L— fuzzy Modules and L—fuzzy quotient modules

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Abstract: In this paper, we studied the homomorphism and isomorphism of L— fuzzy modules, and gave the concept of L—fuzzy quotient modules .
Keywords: complete distributive lattice, L— fuzzy modules, homomorphism, isomorphism , L—fuzzy quotient Module.

1. Preliminaries

In this paper, L、L_1、L_2 and L_3 are all complete distributive lattices with 0 and 1, L^X be the set of whole L—fuzzy subset of X. Let R be a ring with 1, unless specially stated, the module only refers the left module over ring R in this paper.

Definition 1.1 [1]. Let M be left module over ring R, A be a L—fuzzy subset of M, if fo any $x,y \in M$, $r \in R$, we have

1) $A(x+y) \geq A(x) \wedge A(y)$;
2) $A(rx) \geq A(x)$;
3) $A(x) \geq A(-x)$;
4) $A(0)=1$,

then A is called a L—fuzzy submodule of M. In this paper, $F_L(M)$ be the set of whole L—fuzzy submodule of M.

Definition 1.2 If mapping $\tau : L_1 \to L_2$ holds arbitary intersection and union properties, and τ (0)=0, then τ is a homomorphism of lattices. If τ is a homomorphism of lattices, then τ is a isomorphism of lattices.

Definition 1.3 Let f be the mapping: $X_1 \to X_2$, $\tau : L_1 \to L_2$ is a homomorphism of lattices, if A is a L_1—fuzzy subset of X_1, then the L_2—fuzzy subset of X_2 is defined by:

$$(f,\tau)(A)(x_2) = \vee\{\tau(A)(x_1) \mid x_1 \in X_1, f(x_1)=x_2\}, \forall x_2 \in X_2$$

If B is a L_2—fuzzy subset of X_2, then the L_1—fuzzy of X_1 is defined by:

$$(f,\tau)^{-1}(B)(x_1) = \vee\{\beta \mid \tau(\beta) \leq B(f(x_1)), \beta \in L_1\}, \forall x_1 \in X_1$$

Proposition 1.1[3]. Let $f: M_1 \to M_2$ is a homomorphism of modules, and $\tau: L_1 \to L_2$ is a homomorphism of lattices, $A \in F_{L_1}(M_1)$, then $(f,\tau)(A) \in F_{L_2}(M_2)$.

Proposition 1.2[3] Let $f: M_1 \to M_2$ is a homomorphism of modules, and $\tau: L_1 \to L_2$ is a homomorphism of lattices, $B \in F_{L_2}(M_2)$, then $(f,\tau)^{-1}(B) \in F_{L_1}(M_1)$.

2. The Homomorphism and Isomorphism of L— fuzzy Modules

Definition 2.1 Let $A \in F_{L_1}(M_1), B \in F_{L_2}(M_2)$, if the mapping (f,τ) of A into B which satisfies:

1) $f: M_1 \to M_2$ is a homomorphism of modules, ;

2) $\tau: L_1 \to L_2$ is a homomorphism of lattices;

3) $(f,\tau)(A) \leq B$,

then (f,τ) is a homomorphism of A into B.

$hom\ (A,B)$ denotes the set of whole homomorphisms of A into B, then we have $(f,\tau) \in hom\ (A,B)$.

Definition 2.2 Let $A \in F_{L_1}(M_1), B \in F_{L_2}(M_2)$, if the mapping (f,τ) of A into B which satisfies:

1) $f: M_1 \to M_2$ is a iomomorphism of modules ;

2) $\tau: L_1 \to L_2$ is a iomomorphism of lattices;

3) $(f,\tau)(A) = B$,

then (f,τ) is a isomorphism of A into B, written as $A \cong B$.

