)/5]^2} & X > D_3 \end{cases} \qquad \mu_{Very\ Strong} = \begin{cases} 0 & X \le D_2 \\ \dfrac{1}{1 + [(X - D_2)/5]^{-2}} & D_2 < X \le D_3 \\ 1 & X > D_3 \end{cases}$$

From calculating grade of membership for the seven different factors, we can obtain the monomial factor evaluation matrix R and the fuzzy comprehensive evaluation index B, respectively.

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ & \cdots\cdots & \cdots\cdots & \\ r_{71} & r_{72} & r_{73} & r_{74} \end{pmatrix}$$

Where ,

$$r_{ij} = \mu_j^i \qquad i=1,2\ldots7 \qquad j= 1.2.3.4$$

$$B = A \bullet R = (b_1 \quad b_2 \quad b_3 \quad b_4)$$

Where $b_j = \min\{1, \sum_{i=1}^{7} a_i \cdot r_{ij}\}$

And $b_j = \vee (a_i \wedge r_{ij}) \qquad j=1,2,3,4$

Where b_1, b_2, b_3, b_4 indicates four classes of solar activity

The fuzzy operators $\vee$, $\wedge$ indicates the choice of max and min, respectively

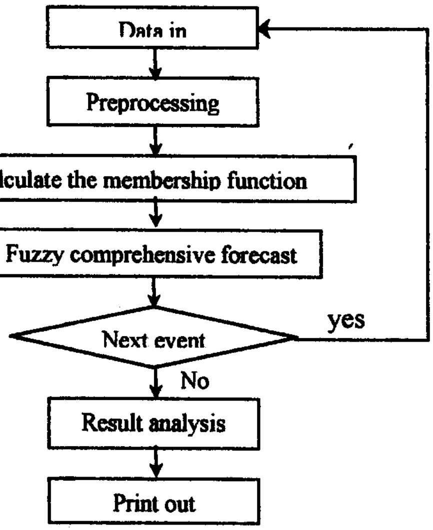
$$\underset{\sim}{B}^{*} = \left(\frac{b_1}{\Sigma b_i} \quad \frac{b_2}{\Sigma b_i} \quad \frac{b_3}{\Sigma b_i} \quad \frac{b_4}{\Sigma b_i} \right) = (b_1^{*} \quad b_2^{*} \quad b_3^{*} \quad b_4^{*})$$

$$(b_1^* \quad b_2^* \quad b_3^* \quad b_4^*)$$

The calculation result indicates the four standard value of membership grade of solar activity for weak, normal, strong, very strong , respectively. In this way the future tendency of solar activity of the active region can be forecasted according to maximum membership principle.

3.Programming and Computational Examples

With the use of the data in preliminary report and forecast of Solar Geophysical Data during Jan.1989--Dec.1992, the theoretical computation for the 24 active regions is performed. The frame of the program structure is shown in Fig 3



Take the example of the active region 6555 crossing the solar disk during March 17th--31th, 1991. According to the original data of the seven factors of March 20,1991 ,We calculate the value of membership grade of different classes with different membership function.. Hence the monomial evaluation factor matrix R is determined.

Then

$$R=\begin{pmatrix} 0.08 & 1 & 0.92 & 0 \\ 0 & 0 & 0.2 & 1 \\ 0 & 0 & 0.74 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.03 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B=A \bullet R =(0.02\ 0.25\ 0.25\ 0.75)$$

Normalize B, we get:

$$B^*= (0.01\ 0.20\ 0.20\ 0.59)$$

According to maximum membership principle, take b_4=0.59,this means that the activity of AR6555 is very strong.

4. Result analysis

According to the membership function and the observational data of the radio and sunspot group in third or fourth day after it being appear and the east limb of the solar disk, we have computed comprehensive evaluation indices for twenty-four active regions. The results are listed in Table 2. For comparison, the real activity of active regions is also listed in Table 2.

Table 2 Counting results of the forecast activity compared with real activities of the active regions

