Countable Paracompactness on L-Fuzzy topological spaces*

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Abstract: Based on III strong fuzzy paracompactness, this paper introduces definition of L-fuzzy countable paracompactness and illustrates its basic characters. Moreover it is proved that L-fuzzy countable paracompactness is $L-good\ extension$. Characters of L-fuzzy countable are studied extensively.

Key words: L-fuzzy countable paracompactness, $\alpha^{\bullet}-$operator, $\alpha-$closed sets, L-fuzzy completely normal spaces, fuzzy lattice, L-fuzzy topological spaces

Introduction

It is necessary to extend countable paracompactness to L-fuzzy topology because countable paracompactness is of importance in general topology. In the recent years there are many kinds of paracompactness in L-fuzzy topology.[1-5] .Among these paracompactness, III strong fuzzy paracomapactness is very popular[6].This paper introduces countable paracompactness in L-fuzzy topological spaces by virtue of III strong fuzzy paracompactness. It is proved that countable paracompactness is $L-good\ extension$. Furthermore, this paper gives a series of characters. In addition three properties are studied. First, the multiplication of a strong fuzzy compact set and a countable paracompact set is a countable paracompact set; Second, completely normal spaces are countable paracompacat; Last, countable paracompactness is genetic for closed sets.

I Preliminary

The detonation of L stands for fuzzy lattice[7] in this paper and $L-fts$ is short for $L-$fuzzy topological spaces. For $A\in L^X$ $\alpha\in M(L)$ and $A\ \mathscr{A}\subset L^X$

$$\tau_a(A)\overset{\Delta}{=}\{x\in X\mid A(x)\geq a\},l_{a'}(A)=\{x\in X\mid A(x)\not\leq a'\}$$

$$\tau_a(\mathscr{A})=\{\tau_a(A)\mid A\in\mathscr{A}\ \},l_{a'}(\mathscr{A})=\{l_{a'}(A)\mid A\in\mathscr{A}\ \}.$$

Other signs are introduced in the paper[7].

Definition 1.1[6] Let Φ be a set family in $L-fts$. $A\in L^X,a\in M(L)$. Φ iscalled $\alpha-$family

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of A , if for every molecule of x_a of A whose height is α ,there exists $Q \in \Phi$ such that $x_a \not\leq Q$. Φ is called $\alpha^{\bullet}$ – covering of A ,if for every molecule x_a of A whose height is α ,there exists $Q \in \Phi$ such that $x_a \leq Q$.

Definition 1.2[7] Let (L^X, δ) be a $L-fts$, $A \in L^X$ and $\mathscr{A} = \{A_t \mid t \in T\} \subset L^X$. $\mathscr{A}$ is $\alpha-$ locally finite in A ,if for every molecule of x_a of A whose height is α ,there exists $P \in \eta^{-}(X_a)$ and finite set $T_0 \subset T$ such that $\forall t \in T - T_0, A_t \leq P$. $\mathscr{A}$ is $\alpha-$ discrete in A ,if T_0 is a unit set.

Lemma 1.3[6] Let $f:(L^{X_1}, \delta_1) \to (L^{X_2}, \delta_2)$ be continuous $L-$ value Zadeh function. and $a \in M(L)$ .If $\mathscr{B} \subset L^{X_2}$ is $\alpha-$ locally finite in D . $f^{-1}(\mathscr{B}) = \{f^{-1}(B) \mid B \in \mathscr{B}\}$ is $\alpha-$ locally finite in $f^{-1}(D)$.

In the following part for $A \in L^X, a \in M(L), \mathscr{A} \in L^X$

$A^{q_a} = \vee\{x_a \in M^{\bullet}(L^X) \mid x_a \not\leq A\}$, $\mathscr{A}^{q_a} = \{A^{q_a} \mid A \in \mathscr{A}\}$.

Lemma 1.4[6] Let (L^X, δ) be $L-fts$. $D \in L^X, a \in M(L)$, and $\mathscr{A} \subset L^X$. $D \wedge \mathscr{A}^{q_a}$. is $\alpha-$ locally finite in D if and only if $\tau_a(D) \cap (\tau_a(\mathscr{A}))'$ is locally finite in the subspace $\tau_a(D)$ of general topological space $(X, l_{a'}(\delta))$.

