

# Countable Paracompactness on $L$ -Fuzzy topological spaces\*

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**Abstract:** Based on *III* strong fuzzy paracompactness, this paper introduces definition of  $L$ -fuzzy countable paracompactness and illustrates its basic characters. Moreover it is proved that  $L$ -fuzzy countable paracompactness is  $L$ -good extension. Characters of  $L$ -fuzzy countable are studied extensively.

**Key words:**  $L$ -fuzzy countable paracompactness,  $\alpha^*$ -operator,  $\alpha$ -closed sets,  $L$ -fuzzy completely normal spaces, fuzzy lattice,  $L$ -fuzzy topological spaces

## Introduction

It is necessary to extend countable paracompactness to  $L$ -fuzzy topology because countable paracompactness is of importance in general topology. In the recent years there are many kinds of paracompactness in  $L$ -fuzzy topology.<sup>[1-5]</sup> Among these paracompactness, *III* strong fuzzy paracompactness is very popular<sup>[6]</sup>. This paper introduces countable paracompactness in  $L$ -fuzzy topological spaces by virtue of *III* strong fuzzy paracompactness. It is proved that countable paracompactness is  $L$ -good extension. Furthermore, this paper gives a series of characters. In addition three properties are studied. First, the multiplication of a strong fuzzy compact set and a countable paracompact set is a countable paracompact set; Second, completely normal spaces are countable paracompact; Last, countable paracompactness is genetic for closed sets.

## I Preliminary

The notation of  $L$  stands for fuzzy lattice<sup>[7]</sup> in this paper and  $L$ -fts is short for  $L$ -fuzzy topological spaces. For  $A \in L^X$ ,  $\alpha \in M(L)$  and  $\mathcal{A} \subset L^X$

$$\tau_\alpha(A) = \{x \in X \mid A(x) \geq \alpha\}, l_{\alpha'}(A) = \{x \in X \mid A(x) \not\geq \alpha'\}$$

$$\tau_\alpha(\mathcal{A}) = \{\tau_\alpha(A) \mid A \in \mathcal{A}\}, l_{\alpha'}(\mathcal{A}) = \{l_{\alpha'}(A) \mid A \in \mathcal{A}\}.$$

Other signs are introduced in the paper<sup>[7]</sup>.

**Definition 1.1**<sup>[6]</sup> Let  $\Phi$  be a set family in  $L$ -fts.  $A \in L^X$ ,  $\alpha \in M(L)$ .  $\Phi$  is called  $\alpha$ -family

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of  $A$ , if for every molecule of  $x_a$  of  $A$  whose height is  $\alpha$ , there exists  $Q \in \Phi$  such that  $x_a \not\leq Q$ .  $\Phi$  is called  $\alpha^*$ -covering of  $A$ , if for every molecule  $x_a$  of  $A$  whose height is  $\alpha$ , there exists  $Q \in \Phi$  such that  $x_a \leq Q$ .

**Definition 1.2**<sup>[7]</sup> Let  $(L^X, \delta)$  be a  $L$ -fts,  $A \in L^X$  and  $\mathcal{A} = \{A_t \mid t \in T\} \subset L^X$ .  $\mathcal{A}$  is  $\alpha$ -locally finite in  $A$ , if for every molecule of  $x_a$  of  $A$  whose height is  $\alpha$ , there exists  $P \in \eta^-(X_a)$  and finite set  $T_0 \subset T$  such that  $\forall t \in T - T_0, A_t \leq P$ .  $\mathcal{A}$  is  $\alpha$ -discrete in  $A$ , if  $T_0$  is a unit set.

**Lemma 1.3**<sup>[6]</sup> Let  $f : (L^{X_1}, \delta_1) \rightarrow (L^{X_2}, \delta_2)$  be continuous  $L$ -value Zadeh function. and  $a \in M(L)$ . If  $\mathcal{B} \subset L^{X_2}$  is  $\alpha$ -locally finite in  $D$ .  $f^{-1}(\mathcal{B}) = \{f^{-1}(B) \mid B \in \mathcal{B}\}$  is  $\alpha$ -locally finite in  $f^{-1}(D)$ .

