Fuzzy rough sets and L-fuzzy sets*

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Abstract The purpose of this paper is to demonstrate the fact that fuzzy rough sets in the sense of Nanda and Majumbar [6] are ,indeed, L-fuzzy sets developed by Goguen[7].

Keywords Fuzzy sets; Rough sets ; Fuzzy rough sets ; Intuitionistic L-fuzzy sets; L-fuzzy sets.

1 Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper in 1965,and later Pawlak give the nation of rough set [4,5] in 1982. In another direction ,Goguen published the paper "L-fuzzy sets" .In subsequent years several generalizations of rough set, including the works of Nanda and Majumbar and Kunchera are appeared in the literature ,the relation between Fuzzy rough sets and Intuitionistic L-fuzzy sets , the relation between L-Fuzzy sets and Intuitionistic L-fuzzy sets are discussed , see[2,3]. In this paper, we shall give the relation between Fuzzy rough sets and L-fuzzy sets, and demonstrate the fact that fuzzy rough sets in the sense of Nanda and Majumbar[] are indeed L-fuzzy sets.

2 Review of Fuzzy lattices, IFS and IFLS

According to [1] the concept of fuzzy lattices is understood as follows:

Definition 2.1 . A fuzzy lattices is a lattice L such that

(i) L is complete

(ii) L is completely distributive.

(iii)There exits an order-reversing involution " $'$ " on L,i.e.,

$$(a')' = a, \quad a \le b \quad \text{iff } b' \le a' \quad (a,b \in L) \tag{1}$$

Example .Suppose that L is a fuzzy lattice, define L^* as follows:

$$L^* = \{(a,b): \quad a,b \in L \text{ and } a \le b'\} \tag{2}$$

And define an order "$\le$" on L^* and an operator " $'$ " on L^* as follows:

$$(a,b) \le (c,d) \text{ iff } a \le c \text{ and } b \ge d \text{ , } ((a,b),(c,d) \in L^*) \tag{3}$$

$$(a,b)' = (b,a) \; ((a,b) \in L^*) \tag{4}$$

Definition 2.2. Let E be a non-empty crisp set . An IFS A^* in E is an object having the form:

$$A^* = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in E\} \tag{5}$$

where μ_A: $E \to [0,1]$, ν_A: $E \to [0,1]$ define the degree of membership and the degree of membership and the degree of nonmembership of the element $x \in E$ to $A \subset E$, respectively ,and

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$$(\forall x \in E)(0 \leq \mu_A(x) + \nu_A(x) \leq 1) \tag{6}$$

Definition 2.3 Let E be a non-empty crisp set . An ILFS A^* in E is an object having the form (5) where $\mu_A: E \to L$, $\nu_A: E \to L$ satisfy the condition $(\forall x \in E)(\mu_A(x) \leq N(\nu_A(x)))$, where N is the order-reversing involution on L.

Theorem 2.1 (see [3]) The concept of IFS ,$ILFS$ and the concept of L-fuzzy sets are equivalent .

3 Fuzzy rough sets in the sense of Nanda and Majumdar

Let U be a nonempty set , $\langle L, \leq \rangle$ a lattice and & a Boolean subalgebra of the set of all subsets of U. Now consider a rough set $X = (X_L, X_U) \in \&^2$ with $X_L \subseteq X_U$. A fuzzy rough set in X is an object of the form $A = (A_L, A_U)$,where A_L and A_U are characterized by a pair of maps $\mu_{A_L}: X_L \to L$, $\mu_{A_U}: X_U \to L$, with the property $\mu_{A_L}(x) \leq \mu_{A_U}(x)$ for all $x \in X_L$.

