Theory of Pointwise Fuzzy Groups

Gu Wen-Xiang
Department Of Computer Science, Northeast Normal University, Chang Chun, JiLin, 130024, China

Abstract: 1. First we point out the following questions in [2,3]: (1) The fundamental theorem (Theorem 2 of [2]) of homomorphism of fuzzy groups is wrong. (2) The fundamental theorem of homomorphism in [3] is not the generalization of the general group's. (3) The definition of the M-fuzzy subgroup in [3](definetin3.1 of [3]) is unreasonable. We proceed to indicate it is necessary to begin the study of the fundamental concepts and results of the pointwise fuzzy groups anew. 2. In this paper the reasonable parts in [2,3] such as the definitions of the pointwise fuzzy group, fuzzy normal subgroup are kept, we also redefine some unreasonable concepts in [2,3] such as the fuzzy quotient and the M-fuzzy subgroup. Based on these we have proved the fundamental theorem of homomorphism of the pointwise fuzzy groups, which is the generalization of the relevant theorem in general group theory.

Keywords: Fuzzy group; pointwise fuzzy group; fuzzy normal subgroup; homomorphism.

1.Introduction

The study of fuzzy group was started with the pioneering paper of Rosenfeld [1], and Qi [2] gave a pointwise definition of fuzzy group by fuzzy point. To distinguish with Rosenfeld's definition, we call it pointwise fuzzy group. Paper [2], which plays an important part in the researching of pointwise fuzzy groups and which [3-7] were based on, also gave some fundamental qualities of the pointwise fuzzy groups. But what is most regret is the important theorem-Theorem2 (Fundamental theorem) has a wrong proof. This is because when proving $\overline{A} \cong A'$ the author used $\phi_1 : (aH)_\mu \to \phi(a_\mu)$ $\overline{A}(aH)_\mu = \lambda'/\lambda\ SupA(x), A(e) = \lambda, H(e) = \lambda'$, x∈ Supp(aH). So when $(aH)_\mu \in \overline{A}$, it's obvious that $a_\mu \in A$ isn't always true. If $a_\mu \notin A$, $\phi(a_\mu)$ has no meaning. Hence ϕ_1 isn't the mapping of $\overline{A} \to A'$, and further a isomorphism.

Although the following theorem had a strict proof in [3] : Let ϕ is a homorphism from A to A', $H = Ker\phi$, then the mapping $v(a_\mu) = (aH)_{\frac{\lambda'}{\lambda}\mu}$ is a homomorphism of $A \to A/H$, and the mapping $\overline{\phi}((aH)_\mu) = \phi(a_\mu)$ is a homomorphism of $\overline{A} \to A'$. But it is regret that when $H(e) = \lambda' \neq \lambda = A(e)$, we have $(\overline{\phi}v)(a_\mu) = \overline{\phi}((aH)_{\frac{\lambda'}{\lambda}\mu}) = \phi(a_{\frac{\lambda'}{\lambda}\mu}) \neq \phi(a_\mu)$ for any a_μ in A and $\overline{\phi}v \neq \phi$. This shows the above theorem in [3] isn't the generalization of the general fundamental theorem of homomorphisms.

In addition, according to Definition 2.1 in [3], if A is an M-fuzzy pointwise group, then A is a closed group. But it is obviously that the subgroup of a closed group isn't

always a closed. So Definition 3.1 in [3] was unreasonable.
The above questions tell us it is necessary to begin the researching of the based concepts and results of the pointwise fuzzy group anew.
In this paper the reasonable parts in [2,3] such as the pointwise fuzzy group, fuzzy normal subgroups are kept. And the unreasonable parts such as fuzzy quotient, homomorphism, M-fuzzy subgroup have been redefined. Based on these improvements, we get the strict proof of the fundamental theorem of homomorphisms of the pointwise fuzzy group which is the generalization of the relevant theorem in general group theory.