II Definitions and characters

Definition 2.1 Let (L^X, δ) be $L-fts$. $D \in L^X, a \in M(L)$. D is $\alpha-III$ countablly paracompact ($\alpha-$ countablly paracompact),if for every $\alpha-$ countable remote domain family Φ of D there exists $\alpha-$ remote domain family Ψ of D satisfying the following conditions:

(i) Ψ is corefine of Φ ,namely $\forall B \in \Psi, \exists A \in \Phi$ such that $A \leq B$.

(ii) $D \wedge \Psi^{q_a}$ is locally finite in D where $\Psi^{q_a} = \{B^{q_a} : B \in \Psi\}$.

D is $\alpha-$ countablly paracompact if D is $\alpha-$ countablly paracompact for $\forall a \in M(L)$ .If D equals to 1_X , (L^X, δ) is $\alpha-$ countablly paracompact or countally paracompact.

Theory 2.2 $\alpha-III$ paracompact set in $L-fts$ is $\alpha-$ countablly paracompact set.

Theory 2.3 Countable strong paracompact set in $L-fts$ is countable paracompact set.

Theory 2.4 Let (L^X, δ) be $L-fts$. $D \in L^X$ and $a \in M(L)$. D is $\alpha-$ countablly paracompact if and only if $\tau_a(D)$ is countablly paracompact in $(X, l_{a'}(\delta))$

Proof Let D be $\alpha-$countablly paracompact in (L^X, δ) and $\mathscr{U}$ countable open covering in the subspace of $\tau_a(D)$ of $(X, l_{a'}(\delta))$.Then there exists countable subfamily $\Psi \subset \delta'$ such that $\tau_a(D) \cap l_{a'}(\psi') = \tau_a(D) \cap (\tau_a(\psi))' = \mathscr{U}$, Also Ψ is $\alpha-$countable remote family of D. From $\alpha-$countable paracompactn- ess of D there exists $\alpha-$remote domain family Φ of D such that Φ is corefine of Ψ and $D \wedge \Phi^{q_a}$ is $\alpha-$locally finite in D.It is easy to know that $\tau_a(D) \cap l_{a'}(\Phi')$ is refine of $\mathscr{U}$ and open covering of subspace $\tau_a(D)$ of $(X, l_{a'}(\delta))$. By Lemma 1.4 it is locally finite in $\tau_a(D)$. Therefore $\tau_a(D)$ is countable paracompact in $(X, l_{a'}(\delta))$.

On the contrary let $\tau_a(D)$ be countable paracompact in $(X, l_{a'}(\delta))$.Because Φ is $\alpha-$countable rmote family of D in (L^X, δ), $\tau_a(D) \cap l_{a'}(\Phi')$ is countable open covering of subspace $\tau_a(D)$ of $(X, l_{a'}(\delta))$. Then there exists $\Psi \subset \delta'$ such that $l_{a'}(\Psi') \cap \tau_a(D)$ is locally finite open covering of subspace $\tau_a(D)$ of $(X, l_{a'}(\delta))$ and corefine of $\tau_a(D) \cap l_{a'}(\Phi')$.So Ψ is $\alpha-$remote family of D in (L^X, δ).By Lemma we know that $D \wedge \psi^{q_a}$ is $\alpha-$locally finite in D.For arbitrary element B of Ψ there exists $A_B \in \Phi$ such that

$\tau_a(D) \cap l_{a'}(B') \subset \tau_a(D) \cap l_{a'}(A'_B)$.Let $\Omega = \{B \vee A_B \mid B \in \Psi\}$.Then

$\tau_a(D) \cap l_{a'}(\Omega') = \{\tau_a(D) \cap l_{a'}((B \vee A_B)') \mid B \in \Psi\} = \{\tau_a(D) \cap l_{a'}(B') \cap l_{a'}(A'_B) \mid B \in \Psi\} =$

$\{\tau_a(D) \cap l_{a'}(B') \mid B \in \Psi\} = \tau_a(D) \cap l_{a'}(\Psi')$.

