VALUE OF BONDS IN THEORY OF FUZZY SETS

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Abstract

This work introduced relating considerations of qualifying bonds values for investor. One accepted, that required by investor one year's foot of manageability is expressed by fuzzy number of type α-β. On entrance one talked over location criterions of financial capital owners. Then one qualified manner of bonds value calculation for investor with utilization of fuzzy numbers. In work one placed also example of calculations according to introduced manner.

1. Location criterions of financial capital owners

Consideration different possibilities of financial investments by owners of economy refers to purchases of stock dependent from their market prices, heights of dividends and of their receipt certainty in futures. Motive of capital owners to executing of investments greatest foot of profit for firms seeking capital. Reason this is getting rid of stock by firms, of which rent ability grows smaller. For this factor giving reasons board of directors at choice most competent controlling personnel is criterion maximize of waited profits sums from placed capital in firm. From regarding hesitancy of future profits level (there are possible situations of lack of profits or losses) partners undertake connected risk with engagement their capitals in production. For benefit of giving reasons to buying stock one should accept waited surplus of dividends sum over incomes, which would be able to receive locating economies in more safe places. In this manner of company participate in competition for inflow of possibly cheaper capitals, gained over thanks to partition of risk between stockholders.

Talking over location criterions of capital one should pay attention on capital owners which investments in concrete organizations determine so much large part of capital given organization, that they can exert large influence on strategy of management. In one's own decisions direct oneself mostly long time criterions of usefulness one's own investments and take under attention eg possibilities of investing in innovation and technological product

development, which bring possible advantages with large postponement. If goes for material aspect of justification their decision this capital locating in innovating investments brings so-called extraordinary profits resulting from certain temporary monopoly, which has at owner disposal of firm exploiting inaccessible for other innovations. These prospective extraordinary profits one can attend too material impulse to executing of very risky innovating investments, because they will demand of long periods researches and evolution - initiation works.

For stockholders important location criterion is manageability of capital identified with cash originating from dividends and, in chance of sale stock by them, from sale these of valuable papers. When stocks are sold, their price represents in this time value all of dividends.

Summing up one should ascertain, that owners of capital undertake decisions about investments base on better or worse definite function of preference, which are dependent of waiting profits sums and from measures of risk.

2. Qualification values of bonds

Other form of locating or gaining over of capital by firm is purchase of bonds or their sale. Bonds determine important source of financing activities of firm, especially, when gaining over of bank- credit is relatively expensive. Value of market bonds one can compare with brought into the sphere of topical questions value all waited financial receipts from bonds in future. Liquid financial centres (cash flow) express in this case size of percentage sum paid to owners of bonds by firm and their nominal values. Value of bonds can express by means following algebraically sums [8]

$$V_b = \sum_{t=1}^{mn} I \cdot \left(1 + \frac{k_b}{m}\right)^{-t} + NV_b\left(1 + \frac{k_b}{m}\right)^{-mn}, \qquad (1)$$

where:

V_b – value of bonds for investor,

I – entire cash - interest paid from bonds in the course of year,

NV_b – value of nominal bonds,

m – length of percentage payment period in years,

n – number of year's payment,

k_b – required by investor one year's foot manageability.

3. Utilization of fuzzy numbers to qualifying values of bonds

In our considerations size k_b is not exactly well known and express her by fuzzy number of type α-β [1], [2], [3], [4], [6].

$$k_b=(a, b, \alpha, \beta) \tag{2}$$

First we will transform example (1) to figure permitting on more easy calculations.

Because I is constant, so we switch off her before sign of sum

$$V_b = I\cdot\sum_{t=1}^{mn}\left(1+\frac{k_b}{m}\right)^{-t} + NV_b\left(1+\frac{k_b}{m}\right)^{-mn} = I\cdot\sum_{t=1}^{mn}\left(\frac{m}{m+k_b}\right)^{t} + NV_b\left(\frac{m}{m+k_b}\right)^{mn}. \tag{3}$$

Then marking

$$q=\frac{m}{m+k_b}, \qquad a_t = q^t, \qquad S=\sum_{t=1}^{mn} a_t \tag{4}$$

We count sum of geometrical progress

$$S=\frac{1-q^{mn}}{1-q}a_1=\frac{1-q^{mn}}{1-q}q. \tag{5}$$

Then example (1) rises adopts form

$$V_b = I\cdot\frac{1-q^{mn}}{1-q}q + NV_b\cdot q^{mn} = I\cdot S + NV_b\cdot q^{mn}. \tag{6}$$

At calculation V_b we can profit from following definition [1]

Definition 1 *(fuzzy number with the minus sign)*

Fuzzy number with the minus sign is defined as follows:

$$-L = (-b, -a, \beta, \alpha). \tag{7}$$

Definition 2 *(inverse of fuzzy number)*

Inverse number is defined as follows:

$$1/L = (1/b, 1/a, \beta/(b(b+\beta)), \alpha/(a(a-\alpha))) \qquad (\text{for } L>0 \text{ or } L<0). \tag{8}$$