2.Pointwise fuzzy groups

Let X be a nonempty set. x_λ is called a fuzzy point on X if the following condition is fitted $x_\lambda(y)=\begin{cases}\lambda, y=x\\ 0, y\neq x\end{cases}$ $x,y\in X, and \lambda\in(0,1]$

x is called a supporting point of x_λ.
Assume A is a fuzzy set of X. If $0<\lambda\leq A(x)$, we call fuzzy point x_λ belong to A and can be written as $x_\lambda\in A$. It's easy to see $x_\lambda=y_\mu$ iff $x=y$ and $\lambda=\mu$; $x_\lambda\subseteq y_\mu$ iff $x=y$ and $\lambda\leq\mu$. A is the union of all fuzzy points $x_\lambda^{'}s$ in A. If $A(a)=\lambda$, we call a_λ a main point of A which can be written as $\dot{a}=a_\lambda$.
Let A be a fuzzy set of X. If there is a rule that, for any $a_\lambda, b_\mu\in A$, there exist only one c_v corresponding to them, denoted $c_v=a_\lambda b_\mu, v=\lambda\wedge\mu$, and for any $a_{\lambda'}\subset a_\lambda, b_{\mu'}\subset b_\mu, a_{\lambda'}b_{\mu'}\subset a_\lambda b_\mu$ is true, we call A a pointwise fuzzy groupoid. If an element e_λ in the fuzzy groupoid A fits $e_\lambda a_\mu=a_\mu$, for any $a_\mu\in A$, then e_λ is called a left identity of A. Similarly we can define a right identify f_μ of A. If A not only has a left identify e_λ, but also has a right identity f_v, it must have $e_\lambda=f_v$, here we call e_λ the identity of the fuzzy groupoid A, and the identify is unique, if it is exists. Assume there is a left identify e_λ in A, $a_\mu\in$ A, if there exits $a'_{\mu'}\in A$ where $a'_{\mu'}a_\mu=e_\mu\subset e_\lambda$, $a'_{\mu'}$ is called a left inverse element of a_μ. Similarly we can define right inverse element of a_μ. If for arbitrary elements a_λ, b_μ, c_v of pointwise fuzzy groupoid A, $(a_\lambda b_\mu)c_v=a_\lambda(b_\mu c_v)$ is true, then A is called a pointwise fuzzy semigroup.
Let A be a pointwise fuzzy semigroup. If A has a left identity e_λ where any $a_\mu\in A$ has a left inverse $a'_{\mu'}$, it's easy to prove e_λ is a right identity of A where $a'_{\mu'}$ is a right inverse of a_μ. Now $a'_{\mu'}$ is called a inverse element of a_μ and denoted it

by $a_{\mu'}^{-1}$. It's obviously that for any $a_{\mu'}^{-1}$ is an inverse of a_μ if it satisfying

$a_\mu^{-1} \subset a_{\mu'}^{-1} \subset \dot{a}^{-1}$. a_μ always has different inverses, and among all the inverses of a_μ, a_μ^{-1} is called $a_\mu^{'}s$ same value inverse element which is sole if exists.

Definition 2.1 Let A be a pointwise fuzzy semigroup of X. A is called a pointwise fuzzy group of X if

(1) There is at least one left identity element e_λ in A;

(2) For any $a_\mu \in A$, there is at least one left inverses element.

Example 2.2 Let X be a general group and A be the characteristic function of X. For any $a_\lambda, b_\mu \in A$, we provide $a_\lambda b_\mu = (ab)_{\lambda\wedge\mu}$, then A is a pointwise fuzzy group on X.

The above example shows the pointwise fuzzy group is the generalization of the general group.

Definition 2.3 Let A be a fuzzy group on X, A is called a fuzzy closed group which can be shortly called a closed group if for any nonempty subset $Y \subset X$, there exits $y \in Y$ satisfying $A(y) = \sup A(x)$, $x \in Y$.

Let A, B be respectively fuzzy sets on nonempty sets X, Y. A rule ϕ is called a mapping from fuzzy set A to fuzzy set B if not only there exists $b_\mu \in B$ corresponding to arbitrary $a_\lambda \in A$, but also is true where $b_{\mu'} \subseteq b_\mu$ is only one corresponding to $a_{\lambda'} \subset a_\lambda$. $b_\mu = \phi(a_\lambda)$ is called the image of a_λ under ϕ where $a_\lambda \in \phi^{-1}(b_\mu)$ is called a preimage of b_μ under ϕ. ϕ is said to be a epimorphism if for any $b_\mu \in B$, $b_{\mu'} \subset b_\mu$ $(\mu' \neq \mu)$ has at least one preimage under ϕ, $B = \phi(A)$ is called a image of A under ϕ. ϕ is called a monomorphism if for arbitrary a_λ, $a_{\lambda'} \in A, a_\lambda \neq a_{\lambda'}$, $\phi(a_\lambda) \neq \phi(a_{\lambda'})$. If ϕ is both an epimorphism and a monomorphism, ϕ is called a bijective mapping. It is naturally to induce a general mapping from supp A to supp B if ϕ is a mapping from A to B, we provide $\phi(x) = y$ if for arbitrary $x \in \text{supp} A$, there exists $x_\lambda \in A$ and $\phi(x_\lambda) = y_\mu$. In addition when ϕ is an epimorphism, B is identical to $\phi(A)$ which is obtained by the general extension theorem.