In the following part for  $A \in L^X, a \in M(L), \mathcal{A} \in L^X$

$$A^{q_a} = \vee \{x_a \in M^*(L^X) \mid x_a \not\leq A\}, \mathcal{A}^{q_a} = \{A^{q_a} \mid A \in \mathcal{A}\}.$$

**Lemma 1.4**<sup>[6]</sup> Let  $(L^X, \delta)$  be  $L$ -fts.  $D \in L^X, a \in M(L)$ , and  $\mathcal{A} \subset L^X$ .  $D \wedge \mathcal{A}^{q_a}$  is  $\alpha$ -locally finite in  $D$  if and only if  $\tau_a(D) \cap (\tau_a(\mathcal{A}))'$  is locally finite in the subspace  $\tau_a(D)$  of general topological space  $(X, l_a(\delta))$ .

## II Definitions and characters

**Definition 2.1** Let  $(L^X, \delta)$  be  $L$ -fts.  $D \in L^X, a \in M(L)$ .  $D$  is  $\alpha$ -III countably paracompact ( $\alpha$ -countably paracompact), if for every  $\alpha$ -countable remote domain family  $\Phi$  of  $D$  there exists  $\alpha$ -remote domain family  $\Psi$  of  $D$  satisfying the following conditions:

(i)  $\Psi$  is corefine of  $\Phi$ , namely  $\forall B \in \Psi, \exists A \in \Phi$  such that  $A \leq B$ .

(ii)  $D \wedge \Psi^{q_a}$  is locally finite in  $D$  where  $\Psi^{q_a} = \{B^{q_a} : B \in \Psi\}$ .

$D$  is  $\alpha$ -countably paracompact if  $D$  is  $\alpha$ -countably paracompact for  $\forall a \in M(L)$ . If  $D$  equals to  $1_X, (L^X, \delta)$  is  $\alpha$ -countably paracompact or countably paracompact.

**Theory 2.2**  $\alpha$ -III paracompact set in  $L$ -fts is  $\alpha$ -countably paracompact set.

**Theory 2.3** Countable strong paracompact set in  $L$ -fts is countable paracompact set.

**Theory 2.4** Let  $(L^X, \delta)$  be  $L$ -fts.  $D \in L^X$  and  $a \in M(L)$ .  $D$  is  $\alpha$ -countably

paracompact if and only if  $\tau_a(D)$  is countably paracompact in  $(X, l_a(\delta))$

**Proof** Let  $D$  be  $\alpha$ -countably paracompact in  $(L^X, \delta)$  and  $\mathcal{U}$  countable open covering in the subspace of  $\tau_a(D)$  of  $(X, l_a(\delta))$ . Then there exists countable subfamily  $\Psi \subset \delta'$  such that  $\tau_a(D) \cap l_a(\Psi') = \tau_a(D) \cap (\tau_a(\Psi))' = \mathcal{U}$ . Also  $\Psi$  is  $\alpha$ -countable remote family of  $D$ . From  $\alpha$ -countable paracompactness of  $D$  there exists  $\alpha$ -remote domain family  $\Phi$  of  $D$  such that  $\Phi$  is corefine of  $\Psi$  and  $D \wedge \Phi^{q_a}$  is  $\alpha$ -locally finite in  $D$ . It is easy to know that  $\tau_a(D) \cap l_a(\Phi')$  is refine of  $\mathcal{U}$  and open covering of subspace  $\tau_a(D)$  of  $(X, l_a(\delta))$ . By Lemma 1.4 it is locally finite in  $\tau_a(D)$ . Therefore  $\tau_a(D)$  is countable paracompact in  $(X, l_a(\delta))$ .

On the contrary let  $\tau_a(D)$  be countable paracompact in  $(X, l_a(\delta))$ . Because  $\Phi$  is  $\alpha$ -countable remote family of  $D$  in  $(L^X, \delta)$ ,  $\tau_a(D) \cap l_a(\Phi')$  is countable open covering of subspace  $\tau_a(D)$  of  $(X, l_a(\delta))$ . Then there exists  $\Psi \subset \delta'$  such that  $l_a(\Psi') \cap \tau_a(D)$  is locally finite open covering of subspace  $\tau_a(D)$  of  $(X, l_a(\delta))$  and corefine of  $\tau_a(D) \cap l_a(\Phi')$ . So  $\Psi$  is  $\alpha$ -remote family of  $D$  in  $(L^X, \delta)$ . By Lemma we know that  $D \wedge \Psi^{q_a}$  is  $\alpha$ -locally finite in  $D$ . For arbitrary element  $B$  of  $\Psi$  there exists  $A_B \in \Phi$  such that

$\tau_a(D) \cap l_a(B') \subset \tau_a(D) \cap l_a(A_B')$ . Let  $\Omega = \{B \vee A_B \mid B \in \Psi\}$ . Then

$\tau_a(D) \cap l_a(\Omega') = \{\tau_a(D) \cap l_a((B \vee A_B)') \mid B \in \Psi\} = \{\tau_a(D) \cap l_a(B') \cap l_a(A_B') \mid B \in \Psi\} =$

$\{\tau_a(D) \cap l_a(B') \mid B \in \Psi\} = \tau_a(D) \cap l_a(\Psi')$ .