Region No.	Date	Original data	Counting result	Producing activity	Real activity	
		$u_1, u_2, u_3, u_4, u_5, u_6, u_7$	$b^*_1, b^*_2, b^*_3, b^*_4$		Numbers of H_α flare	Peak flux of proton event
5323	1989 01 18	0, 27, 7, 210, 3, 2, 0	0. 35, 0. 33, 0. 20, 0. 12	weak	5/0/0/0	
5324	1989 01 20	0, 21, 1, 30, 1, 1, 0	0. 51, 0. 28, 0. 19, 0. 02	weak	3/0/0/0	
5395*	1989 03 19	16. 3, 39, 15, 2000, 21, 4,	0, 0. 01, 0. 26, 0. 72	Strong	132/37/21/5	3500, 2100
5451	1989 04 16	10, 37, 14, 810, 23, 3, 0	0, 0. 14, 0. 48, 0. 37	Stronger	63/3/1/0	
5528	1989 06 10	14, 40, 22, 1660, 60, 4, 0	0, 0. 03, 0. 23, 0. 74	Strong	99/5/1/1	
5698*	1989 09 19	0, 37, 12, 930, 44, 2, 1	0. 12, 0. 13, 0. 22, 0. 54	Strong	61/7/0/0 +1	4500, X9. 8/EPL
5728	1989 10 08	0, 31, 4, 150, 6, 2, 0	0. 37, 0. 31, 0. 15, 0. 17	Weak	27/0/0/0	
5800*	1989 11 22	3. 5, 30, 10, 440, 16, 2, 1	0. 09, 0. 28, 0. 31, 0. 32	Strong	42/2/2/1	380, 7300
5927	1990 02 10	2, 9, 4, 420, 6, 2, 0	0. 43, 0. 49, 0. 08, 0	General	21/1/0/0	
6049	1990 05 05	1. 6, 29, 16, 290, 14, 2, 0	0. 12, 0. 26, 0. 38, 0. 25	Stronger	49/3/0/0	
6063	1990 05 15	14, 37, 12, 940, 30, 4, 0	0, 0. 08, 0. 40, 0. 52	Strong	48/6/6/1	410, 180
6064	1990 05 15	13. 8, 37, 14, 620, 55, 3, 0	0, 0. 14, 0. 44, 0. 42	Stronger	129/11/2/0	
6272	1990 09 17	0, 30, 8, 510, 25, 2, 0	0. 15, 0. 25, 0. 37, 0. 23	Stronger	25/5/0/0	
6497	1991 02 16	12, 31, 13, 170, 19, 2, 0	0. 07, 0. 25, 0. 38, 0. 29	Stronger	21/0/1/0	13
6555*	1991 03 20	9. 6, 38, 20, 2270, 64, 2, 1	0. 01, 0. 20, 0. 20, 0. 59	Strong	109/40/10/5	43000
6615*	1991 05 06	29. 9, 37, 14, 870, 37, 4, 1	0, 0. 01, 0. 26, 0. 73	Strong	68/7/0/0 +1	350
6619*	1991 05 07	29. 9, 26, 8, 1030, 5, 4, 1	0. 10, 0. 16, 0. 25, 0. 50	Strong	44/3/2/0	
6659*	1991 06 05	14. 8, 39, 11, 2100, 31, 3,	0, 0. 07, 0. 34, 0. 59	Strong	75/20/6/6	3000, 1400
6734	1991 07 21	14, 28, 14, 870, 11, 4, 0	0, 0. 14, 0. 46, 0. 40	Stronger	46/5/2/0	
7100*	1992 03 13	0, 30, 10, 290, 15, 3, 1	0. 10, 0. 17, 0. 40, 0. 33	Stronger	12/0/1/1	10
7154*	1992 05 05	0, 33, 9, 390, 37, 2, 1	0. 12, 0. 21, 0. 29, 0. 38	Strong	32/3/0/0/1	4600
7205*	1992 07 23	8, 36, 8, 780, 17, 3, 1	0. 07, 0. 19, 0. 35, 0. 40	Strong	51/5/2/0	390
7248	1992 08 04	19, 30, 10, 410, 19, 4, 0	0, 0. 16, 0. 45, 0. 39	Stronger	88/7/0/0	14
7321*	1992 10 25	16. 3, 26, 6, 470, 23, 4, 1	0. 05, 0. 18, 0. 34, 0. 43	Strong	62/7/1/0 +1	2700, 790

The asterisks "*" in Table 2 indicate that the morphology of the Sunspot group is a -typed one with a single multiple structure. The symbols "+1" in table 2 represents the limb X-ray flare.

Some research shows that the activity of the active region having δ-type sunspots with a single multiple structure generally is very strong. Many proton flares relate to these active regions.[6,7,8]

Although the area of δ-type sunspot is sometimes very small, it can produce large flare. For example, in the active region 5800, the area of the sunspot group only is 440×10^{-6} hemi, but its activity is very strong. Two X class X rays were produced by AR5800, and two strong proton events with 380 p.f.u. and 7300 p.f.u. (1p.f.u.=proton $cm^{-2}s^{-1}$ $ster^{-1}$) Peak fluxes are accompanied. The AR7154 also has a δ-type sunspot with an area of 390×10^{-6} hemi, but it produced a 4B class H_α flare, and accompanied by a strong proton event with 4600 p.f.u. peak flux.

It can be seen from Table 2 that the forecasting result for AR6619 is a grade higher than real activity. However, considering AR6610and AR6659 in next solar rotation cycle is in the same active region, and the AR6659 has very strong activity, the predicted activity of AR6619 is basically consistent in principle with the real situation.

Among the predicting results of the 24 active regions, the results for 23 sunspot groups are consistent with their real activity, so the predicting accuracy of the solar activity can reach 95.8%. This proves that the method of the fuzzy comprehensive evaluation increases the forecast accuracy a lot and thus is an ideal method for the solar activity prediction.

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