Definition2.4 Let A, B be respectively pointwise fuzzy groupoids on X, Y, ϕ be a mapping from A to B. If for any $a_\lambda, a_{\lambda'} \in A, b_\mu = \phi(a_\lambda), b_{\mu'} = \phi(a_{\lambda'})$, the following conditions are satisfied:

(1) $\phi(a_\lambda a_{\lambda'}) = \phi(a_\lambda)\phi(a_{\lambda'})$;

(2) $\lambda = \lambda'$ iff $\mu = \mu'$,

then ϕ is called a homomorphism from A to B.

ϕ is a homomorphism from A to B if ϕ is a epimorphism that can be written as $A \sim B$ where ϕ is a isomorphism from A to B if ϕ is a bijective mapping that can be written as $A \cong B$.

In general, although ϕ is a epimorphism from fuzzy group A onto fuzzy group A', it isn't always true that for arbitrary $a'_{\lambda'} \in A'$ there exists $a_\lambda \in A$ that $\phi(a_\lambda) = a'_{\lambda'}$.

But, we can easily prove that ϕ is a epimorphism iff for any $a'_{\lambda'} \in A'$ there exists $a_\lambda \in A$ that $\phi(a_\lambda) = a'_{\lambda'}$ where ϕ is a homomorphism from the closed group A to fuzzy groupoid A', and A is a fuzzy (semi) group iff A' is a fuzzy (semi)group where ϕ is a homomorphism from the fuzzy groupoid A onto A'. The homomorph of an inverse of a_μ is the inverse of the homomorph of a_μ where the homomorph of a identity is a identity.

B is called a fuzzy subgroup of the fuzzy group A if B is a nonempty fuzzy subset of A and forms a fuzzy group with the operation of A.

Propositon 2.5 Let B be a subset of the fuzzy group A, then the following statements are equivalence:

(1) B is a fuzzy subgroup of A;

(2) B is closed under the operation of A and B includes the inverse elements of all elements in B;

(3) For arbitrary $a_\lambda, b_\mu \in B, a_\lambda b_\mu^{-1} \in B$ is true.

3. Pointwise fuzzy normal subgroup and pointwise fuzzy quotients

Definition3.1 Let H be a pointwise fuzzy subgroup of the pointwise fuzzy group A, $a_\lambda \in A$. Then $a_\lambda H = \{a_\lambda h_\mu | h_\mu \in H\}$ is called a left subcoset of H with respect of a_λ. aH is called a left coset of H with respect to a. Where aH is the union of all the left subcosets whose intersection with $a_\lambda H$ is nonempty. Similarly, the right subcoset and right coset can be easily defined.

If $a_{\lambda'} \subset a_\lambda$, then $a_{\lambda'} H \subset a_\lambda H$, and for two left cosets are equivalence otherwise their intersection is empty.

proposition 3.2 Let H be a pointwise fuzzy subgroup of the pointwise fuzzy group A, then the following statements are equivalence:

(1) aH = bH;

(2) $b \in \mathrm{supp}(aH)$;

(3) There exists $v \in (0,1]$ and $a_v = b_v h_v$, $h_v \in H$, is true.

Proof (2)$\Rightarrow$(1) Since $b \in \mathrm{supp}(aH)$, there exists $\lambda > 0$ such that $b_\lambda \in aH$. Assume $v = \min(\lambda, H(e))$, then we have $b_v \subset b_\lambda \in aH$ and $b_v \in b_v H$, by these we know $b_v \in aH \cap bH \neq \phi$, hence aH = bH.

(1) $\Rightarrow$ (2) Is clear.

(1) $\Rightarrow$ (3) For $b_\lambda \subset \dot{b}, b_\lambda H \subset bH = aH$ is obviously, and by the definition of aH, we know there must exist $\mu > 0$ where $a_\mu H \cap b_\lambda H \neq \phi$. Hence there has $\nu > 0$ and $c_\nu \in a_\mu H \cap b_\lambda H$, by this we can get $h'_\zeta, h''_\xi \in H$ where $a_\mu h'_\zeta = c_\nu = b_\lambda h''_\xi$.

