Property of Fuzzy sub-transposed Matrix and sub-symmetric Matrix

Shi Junxian **Wang Hongxu**

(Liaoyang College of Petrochemical Technology,China, Liaoning, Liaoyang 111003)

Abstract: In reference [1] the concepts of fuzzy sub-transposed matrix and fuzzy sub-symmetric matrix has been presented. In this paper, their properties will been expounded.

Key words: fuzzy matrix, fuzzy sub-transposed matrix, and fuzzy sub-symmetric matrix.

0. Introduction

[1] has raised concepts for fuzzy sub-transposed matrix and fuzzy sub-symmetric matrix.. In this paper, we discuss their properties.

Let L=[0,1] matrix A=$(a_{ij})_{n\times m}, a_{ij} \in L$ is called $n\times m$ fuzzy matrix. And $L^{n\times m}$ is a set of all $n\times m$ fuzzy matrixes. Concepts and signs, which aren't particularly pointed out in this paper, will be found in reference [1].

1.Fuzzy sub-transposed matrix and its properties

Definition 1[1]: Let A=$(a_{ij})_{n\times m}, a_{ij} \in L$, $A^{ST} = (a_{kh}^{ST}) \in L^{m\times n}$ is called fuzzy sub-transposed matrix of A, where $a_{kh}^{ST} = a_{n-h+1,m-k+1}$; $i,h = 1,2,\cdots n.$ $j,k = 1,2,\cdots m.$

Theorem 1 Fuzzy sub-transposed matrix satisfies the following properties:

(1) $(A^{ST})^{ST} = A$

(2) $(A^{C})^{ST} = (A^{ST})^{C}$ where A^{C} is complement of matrix A.

(3) $(\lambda A)^{ST} = \lambda(A^{ST})$ where $\lambda \in L$, λA is equal to the product of the number λ and the fuzzy matrix A.

(4) If $A,B \in L^{n\times m}$ then $(A\cup B)^{ST} = A^{ST} \cup B^{ST}, (A\cap B)^{ST} = A^{ST} \cap B^{ST}$.

(5) If $A,B \in L^{n\times m}$ then $A \le B$ if and only if $A^{ST} \le B^{ST}$.

(6)[1] If $A \in L^{n\times m}$, $B \in L^{m\times t}$ then $(A\circ B)^{ST} = B^{ST} \circ A^{ST}$. Where "$\circ$" express the composite operation for fuzzy matrix.

(7) Under composite operation conditions of fuzzy matrices,

$$(A_1 \circ A_2 \circ \cdots \circ A_k)^{ST} = A_k^{ST} \circ A_{k-1}^{ST} \circ \cdots \circ A_2^{ST} \circ A_1^{ST}$$

(8) If $A \in L^{n\times n}$ then $(A^k)^{ST} = (A^{ST})^k$, where k is positive integer.

2. Fuzzy sub-symmetric matrix and its properties

Definition 2[1]: Let $A = (a_{ij}) \in L^{n\times n}$ and $A^{ST} = A$, the $A^{ST} = A$ means $a_{n-j+1,n-i+1} = a_{ij}$,

$i, j = 1,2,\cdots n.$ then A is called n-order fuzzy sub-symmetric matrix.

From the definition 2,we list n-order fuzzy sub-symmetric matrix as fellows:

$$B_n = \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n-2} & b_{1n-1} & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2n-2} & b_{2n-1} & b_{1n-1} \\ b_{31} & b_{32} & b_{33} & \cdots & b_{3n-1} & b_{2n-2} & b_{1n-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{n-2,1} & b_{n-2,2} & b_{n-2,3} & \cdots & b_{33} & b_{23} & b_{13} \\ b_{n-1,1} & b_{n-1,2} & b_{n-2,2} & \cdots & b_{32} & b_{22} & b_{12} \\ b_{n,1} & b_{n-1,1} & b_{n-2,1} & \cdots & b_{31} & b_{21} & b_{11} \end{pmatrix}$$

Definition 3 The connecting line of element b_{1n} and b_{n1} of the n-order fuzzy matrix is called auxiliary diagonal.