Proposition 2.1 a) Let $f: M_1 \to M_2$ homomorphism of modules, $\tau: L_1 \to L_2$ is a homomorphism of lattices, then $(f,\tau) \in hom\ (A,B)$ iff for any $\lambda \in L_1, f(A_\lambda) \subseteq B_{\tau(\lambda)}$.

b) Let $f: M_1 \to M_2$ is a iomomorphism of modules, $\tau: L_1 \to L_2$ is a iomomorphism of lattices, then (f,τ) is a isomorphism of A into B iff for any $\lambda \in L_1, f(A_\lambda) = B_{\tau(\lambda)}$.

Proposition 2.1 can be easily established by proposition 2 of [2].

Proposition 2.2 Let $(f,\tau)\in hom(A,B)$, $(g,\phi)\in hom(B,C)$, " $\circ$ " is defined by $(g,\phi)\circ(f,\tau)=(g\circ f,\phi\circ\tau)$,and $A\in F_{L_1}(M_1)$, $B\in F_{L_2}(M_2)$, $C\in F_{L_3}(M_3)$,then $(g\circ f,\phi\circ\tau)\in hom(A,C)$, and " $\circ$ " satisfies associative law .

Proof: It is quite evident that $g\circ f$ is a homomorphism of modules,and $\phi\circ\tau$ is a homomorphism of lattices, because $(f,\tau)\in hom(A,B)$, $(g,\phi)\in hom(B,C)$, so $(f,\tau)(A)\le B$,$(g,\phi)(B)\le C$,then we have

$$\begin{aligned}(g\circ f,\phi\circ\tau)(A)&=((g,\phi)\circ(f,\tau))(A)\\&=(g,\phi)\circ((f,\tau)(A))\\&\le(g,\phi)(B)\le C\end{aligned}$$

then $(g\circ f,\phi\circ\tau)\in hom(A,C)$.

It is quite evident that " $\circ$ " satisfies associative law .

3. L—fuzzy quotient modules

Proposition 3.1 Let $A\in F_L(M)$, N be a submodule of M , ${}^{A}Z_N$ is defined by ${}^{A}Z_N(x+N)=\vee_{n\in N}A(x+n)$, then ${}^{A}Z_N$ is a L—fuzzy submodule of M/N, and ${}^{A}Z_N$ is called a L—fuzzy quotient module.

Proof: Let $f: M\to M/N$, $x\to x+N$, it is every clear that f is a surjective homomorphism of M into M/N , for any $x+N\in M/N$

$$\begin{aligned}f(A)(x+N)&=\vee_{f(y)=x+N}A(y)=\vee_{y+N=x+N}A(y)=\vee_{y\in x+N}A(y)\\&=\vee_{n\in N}A(x+n)={}^{A}Z_N(x+N)\end{aligned}$$

that is $f(A)={}^{A}Z_N$,and ${}^{A}Z_N$ is a L—fuzzy submodule of M/N by proposition 1.1 .

Proposition 3.2 Let f is a surjective homomorphism of M_1 into M_2, $N=\ker f$, f^{*} is a iomomorphism of M_1/N into M_2 by $x+N\to f(x)$,if (f,τ) is a surjective homomorphism of A into B , τ is a iomomorphism of L_1 into L_2, then (f^{*},τ) is a iomomorphism of ${}^{A}Z_N$ into B.

Proof: For any $x_2 \in M_2$

$$(f^*,\tau)({}^{A}Z_N)(x_2)=\bigvee_{f^*(x_1+N)=x_2} \tau({}^{A}Z_N)(x_1+N)$$

$$=\bigvee_{f(x_1)=x_2} \tau(\bigvee_{n\in N} \tau(A(x_1+n)))$$

$$=\bigvee_{\substack{f(x_1)=x_2\\ n\in N}} \tau(A(x_1+n))$$

$$=\bigvee_{f(u)=x_2} \tau(A(u))$$

$$=(f,\tau)\, A(x_2)=B(x_2)$$

That is $(f^*,\tau)({}^{A}Z_N)=B$.

So (f^*,τ) is a iomomorphism of ${}^{A}Z_N$ into B by definition 2.2.

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