It is easy to prove that  $\Omega$  is  $\alpha$ -remote family of  $D$  in  $(L^X, \delta)$  and corefine of  $\Phi$ . For arbitrary  $P \in \Omega$  there exists  $B \in \Psi$  such that  $P = B \vee A_B$ . Therefore  $P^{q_a} \leq B^{q_a}$ . Then  $D \wedge \Omega^{q_a}$  is  $\alpha$ -locally finite if  $D$ . So  $D$  is  $\alpha$ -countable paracompact in  $(L^X, \delta)$ .

If  $(L^X, \delta)$  is a weakly induced space<sup>[7]</sup>,  $l_a(\delta) = [\delta]$  for arbitrary  $a \in M(L)$ .

**Corollary 2.5** Let  $(L^X, \delta)$  be weakly induced  $L$ -tfs. The following conditions are equivalent:

(i)  $(L^X, \delta)$  is countable paracompact.

(ii) there exists  $a \in M(L)$  such that  $(L^X, \delta)$  is  $\alpha$ -countable paracompact.

(iii)  $(X, [\delta])$  is countable paracompact.

**Corollary 2.6** Countable paracompactness is  $L$ -good extension.

In the following part we introduce a new operator. By this new operator we can show countable paracompactness in a novel way.

**Definition 2.7** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ . A new operator is defined by

$$a^\bullet : L^X \rightarrow \delta', \forall A \in L^X, a^\bullet(A) = \bigwedge \{ G : G \in \delta' \text{ and } G \vee A \geq a \}.$$

**Proposition 2.8** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ ,  $A, B \in L^X$ , there are following remarks:

(i)  $a^\bullet(A) \in \delta'$ ;

(ii)  $A \vee a^\bullet(A) \geq a$ ;

(iii)  $A \leq B \Rightarrow a^\bullet(B) \leq a^\bullet(A)$ ;

(iv) If  $A \in \delta'$   $A \vee B \geq a \Leftrightarrow a^\bullet(B) \leq A$ ;

(v) If  $A$  and  $B \in \delta'$ ,  $A \vee B \geq a \Leftrightarrow a^\bullet(B) \leq A \Leftrightarrow a^\bullet(A) \leq B$ .

It is easy to know that operator  $a^\bullet$  is extension of the operator of the paper<sup>[5]</sup>.

**Theory 2.9** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ .  $(L^X, \delta)$  is  $\alpha$ -countable paracompact if and only if for every  $\alpha$ -countable remote domain family  $\Omega$  of  $1_X$  there exists  $\alpha$ -remote domain family

$\Psi$  of  $1_X$  such that  $\Psi$  is corefine of  $d\Omega$  and  $a^\bullet(\Psi) = \{a^\bullet(A) : A \in \Psi\}$  is  $\alpha$ -locally finite in  $1_X$ .

**Proof** From theory 2.4 it is enough to prove that conditions of theory 2.9 are equivalent to countable paracompactness of  $(X, L_a(\delta))$ .

Let  $(X, L_a(\delta))$  be countable paracompact and  $\Omega$  be  $\alpha$ -countable remote domain family of  $1_X$ .

Then  $L_a(\Omega') = \{l_a(Q') : Q' \in \Omega\}$  is countable open covering of  $(X, L_a(\delta))$ . Therefore there exists open refinement  $L_a(\Delta)$  ( $\Delta \subset \delta$ ) of  $L_a(\Omega')$  is locally finite in  $(X, L_a(\delta))$ . For arbitrary  $B$  of  $\Delta$  there exists

$Q_B \in \Omega$  such that  $l_a(B) \subset l_a(Q_B)$ . Let  $\Psi = \{B' \vee Q_B : B \in \Delta\}$ . Then  $\Psi$  is corefine of  $\Omega$ . In addition,

$\Delta'$  is  $\alpha$ -remote domain family of  $1_X$ . So for  $\forall x \in X$  there exists  $B \in \Delta$  such that  $x_a \notin B'$ , namely

$x \in l_{a'}(B)$ . Therefore  $x \in l_{a'}(Q'_B)$ , namely  $x_a \notin Q_B$ . This shows that  $\Psi$  is  $\alpha$ -remote domain family of  $1_X$ . In the following part we only prove that  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ .

For  $\forall x \in X$  there exists open domain  $l_{a'}(W)$  of  $x$  and finite subfamily  $\Delta_0$  of  $\Delta$  such that  $l_{a'}(B) \cap l_{a'}(W) = l_{a'}(B \wedge W) = \emptyset$ . for  $\forall B \in \Delta - \Delta_0$ . It is easy to know that for  $W' \in \eta(x_a)$  and  $\forall B \in \Delta - \Delta_0, B \wedge W \leq a'$  and  $B' \vee W' \geq a$ . Furthermore,  $B' \vee Q_B \vee W' \geq a$ . By proposition 2.8  $a^*(B' \vee Q_B) \leq W'$ . Therefore  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ .

On the contrary, let  $l_{a'}(\Omega') (\Omega \in \delta')$  be countable open covering of  $(X, l_{a'}(\delta))$ . So  $a^*(\Psi)$  is  $\alpha$ -countable remote family of  $1_X$ . Therefore there exists  $\alpha$ -remote family  $\Psi'$  of  $1_X$  such that  $\Psi$  is corefine of  $\Omega$  and  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ . It is easy to know that  $l_{a'}(\Psi')$  is open refinement of  $l_{a'}(\Omega')$ . For  $\forall x \in X$  there exists  $P \in \eta^-(x_a)$  and finite subfamily  $\Psi_0$  of  $\Psi'$  such that  $a^*(B) \leq P$  for  $\forall B \in \Psi - \Psi_0$ . So  $B \vee P \geq a$  and  $l_{a'}(B' \wedge P') = l_{a'}(B') \cap l_{a'}(P') = \emptyset$ . From  $l_{a'}(P')$  is open domain of  $x$  we know that  $l_{a'}(\Psi')$  is locally finite in  $(X, l_{a'}(\delta))$ . Therefore  $(X, l_{a'}(\delta))$  is countable paracompact.

End of proof.

In the following part we show  $\alpha$ -countable paracompactness via  $\alpha$ -family consisting of  $\alpha$ -closed sets.

**Definition 2.10** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$  and  $A \in L^X$ .  $A$  is  $\alpha$ -closed if

$$\forall x_a \in M^*(L^X), x_0 \in A^- \Rightarrow x_a \in A.$$

**Theory 2.11** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ .  $(L^X, \delta)$  is  $\alpha$ -countable paracompact if and only if for every  $\alpha$ -closed  $\alpha$ -countable family  $\Omega$  of  $1_X$  there exists  $\alpha$ -closed  $\alpha$ -family  $\Psi$  of  $1_X$  such that  $\Psi$  is corefine of  $\Omega$  and  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ .

**Proof** Let  $(L^X, \delta)$  be  $\alpha$ -countable paracompact and  $\Omega$  be  $\alpha$ -closed  $\alpha$ -countable family of  $1_X$ . From definition of  $\alpha$ -closed set we know that  $\Omega^- = \{Q^- : Q \in \Omega\}$  is  $\alpha$ -countable remote domain family of  $1_X$ . Therefore there exists  $\alpha$ -remote domain family  $\Psi$  of  $1_X$  such that  $\Psi$  is corefine of  $\Omega^-$  and  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ . Because closed set is  $\alpha$ -closed set,  $\Psi$  above

mentioned satisfied requirement of theory.

On the contrary, let  $\Omega$  be  $\alpha$ -remote domain family of  $1_X$ . So  $\Omega$  is  $\alpha$ -closed and  $\alpha$ -countable family. Therefore there exists  $\alpha$ -closed and  $\alpha$ -family  $\Psi$  such that  $\Psi$  is corefine of  $\Omega$  and  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ . Consider  $\Psi^-, \Psi^-$  is corefine of  $\Omega$ . By  $\alpha$ -closeness of  $\Psi$  it is easy to know that  $\Psi^-$  is  $\alpha$ -remote domain family of  $1_X$ . from proposition 2.8 (iii)  $a^*(\Psi^-)$  is  $\alpha$ -locally finite in  $1_X$ . Therefore  $(L^X, \delta)$  is  $\alpha$ -countable paracompact.