It is easy to prove that Ω is $\alpha-$remote family of D in (L^X, δ) and corefine of Φ. For arbitrary $P \in \Omega$ there exists $B \in \Psi$ such that $P = B \vee A_B$.Therefore $P^{q_a} \le B^{q_a}$.Then $D \wedge \Omega^{q_a}$ is $\alpha-$locally finite if D. So D is $\alpha-$countable paracompact in (L^X, δ).

If (L^X, δ) is a weakly induced space[7], $l_{a'}(\delta) = [\delta]$ for arbitrary $a \in M(L)$.

Corollary 2.5 Let (L^X, δ) be weakly induced $L-tfs$. The following conditions are equivalent:

(i) (L^X, δ) is countable paracompact.

(ii) there exists $a \in M(L)$ such that (L^X, δ) is $\alpha-$ countable paracompact.

(iii) $(X, [\delta])$ is countable paracompact.

Corollary 2.6 Countable paracompactness is $L-good\ extension$.

In the following part we introduce a new operator. By this new operator we can show countable paracompactness in a novel way.

Definition 2.7 Let (L^X, δ) be $L-fts$. $a \in M(L)$. A new operator is defined by

$$a^{\bullet}: L^X \to \delta', \forall A \in L^X, a^{\bullet}(A) = \wedge\{\ G : G \in \delta' \text{ and } G \vee A \geq a\}.$$

Proposition 2.8 Let (L^X, δ) be $L-fts$. $a \in M(L)$, $A, B \in L^X$, there are following remarks:

(i) $a^{\bullet}(A) \in \delta'$;

(ii) $A \vee a^{\bullet}(A) \geq a$;

(iii) $A \leq B \Rightarrow a^{\bullet}(B) \leq a^{\bullet}(A)$;

(iv) If $A \in \delta'$ $A \vee B \geq a \Leftrightarrow a^{\bullet}(B) \leq A$;

(v) If A and $B \in \delta'$, $A \vee B \geq a \Leftrightarrow a^{\bullet}(B) \leq A \Leftrightarrow a^{\bullet}(A) \leq B$.

It is easy to know that operator $a^{\bullet}$ is extension of the operator of the paper[5].

Theory 2.9 Let (L^X, δ) be $L-fts$. $a \in M(L)$. (L^X, δ) is $\alpha-$ countable paracompact if and only if for every $\alpha-$ countable remote domain family Ω of 1_X there exists $\alpha-$ remote domain family Ψ of 1_X such that Ψ is corefine of dΩ and $a^{\bullet}(\Psi) = \{a^{\bullet}(A) : A \in \Psi\}$ is $\alpha-$ locally finite in 1_X.

Proof From theory 2.4 it is enough to prove that conditions of theory 2.9 are equivalent to countable paracomapctness of $(X, L_{a'}(\delta))$.

Let $(X, l_{a'}(\delta))$ be countable paracompact and Ω be $\alpha-$ countable remote domain family of 1_X. Then $l_{a'}(\Omega') = \{l_{a'}(Q') : Q \in \Omega\}$ is countable open covering of $(X, l_{a'}(\delta))$. Therefore there exists open refinement $l_{a'}(\Delta)(\Delta \subset \delta)$ of $l_{a'}(\Omega')$ is locally finite in $(X, l_{a'}(\delta))$. For arbitrary B of Δ there exists $Q_B \in \Omega$ such that $l_{a'}(B) \subset l_{a'}(Q'_B)$. Let $\Psi = \{B' \vee Q_B : B \in \Delta\}$. Then Ψ is corefine of Ω. In addition, Δ' is $\alpha-$ remote domain family of 1_X. So for $\forall x \in X$ there exists $B \in \Delta$ such that $x_a \not\leq B'$, namely

$x \in l_{a'}(B)$.Therefore $x \in l_{a'}(Q'_B)$,namely $x_a \not\leq Q_B$.This shows that Ψ is $\alpha -$ remote domain family of 1_X.In the following part we only prove that $a^\bullet(\Psi)$ is $\alpha -$ locally finite in 1_X.