Definition 3 *(the of sums of fuzzy numbers)*

Let us assume L1 =(a1, b1, a1, b1) and L2 =(a2, b2, a2, b2). The of sums of fuzzy numbers is defined as follows:

$$L_1+L_2 = (a_1+a_2, b_1+b_2, \alpha_1+\alpha_2, \beta_1+\beta_2). \tag{9}$$

Definition 4 *(the difference of fuzzy numbers)*

The difference of fuzzy numbers is defined as follows:

$$L_1-L_2 = (a_1-b_2, b_1-a_2, \alpha_1+\beta_2, \alpha_2+\beta_1). \tag{10}$$

Definition 5 *(the product of fuzzy numbers)*

The product of fuzzy numbers is defined as follows:

$$L_1 \cdot L_2 = \begin{cases} (a_1a_2, b_1b_2, a_1\alpha_2 + a_2\alpha_1 - \alpha_1\alpha_2, b_1\beta_2 + b_2\beta_1 + \beta_1\beta_2) & \text{for } L_1 > 0, L_2 > 0, \\ (a_1b_2, b_1a_2, b_2\alpha_1 - a_1\beta_2 + \alpha_1\beta_2, -b_1\alpha_2 + a_2\beta_1 - \beta_1\alpha_2) & \text{for } L_1 < 0, L_2 > 0, \\ (b_1a_2, a_1b_2, b_1\alpha_2 - a_2\beta_1 + \beta_1\alpha_2, -b_2\alpha_1 + a_2\beta_2 - \alpha_1\beta_2) & \text{for } L_1 > 0, L_2 < 0, \\ (b_1b_2, a_1a_2, -b_1\beta_2 - b_2\beta_1 - \beta_1\beta_2, -a_1\alpha_2 - a_2\alpha_1 + \alpha_1\alpha_2) & \text{for } L_1 < 0, L_2 < 0. \end{cases} \tag{11}$$

Definition 6 *(the quotient of fuzzy numbers)*

The quotient of fuzzy numbers is defined as follows:

$$L_1 / L_2 = \begin{cases} (a_1/b_2, b_1/a_2, (a_1\beta_2 + b_2\alpha_1)/(b_2(b_2+\beta_2)), (b_1\alpha_2 + a_2\beta_1)/(a_2(a_2-\alpha_2))) \\ \text{for } L_1 > 0, \quad L_2 > 0, \\ (a_1/a_2, b_1/b_2, (a_2\alpha_1 - a_2\alpha_2)/(a_2(a_2-\alpha_2)), (b_2\beta_1 - b_1\beta_2)/(b_2(b_2+\beta_2))) \\ \text{for } L_1 < 0, \quad L_2 > 0, \\ (b_1/b_2, a_1/a_2, (b_1\beta_2 - b_2\beta_1)/(b_2(b_2+\beta_2)), (a_1\alpha_2 - a_2\alpha_1)/(a_1(a_1-\alpha_1))) \\ \text{for } L_1 > 0, \quad L_2 < 0, \\ (b_1/a_2, a_1/b_2, (-b_1\alpha_2 - a_2\beta_1)/(a_2(a_2-\alpha_2)), (-a_1\beta_2 - b_2\alpha_1)/(b_2(b_2+\beta_2))) \\ \text{for } L_1 < 0, \quad L_2 < 0. \end{cases} \tag{12}$$

In aim of fuzzy number square calculation many times we will use example (11).

Example

One should find value of bonds V_b knowing, that NV_b=1000 zloty, I=88 zloty, m=5 years, n=5 years, k_b is fuzzy number in form k_b=(0. 2, 0. 3, 0. 01, 0. 03).

Counting V_b on the ground of example (6) in turn we receive:

$m+k_b$=(5.2, 5.3, 0.01, 0.03), q=(0.9434, 0.9615, 0.0053, 0.00185),

q^{25}=(0.233, 0.375, 0.03066, 0.01849), $NV_b \cdot q^{25}$=(232.9989, 375.1170, 30.6642, 18.4932),

1-q=(0.03846, 0.0566, 0.00185, 0.0053), $1-q^{25}$=(0.625, 0.767, 0.01849, 0.3066),

I·S=(916.495, 1687.403, 107.975, 159.822), V_b=(1149.49, 2062.52, 138.64, 178.32).

Obtained sum Vb marks value of bonds for investor, which would buy them, if this value were at least so high as market price. These values of bonds are from section [1149. 49, 2062. 52] zloty and can be diminished (with certain probability of risk) to sums 1010. 85 zloty or can be enlarged (with certain probability of risk) to sums 2240. 84 zloty.

References

[1] Bolc L., Borodziewicz W., Wójcik M.: *Podstawy przetwarzania informacji niepewnej i niepełnej*. PWN, Warszawa 1991.

[2] Czogała E., Pedrycz W.: *Elementy i metody teorii zbiorów rozmytych*. PWN, Warszawa 1985.

[3] Drewniak J.: *Fuzzy relation calculus*. Scientific Works of Silesian University No 1063, Katowice 1989.

[4] Dubois D., Prade H. *Operations on fuzzy numbers*, Int. J. Syst. Sci., vol. 9, 1978.

[5] Hongern Ch.: *Cost Accounting*. A Managerial Emphasis, New Yersey 1990.

[6] Kacprzyk J. *Zbiory rozmyte w analizie systemowej*. PWN, Warszawa 1983.

[7] Tiedtke J.R.: *Finansowanie i inwestycje*. Die Akademie, Hamburg-Katowice 1991, s. 79.

[8] Woźniak-Sobczak A.: *Gospodarowanie kapitałem w firmie*. AE Katowice, Katowice 1996.