**Theory 2.12** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ . So  $(L^X, \delta)$  is  $\alpha$ -countable paracompact if and only if for every  $\alpha$ -remote domain family of  $1_X$   $\Phi = \{A_i : i \in N\}$  there exists  $\alpha$ -remote domain family of  $1_X$   $\Psi = \{B_i : i \in N\}$  such that  $A_i \leq B_i$  and  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$  for  $\forall i \in N$ .

**Proof** Let  $(L^X, \delta)$  be  $\alpha$ -countable paracompact and  $\Phi = \{A_i : i \in N\}$  be  $\alpha$ -countable remote domain family of  $1_X$ . There exists corefine  $\Omega$  of  $\alpha$ -remote domain family  $\Omega$ .  $a^*(\Omega)$  is  $\alpha$ -locally finite in  $1_X$ . For  $\forall B \in \Omega$  there exists  $i(B) \in N$  such that  $A_{i(B)} \leq B$ .

Let  $B_i = \bigvee_{i(B)=i} B$ . Then  $A_i \leq B_i$ . Let  $\Psi = \{B_i : i \in N\}$ . Then  $\Psi$  is  $\alpha$ -countable remote domain family of  $1_X$  and corefine of  $\Phi$ . In the following part it is enough to prove that  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ . Because  $a^*(\Omega)$  is  $\alpha$ -locally finite in  $1_X$  for  $\forall x_a \in M^*(L^X)$  there exists  $P \in \eta(x_a)$  and finite subfamily  $\Omega_0$  of  $\Omega$  such that  $a^*(B) \leq P$  for  $\forall B \in \Omega - \Omega_0$ . By proposition 2.8  $P \vee B \geq a$ . Then  $P \vee B_i \geq a$ , namely  $a^*(B_i) \leq P$ . Because  $\Omega_0$  is finite, The number of  $B_i$  whose elements belong to  $\Omega_0$  must be finite. Therefore  $a^*(\Psi)$  is  $\alpha$ -locally finite in  $1_X$ .

It is easy to prove sufficiency.

End of proof.

**Definition 2.13** Let  $\{F_i\}_{i \in N} \subset L^X$  and  $a \in M(L)$ .  $\{F_i\}_{i \in N}$  is  $\alpha$ -decreasing if  $\tau_a(F_1) \supset \tau_a(F_2) \supset \dots$

**Theory 2.14** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ . Then  $(L^X, \delta)$  is countable paracompact if and only if for every  $\alpha$ -decreasing  $\alpha$ -remote domain family  $\{F_i\}_{i \in N}$  there exists  $\{W_i\}_{i \in N} \subset \delta$  such that

$$W_i' \in \eta_a(F_i), i \in N \text{ and } \bigcap_{i=1}^{\infty} \overline{l_{a'}(W_i)} = \emptyset$$

**Proof** It is easy to be proved by theory 2.4 and results of general topology.

### III Properties of countable paracompactness

The following theories show that  $(\alpha-)$  is closely genetic

**Theory 3.1** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ .  $A \in L^X$  and  $B \in \delta'$ .

(i) If  $A$  is  $\alpha$ -countable paracompact,  $A \wedge B$  is  $\alpha$ -countable paracompact, too.

(ii) If  $A$  is countable paracompact,  $A \wedge B$  is countable paracompact, too.

**Proof** It is enough to prove that (i) is right. Let  $\Phi$  be  $\alpha$ -countable remote family of  $A \wedge B$ . Then  $\Phi \cup \{B\}$  be  $\alpha$ -countable remote family of  $A$ . Because  $A$  is  $\alpha$ -countable paracompact, there exists  $\alpha$ -countable remote family  $\Psi$  of  $A$  such that  $\Psi$  is corefine of  $\Phi \cup \{B\}$  and  $A \wedge \Psi^{q_a}$  is  $\alpha$ -locally finite in  $A$ . Let  $\Omega = \{P \in \Psi \mid B \not\subseteq P\}$ , then  $\Omega$  is  $\alpha$ -countable remote domain family of  $A \wedge B$  and corefine of  $\Phi$ . It is easy to know that  $A \wedge \Omega^{q_a}$  is  $\alpha$ -locally finite in  $A \wedge B$ . So  $A \wedge B \wedge \Omega^{q_a}$  is  $\alpha$ -locally finite in  $A \wedge B$ . Therefore  $A \wedge B$  is  $\alpha$ -countable paracompact.