It is clear that $\nu = \min(\mu, \lambda, \xi, \zeta)$, hence $a_\nu h'_\nu = b_\nu h''_\nu$, so

$a_\nu = b_\nu h_\nu, h_\nu = h''_\nu h'^{-1}_\nu \in H$.

(3) $\Rightarrow$ (1) Is clear.

Definition3.3 Let H be a pointwise fuzzy subgroup of the pointwise fuzzy group A. If for arbitrary $a_\lambda \in A, a_\lambda H = Ha_\lambda$ is always true, then H is called a pointwise fuzzy normal subgroup of A.

It's clear that $aH = Ha$ for any $a \in A$ if H is a pointwise fuzzy normal subgroup of A.

By Definition 3.3 It is easy to prove the following proposition.

Proposition3.4 Let H be a pointwise fuzzy subgroup of the pointwise fuzzy group A, then the following statements are equivalence:

(1) H is a pointwise fuzzy normal subgroup of A;

(2) For arbitrary $a_\lambda \in A, h_\mu \in H, a_\lambda h_\mu a_\lambda^{-1} \in H$ is always true;

(3) For arbitrary $a_\lambda, b_\mu \in A$, if $a_\lambda b_\mu \in H$, then $b_\mu a_\lambda \in H$.

Assume H is a pointwise fuzzy normal subgroup of the pointwise fuzzy group A, K is a general set which is formed by all the left cosets of H. We define a fuzzy set $\bar{A}$ on K: $\bar{A}(aH) = \sup A(x), x \in \text{supp}(aH)$.

We can prove the following proposition similarly to proposition4.3 in [2].

Proposition 3.5 $\bar{A}$ be a pointwise fuzzy group with the product: $(aH)_\lambda (bH)_\mu = (cH)_\zeta$, here $\zeta = \min(\lambda, \mu)$, c is the supporting point of ab, $(aH)_\lambda$ and $(bH)_\mu$ are arbitrary elements in $\bar{A}$.

Definition3.6 The above $\bar{A}$ is called a fuzzy quotient with respect to H, written as $\bar{A} = A/H$.

Definition3.7 Assume H is a pointwise fuzzy normal subgroup of a closed group A, for $\mu \in \left(0, \bar{A}(aH)\right]$, let $D_\mu(aH) = \{x : x \in \text{supp}(aH), A(x) \geq \mu\}$ and we call it a μ – degree set of aH.

It is clear $b_\mu \subseteq b$ is always true for any $b \in D_\mu(aH)$.

Proposition3.8 If $(aH)_\mu \in \bar{A}$, then $D_\mu(aH) \neq \phi$.

Proof: Since $(aH)_\mu \in \bar{A}$, then $\mu \le \bar{A}(aH) = \sup A(x)$, $x \in$ supp(aH). And supp $aH \neq \phi$ $x \in$ supp aH . For A is a closed group, there exists $y \in$ supp(aH),A(y)=SupA(x),$x \in$ Supp(aH),so A(y) $\ge \mu$. Hence $y \in D_\mu(aH)$ and $D_\mu(aH) \neq \phi$.

Proposition3.9 If $(aH)_\lambda, (bH)_\mu \in \bar{A}, a \in D_\lambda(aH), b \in D_\mu(bH)$ and $a_\lambda b_\mu = d_v$, then $(aH)_\lambda (bH)_\mu = (dH)_v$.

Proposition3.10 Let $(aH)_\mu$ is an arbitrary element in $\bar{A}$, then there must exist $b \in D_\mu(aH)$ satisfying $b_\mu \subset \dot{b}$ and $(aH)_\mu = (bH)_\mu$.

Proof. Since $(aH)_\mu \in \bar{A}$, then $D_\mu(aH) \neq \phi$. Assume $b \in D_\mu(aH) \subseteq$ supp(aH), then $b_\mu \subset \dot{b}$ and $(aH)_\mu = (bH)_\mu$.

For any $(aH)_\mu \in \bar{A}$, it is clear that $a_\mu \subset \dot{a}$ isn't always true, that is to say $a \in D_\mu(aH)$ isn't always true. But by Proposition3.10, for any $(aH)_\mu \in \bar{A}$ we can assume $a_\mu \subset \dot{a}(a \in D_\mu(aH))$, without losing the generalization.