For $\forall x \in X$ there exists open domain $l_{a'}(W)$ of x and finite subfamily Δ_0 of Δ such that $l_{a'}(B) \cap l_{a'}(W) = l_{a'}(B \wedge W) = \emptyset$. for $\forall B \in \Delta - \Delta_0$.It is easy to know that for $W' \in \eta(x_a)$ and $\forall B \in \Delta - \Delta_0, B \wedge W \leq a'$ and $B' \vee W' \geq a$.Furthermore, $B' \vee Q_B \vee W' \geq a$.By proposition 2.8 $a^\bullet(B' \vee Q_B) \leq W'$.Therefore $a^\bullet(\Psi)$ is $\alpha -$ locally finite in 1_X.

On the contrary, let $l_{a'}(\Omega')(\Omega \in \delta')$ be countable open covering of $(X, l_{a'}(\delta))$.So $a^\bullet(\Psi)$ is $\alpha -$ countable remote family of 1_X.Therefore there exists $\alpha -$ remote family Ψ of 1_X such that Ψ is corefine of Ω and $a^\bullet(\Psi)$ is $\alpha -$ locally finite in 1_X.It is easy to know that $l_{a'}(\Psi')$ is open refinement of $l_{a'}(\Omega')$ For $\forall x \in X$ there exists $P \in \eta^-(x_a)$ and finite subfamily Ψ_0 of Ψ such that $a^\bullet(B) \leq P$ for $\forall B \in \Psi - \Psi_0$.So $B \vee P \geq a$ and $l_{a'}(B' \wedge P') = l_{a'}(B') \cap l_{a'}(P') = \emptyset$.From $l_{a'}(P')$ is open domain of x we know that $l_{a'}(\Psi')$ is locally finite In $(X, l_{a'}(\delta))$.Therefore $(X, l_{a'}(\delta))$ is countable paracompact.

End of proof.

In the following part we show $\alpha -$ countable paracompactness via $\alpha -$ family consisting of $\alpha -$ closed sets.

Definition 2.10 Let (L^X, δ) be $L - fts$. $a \in M(L)$ and. $A \in L^X$ A is $\alpha -$ closed if

$\forall x_a \in M^\bullet(L^X), x_0 \in A^- \Rightarrow x_a \in A$.

Theory 2.11 Let (L^X, δ) be $L - fts$. $a \in M(L)$. (L^X, δ) is $\alpha -$ countable paracompact if and only if for every $\alpha -$ closed $\alpha -$ countable family Ω of 1_X there exists $\alpha -$ closed $\alpha -$ family Ψ of 1_X such that Ψ is corefine of Ω and $a^\bullet(\Psi)$ is $\alpha -$ locally finite in 1_X.

Proof Let (L^X, δ) be $\alpha -$ countable paracompact and Ω be $\alpha -$ closed $\alpha -$ countable family of 1_X. From definition of $\alpha -$ closed set we know that $\Omega^- = \{Q^- : Q \in \Omega\}$ is $\alpha -$ countable remote domain family of 1_X.Therefore there exists $\alpha -$ remote domain family Ψ of 1_X such that Ψ is corefine of Ω^- and $a^\bullet(\Psi)$ is $\alpha -$ locally finite in 1_X.Because closed set is $\alpha -$ closed set , Ψ above mentioned satisfied requirement of theory.

On the contrary, let Ω be $\alpha-$remote domain family of 1_X .So Ω is $\alpha-$closed and $\alpha-$countable family. Therefore there exists $\alpha-$closed and $\alpha-$family Ψ such that Ψ is corefine of Ω and $a^{\bullet}(\Psi)$ is $\alpha-$locally finite in 1_X .Consider Ψ^-, Ψ^- is corefine of Ω .By $\alpha-$closeness of Ψ it is easy to know that Ψ^- is $\alpha-$remote domain family of 1_X .from proposition 2.8 (iii) $a^{\bullet}(\Psi^-)$ is $\alpha-$locally finite in 1_X . Therefore (L^X,δ) is $\alpha-$countable paracompact.