For any two fuzzy rough sets $A = (A_L, A_U), B = (B_L, B_U)$ in X, we define

(i) $A \subseteq B$ *iff* $\mu_{A_L}(x) \leq \mu_{B_L}(x)$ for all $x \in X_L$ and $\mu_{A_U}(x) \leq \mu_{B_U}(x)$ for all $x \in X_U$;

(ii) A=B iff $A \subseteq B$ and $B \subseteq A$;

(iii) $C = A \cup B$, where $\mu_{C_L}(x) = \mu_{A_L}(x) \vee \mu_{B_L}(x)$ for all $x \in X_L$, $\mu_{C_U}(x) = \mu_{A_U}(x) \vee \mu_{B_U}(x)$ for all $x \in X_U$;

(iv) $D = A \cap B$, where $\mu_{D_L}(x) = \mu_{A_L}(x) \wedge \mu_{B_L}(x)$ for all $x \in X_L$, $\mu_{D_U}(x) = \mu_{A_U}(x) \wedge \mu_{B_U}(x)$ for all $x \in X_U$;

(v) $\overline{A} = (\overline{A}_L, \overline{A}_U)$, where $\mu_{\overline{A}_L}(x) = \mu_{A_U}(x)'$ for all $x \in X_U$, $\mu_{\overline{A}_U}(x) = \mu_{A_L}(x)'$ for all $x \in X_L$.

(Note that $'$: $L \to L$ is an involutive order-reversing operation in $\langle L, \leq \rangle$.)

If ,in general, L is a complete lattice, and $\{A_i : i \in J\}$ is an arbitrary family of fuzzy rough sets in X, where $A_i = (A_{iL}, A_{iU})$,then

(vi) $E = \cup A_i$, where $\mu_{E_L}(x) = \vee \mu_{A_{iL}}(x)$ for all $x \in X_L$ and $\mu_{E_L}(x) = \vee \mu_{A_{iL}}(x)$ for all $x \in X_U$;

(vii) $F = \cap A_i$, where $\mu_{F_L}(x) = \wedge \mu_{A_{iL}}(x)$ for all $x \in X_L$ and $\mu_{F_L}(x) = \wedge \mu_{A_{iL}}(x)$ for all $x \in X_U$;

4 Fuzzy rough sets are L-fuzzy sets

Let U be a nonempty set, $\langle L, \leq \rangle$ a complete distributive lattice whose least element and greatest element are denoted by 0 and 1, respectively, with an involutive order-reversing operation $' : L \to L$, and & a Boolean subalgebra of the set of all subsets of U. Now consider a fuzzy rough set $A = (A_L, A_U)$ in $X = (X_L, X_U) \in \&^2$,where the functions $\mu_{A_L}: X_L \to L$, $\mu_{A_U}: X_U \to L$

Do satisfy the property $\mu_{A_L}(x) \le \mu_{A_U}(x)$ for all $x \in X_L$. Without loss of generality ,we may assume that $\mu_{A_L}(x)$ and $\mu_{A_U}(x)$are defined on all of U by assigning

$$\tilde{\mu}_{A_L}(x) = \begin{cases} \mu_{A_L}(x) & \text{if } x \in X_L, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \tilde{\mu}_{A_U}(x) = \begin{cases} \mu_{A_U}(x) & \text{if } x \in X_U, \\ 0 & \text{otherwise,} \end{cases}$$

Under these extensions ,we may form the intuitioinstic L-fuzzy set $F(A)$ in U, where $F(A) = \langle x, \mu_{F(A)}, \gamma_{F(A)} \rangle$ and $\mu_{F(A)}(x) = \tilde{\mu}_{A_L}(x)$, $\gamma_{F(A)}(x) = (\tilde{\mu}_{A_U}(x))'$ for all $x \in U$, and hence our final result follows.

Theorem 4.1 (see [2]) Let U be a nonempty set, $\langle L, \le \rangle$ a complete distributive lattice whose least element and greatest element are denoted by 0 and 1, respectively, with an involutive order-reversing operation $' : L \to L$, and & a Boolean subalgebra of the set of all subsets of U. Then any fuzzy rough set in $X = (X_L, X_U) \in \&^2$ is an intuitionistic L-fuzzy in U .

Therefore ,we can obtain the following theorem.

Theorem 4.2 Let U be a nonempty set, $\langle L, \le \rangle$ a complete distributive lattice whose least element and greatest element are denoted by 0 and 1, respectively, with an involutive order-reverding operation $' : L \to L$, and & a Boolean subalgebra of the set of all subsets of U. Then any fuzzy rough set in $X = (X_L, X_U) \in \&^2$ is an L-fuzzy in U .

Proof. Recall the theorem 2.1 and theorem 4.1, we can obtain the result .

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