**Theory 3.2** Let  $(L^X, \delta)$  and  $(L^Y, \mu)$  be  $L$ -fts and  $A$  be a countable paracompact set of  $(L^X, \delta)$ . And  $B$  is a strong  $F$  paracompact set of  $(L^Y, \mu)$ . Then  $A \times B$  is a countable paracompact set of  $(L^{X \times Y}, \delta \times \mu)$ .

**Proof** By theory 2.4  $\tau_a(A)$  is a countable paracompact set of  $(X, l_{a'}(\delta))$  for  $\forall a \in M(L)$ . It is easy to know that  $B$  strong  $F$  compact in  $(L^Y, \mu)$  if and only if  $\tau_a(B)$  is compact in  $(Y, l_{a'}(\mu))$  for  $\forall a \in M(L)$ . From some results of general topology  $\tau_a(A \times B) = \tau_a(A) \times \tau_a(B)$  is countable paracompact in  $(X \times Y, l_{a'}(\delta \times \mu))$ . From theory 2.4 this theory is right.

**Definition 3.3** Let  $(L^X, \delta)$  be  $L$ -fts.  $a \in M(L)$ .  $(L^X, \delta)$  is a  $\alpha$ -completely normal space if  $(L^X, \delta)$  is  $\alpha$ -normal, and for  $\forall F \in \delta'$  there exist  $G_i \in \delta, i = 1, 2, \dots$  such that  $\tau_a(F) = \bigcap_{i \in N} l_{a'}(G_i)$ . If

for  $a \in M(L)$   $(L^X, \delta)$  is  $\alpha$  – completely normal,  $(L^X, \delta)$  is completely normal.

It is to know that  $(L^X, \delta)$  is  $\alpha$  – completely normal if and only if  $(L^X, \delta)$  is  $\alpha$  – normal and for  $G \in \delta$  there exist  $F_i \in \delta$ ,  $i=1,2,\dots$  such that  $l_a(G) = U_{\tau_a}(F_i)$ .

**Theory 3.4** Let  $(L^X, \delta)$  be  $L$  – fts.  $a \in M(L)$ .  $(L^X, \delta)$  is  $\alpha$  – completely normal if and only if  $(X, l_a(\delta))$  is completely normal.

**Theory 3.5** Let  $(L^X, \delta)$  be  $L$  – fts.  $a \in M(L)$ . If  $(L^X, \delta)$  is  $\alpha$  – completely normal  $(L^X, \delta)$  is  $\alpha$  – countable paracompact.

**Proof** It is proved by theory 3.4 and some results of general topology.

**Theory 3.6** Let  $(L^X, \delta)$  and  $(L^Y, \mu)$   $L$  – fts which are induced. If  $(L^X, \delta)$  is a countable paracompact level normal space<sup>[6]</sup> and  $(L^Y, \mu)$  is a strong  $F$  compact  $II$  countable space,  $(L^{X \times Y}, \delta \times \mu)$  is level normal.

**Proof** From theory 2.4  $(X, [\delta])$  is a countable paracompact normal space. By [7]  $(Y, [\mu])$  is a compact  $II$  countable space. So from some results of general topology ([8] theory 5.2.7)  $X \times Y$  is normal. By [7]  $(L^{X \times Y}, \delta \times \mu)$  is generated by  $X \times Y$ . Therefore  $(L^{X \times Y}, \delta \times \mu)$  is a level normal space.

**Theory 3.7** Let  $(L^X, \delta)$  and  $(L^Y, \mu)$   $L$  – fts weakly which are induced weakly.  $f : (L^X, \delta) \rightarrow (L^Y, \mu)$  is continuously closed and full  $L$  – value Zadeh function. If  $(L^X, \delta)$  is a countable paracompact level normal space,  $(L^Y, \mu)$  is a countable paracompact level normal space.

**Proof** Because  $(L^X, \delta)$  is a weakly induced space, from theory 2.5  $(X, [\delta])$  is a countable paracompact space. By [6]  $(X, [\delta])$  is a level normal space. Because  $f : (L^X, \delta) \rightarrow (L^Y, \mu)$  is continuously closed and full  $L$  – value Zadeh function, general mapping  $f : (X, [\delta]) \rightarrow (Y, [\mu])$  is continuous closed full mapping. From some results of general topology  $(Y, [\mu])$  is countable paracompact level normal space. It is proved by theory 2.5  $(L^Y, \mu)$  is a countable paracompact level normal space.

#### Reference:

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