Theory 2.12 Let (L^X,δ) be $L-fts$. $a \in M(L)$ .So (L^X,δ) is $\alpha-$countable paracompact if and only if for every $\alpha-$remote domain family of 1_X $\Phi=\{A_i : i \in N\}$ there exists $\alpha-$remote domain family of 1_X $\Psi=\{B_i : i \in N\}$ such that $A_i \le B_i$ and $a^{\bullet}(\Psi)$ is $\alpha-$locally finite in 1_X for $\forall i \in N$.

Proof Let (L^X,δ) be $\alpha-$countable paracompact and $\Phi=\{A_i : i \in N\}$ be $\alpha-$countable remote domain family of 1_X .There exists corefine Φ of $\alpha-$remote domain family Ω . $a^{\bullet}(\Omega)$ is $\alpha-$locally finite in 1_X . For $\forall B \in \Omega$ there exists $i(B) \in N$ such that $A_{i(B)} \le B$.

Let $B_i = \bigvee_{i(B)=i} B$ .Then $A_i \le B_i$. Let $\Psi=\{B_i : i \in N\}$ .Then Ψ is $\alpha-$countable remote domain family of 1_X and corefine of Φ .In the following part it is enough to prove that $a^{\bullet}(\Psi)$ is $\alpha-$locally finite in 1_X .Because $a^{\bullet}(\Omega)$ is $\alpha-$locally finite in 1_X for $\forall x_a \in M^{\bullet}(L^X)$ there exists $P \in \eta(x_a)$ and finite subfamily Ω_0 of Ω such that $a^{\bullet}(B) \le P$ for $\forall B \in \Omega-\Omega_0$ .By proposition 2.8 $P \vee B \ge a$ .Then $P \vee B_i \ge a$,namely $a^{\bullet}(B_i) \le P$ .Because Ω_0 is finite ,The number of B_i whose elements belong to Ω_0 must be finite. Therefore $a^{\bullet}(\Psi)$ is $\alpha-$locally finite in 1_X .

It is easy to prove sufficienc.

End of proof.

Definition 2.13 Let $\{F_i\}_{i\in N} \subset L^X$ and $a \in M(L)$. $\{F_i\}_{i\in N}$ is $\alpha-$decreasing if $\tau_a(F_1) \supset \tau_a(F_2) \supset \dots\dots\dots$

Theory 2.14 Let (L^X,δ) be $L-fts$. $a \in M(L)$ .Then (L^X,δ) is countable paracompact if and only if for every $\alpha-$decreasing $\alpha-$remote domain family $\{F_i\}_{i\in N}$ there exists $\{W_i\}_{i\in N} \subset \delta$ such that

$W_i' \in \eta_a(F_i), i \in N$ and $\bigcap_{i=1}^{\infty} \overline{l_{a'}(W_i)} = \emptyset$

Proof It is easy to be proved by theory 2.4 and results of general topology.

III Properties of countable paracompactness

The following theories show that $(\alpha-)$ is closely genetic

Theory 3.1 Let (L^X, δ) be $L-fts$. $a \in M(L)$. $A \in L^X$ and $B \in \delta'$.

(i) If A is $\alpha-$ countable paracompact, $A \wedge B$ is $\alpha-$ countable paracompact,too.

(ii) If A iscountable paracompact, $A \wedge B$ is countable paracompact,too.

Proof It is enough to prove that (i) is right.Let Φ be $\alpha-$ countable remote family of $A \wedge B$ .Then $\Phi \bigcup \{B\}$ be $\alpha-$ countable remote family of A. Because A is $\alpha-$ countable paracompact, there exists $\alpha-$ countable remote family Ψ of A such that Ψ is corefine of $\Phi \bigcup \{B\}$ and $A \wedge \Psi^{q_a}$ is $\alpha-$ locally finite in A .Let $\Omega = \{P \in \psi \mid B \not\leq P\}$,then Ω is α -countable remote domain family of A ∧ B and corefine of Φ .It is easy to know that $A \wedge \Omega^{q_a}$ is $\alpha-$ locally finite in $A \wedge B$ .So $A \wedge B \wedge \Omega^{q_a}$ is $\alpha-$ locally finite in $A \wedge B$ .Therefore $A \wedge B$ is $\alpha-$ countable paracompact.