Definition 4 Let B=$(b_{ij}) \in L^{n \times n}$, if b_{ij} and b_{kh} satisfy $b_{ij} = b_{kh}$ and $j+k=i+h=n+1$,then b_{ij} and b_{kh} are symmetric on the auxiliary diagonal. Where b_{ij}, b_{kh} are elements on both sides of auxiliary diagonal of matrix B.

Theorem 2 N-order fuzzy sub-symmetric matrix is a symmetric n-order fuzzy matrix on the auxiliary diagonal.

Theorem 3 $\quad b_{1n}, b_{2,n-1}, \cdots, b_{n-1,2}, b_{n1}$ ......(2-1)

are elements on the auxiliary diagonal of the n-order sub-symmetric fuzzy matrix B=(b_{ij}), here suffix sum of arbitrary b_{ij} on the auxiliary diagonal of B is n+1.

In fact, for B is a sub-symmetric fuzzy matrix, then $b_{ij} = b_{n-j+1,n-i+1}$ (for all b_{ij} on the auxiliary diagonal of the B) and for b_{ij} is an element on auxiliary diagonal of the B, then $i = n - j + 1$, $j = n - i + 1$ so $i + j = n + 1$. If $i = 1,2,\cdots,n$, then $j = n, n-1,\cdots,2,1$. So the elements on the auxiliary diagonal of the n-order sub-symmetric fuzzy matrix B=(b_{ij}) is $b_{1n}, b_{2,n-1}, \cdots, b_{n-1,2}, b_{n1}$. where the element's suffix sum is n+1.

Theorem 4 The number of different elements in the fuzzy sub-symmetric matrix is $\dfrac{n(n+1)}{2}$.

Notice that in the n-order fuzzy sub-symmetric matrix, there are two types of elements. One type is that B contains two such elements, for all b_{ij} in the B its feature is $b_{ij} = b_{n-j+1,n-i+1}$ and $i \neq n - j + 1$ or $j \neq n - i + 1$ at least one of two exists. Another is that B only contains one such kind elements, which are the elements on the auxiliary diagonal. The number of them is n, as formula (2-1). Then, definition can be derived as follows.

Definition 5 In the n-order fuzzy sub-symmetric matrix B, when B contains two b_{ij}, b_{ij} can be called as double-element type. When B only contains one b_{ij}, b_{ij} can be called as single-element type.

Theorem 5 Let B=$(b_{ij}) \in L^{n\times n}$ is fuzzy sub-symmetric matrix, then the elements of formula (2-1) listed are all single elements for B, and the number of double elements for B is $\frac{n(n-1)}{2}$.

Definition6 Let B=$(b_{ij}) \in L^{n\times n}$,if i row vector of B can be signed as $(b_{i1}, b_{i2}, \cdots, b_{in})$, $i \in \{1,2,\cdots,n\}$, and n-i+1 column vector of B can be signed as $\begin{bmatrix} b_{in} \\ \vdots \\ b_{i2} \\ b_{i1} \end{bmatrix}$, then i row vector of B and n-i+1 column vector of B are anti-symmetric each other. It can be noted as $(b_{i1}, b_{i2}, \cdots, b_{in})^{ST} = \begin{bmatrix} b_{in} \\ \vdots \\ b_{i2} \\ b_{i1} \end{bmatrix}$.

Theorem 6 Let B=$(b_{ij}) \in L^{n\times n}$ is fuzzy sub-symmetric matrix, for i=1,2, ...,n, i row vector of B and n-i+1 column vector are anti-symmetry each other.

It is can be seen that in the structure of fuzzy sub-symmetric, the auxiliary diagonal is symmetric axis, i row and n-i+1 column are anti-symmetry each other.

References

[1] Zhang San-hua, Chen Gou-shu . The sub-realizable Problem for Fuzzy sub-symmetric Matrices and sub-realization Conditions. Journal of Sichuan Normal University (Natural Science) May,2000, Vol.23,242~246