Theory 3.2 Let (L^X, δ) and (L^Y, μ) be $L-fts$ and A be a countable paracompact set of (L^X, δ) .And B is a strong F paracompact set of (L^Y, μ). Then $A \times B$ is a countable paracompact set of $(L^{X \times Y}, \delta \times \mu)$.

Proof By theory 2.4 $\tau_a(A)$ is a countable paracompact set of $(X, l_{a'(\delta)})$for $\forall a \in M(L)$ .It is easy to know that B strong F compact in (L^Y, μ) if and only if $\tau_a(B)$ is compact in $(Y, l_{a'}(\mu))$ for $\forall a \in M(L)$. From some results of general topology $\tau_a(A \times B) = \tau_a(A) \times \tau_a(B)$ is countable paracompact in $X \times Y, l_{a'}(\delta \times \mu))$ .From theory 2.4 this theory is right.

Definition 3.3 Let (L^X, δ) be $L-fts$. $a \in M(L)$. (L^X, δ) is a $\alpha-$ completely normal space if (L^X, δ) is $\alpha-$ normal.and for $\forall F \in \delta'$ there exist $G_i \in \delta, i = 1,2, \ldots$.such that $\tau_a(F) = \bigcap_{i \in N} l_{a'}(G_i)$. If for $a \in M(L)$ (L^X, δ) is $\alpha -$ completely normal, (L^X, δ) is completely normal.

It is to know that (L^X, δ) is $\alpha -$ completely normal if and only if (L^X, δ) is $\alpha -$ normal and for $G \in \delta$ there exist $F_i \in \delta^{\prime -}$ $, i=1,2,\ldots$ such that $l_{a'}(G) = U_{\tau a}(F_i)$.

Theory 3.4 Let (L^X, δ) be $L - fts$. $a \in M(L)$. (L^X, δ) is $\alpha -$ completely normal if and only if $(X, l_{a'}(\delta))$ is completely normal.

Theory 3.5 Let (L^X, δ) be $L - fts$. $a \in M(L)$ .If (L^X, δ) is $\alpha -$ completely normal (L^X, δ) is $\alpha -$ countable paracompact.

Proof It is proved by theory 3.4 and some results of general topology.

Theory 3.6 Let (L^X, δ) and (L^Y, μ) $L - fts$ which are induced. If (L^X, δ) is a countable paracompact level normal space[6] and (L^Y, μ) is a strong F compact II countable space, $(L^{X \times Y}, \delta \times \mu)$ is level normal .

Proof From theory 2.4 $(X, [\delta])$ is a countable paracompact normal space.By [7] $(Y, [\mu])$ is a compact II countable space. So from some results of general topology([8]theory 5.2.7) $X \times Y$ is normal. By [7] $(L^{X \times Y}, \delta \times \mu)$ is generated by $X \times Y$ .Therefore $(L^{X \times Y}, \delta \times \mu)$ is a level normal space.

Theory 3.7 Let (L^X, δ) and (L^Y, μ) $L - fts$ weakly which are induced weakly. $f : (L^X, \delta) \to (L^Y, \mu)$ is continuously closed and full L-value Zadeh function. If (L^X, δ) is a countable paracompact level normal space, (L^Y, μ) is a countable paracompact level normal space.

Proof Because (L^X, δ) is a weakly induced space, from theory 2.5 $(X, [\delta])$ is a countable paracompact space. By[6] $(X, [\delta])$ is a level normal space. Because $f : (L^X, \delta) \to (L^Y, \mu)$ is continuously closed and full L-value Zadeh function, general mapping $f : (X, [\delta]) \to (Y, [\mu])$ is continuous closed full mapping. From some results of general topology $(Y, [\mu])$ is countable paracompact level normal space. It is provedby theory2.5 (L^Y, μ) is a countable paracompact level normal space.

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