Theorem3.11 If A is a closed group on X and H is a pointwise fuzzy normal subgroup of A, then the fuzzy quotient $\bar{A} = A/H$ is a closed group.

Proof. For any nonempty subset B of K, we shall prove that there must have a $aH \in B$ satisfying $\bar{A}(aH) = \sup \bar{A}(bH)$, $bH \in B$. In case of A is a closed group there exists $y \in$ supp(aH) satisfying

$$A(y) = \sup A(x), x \in \text{supp}(bH).$$

Hence $\bar{A}(bH) = \sup A(x) = A(y)$ (*)

Let Y is the set of all y satisfying (*) when bH runs over B.

It is clear that Y is a nonempty subset of X and

$$\{\text{Sup}(\bar{A}(bH)), bH \in B\} = \{\text{Sup}(A(y)), y \in Y\}$$

But A is closed, hence there is a $a \in Y$ satisfying $A(a) = \sup A(y)$, $y \in Y$. So $A(a) = \sup(\bar{A}(bH))$, $bH \in B$.

Now let's prove $aH \in B$.

Since $a \in Y$, there exists $bH \in B$ satisfying $a \in \text{supp}(bH)$ and $A(a) = \sup A(x)$, $x \in$ supp(bH).

By propsition3.2 we know $aH = bH \in B$.

Now we can get

$$\bar{A}(aH) = \sup_{x \in \operatorname{supp} aH} A(x) \geq A(a) = \sup_{bH \in B} \bar{A}(bH) \geq \bar{A}(aH)$$

in case of $a \in \operatorname{supp} aH$. Hence $\bar{A}(aH) = \sup \bar{A}(bH)$, $bH \in B$.

We have proved $\bar{A} = A / H$ is closed.

Proposition3.12 Let ϕ be a homomorphism from the fuzzy group A onto the fuzzy group $A^{'}$, then $H = \left\{ x_\mu \in A : \phi(x_\mu) = e^{'}_{\mu^{'}} \subset e^{'} \right\}$ is a fuzzy normal subgroup of A, where $e^{'}$ is the unit of $A^{'}$. From now we call H the kernel of ϕ and write it as $H = \ker \phi$.

Theorem 3.13 Let ϕ be a homomorphism from the closed group A onto the fuzzy group $A^{'}$, $H = \ker \phi$, then

(1) The mapping $\nu(a_\mu) = (aH)_\mu, a_\mu \in A$, is a homomorphism from A to $\bar{A} = A / H$, we call ν the natural homomorphism from now;

(2) There exists a isomorphism $\bar{\phi}$ from $\bar{A}$ to $A^{'}$;

(3) $\phi = \bar{\phi} \nu$.

Proof. (1) ν is clearly a one-velued mapping from A to $\bar{A}$. For any $a_\lambda, b_\mu \in A$,

$\nu(a_\lambda) = (aH)_\lambda, \nu(b_\mu) = (bH)_\mu$ and $a \in D_\lambda(aH), b \in D_\mu(bH)$

Let $a_\lambda b_\mu = c_{\lambda \wedge \mu}$, then $\nu(a_\lambda b_\mu) = \nu(c_{\lambda \wedge \mu}) = (cH)_{\lambda \wedge \mu}$,

$\nu(a_\lambda)\nu(b_\mu) = (aH)_\lambda (bH)_\mu = (cH)_{\lambda \wedge \mu}$.

Hence $\nu(a_\lambda b_\mu) = \nu(a_\lambda)\nu(b_\mu)$

And it's clear that $\lambda = \mu$ iff the value of $\nu(a_\lambda)$ and $\nu(b_\mu)$ is equal, hence ν is the homomorphism from A to $\bar{A}$.

For arbitrary $(aH)_\mu \in \bar{A}$, assume $(aH)_{\mu^{'}} \subset (aH)_\mu, \mu^{'} \neq \mu$, then let

$\bar{A}(aH) = \sup A(x) = \xi$, $x \in \operatorname{supp}(aH)$.

In case of A being closed, there exists $y \in \operatorname{supp}(aH)$ satisfying

$A(y) = \sup A(x) = \xi$, $x \in \operatorname{supp} aH$.

So $\nu(y_\xi) = (yH)_\xi = (aH)_\xi = (aH)$,

and so $\mu^{'} < \mu \leq \xi$.

Hence there must exist $y_{\mu'} \subset y_{\mu} \subset y_{\xi}$ satisfying $\nu(y_{\mu'}) = (yH)_{\mu'} = (aH)_{\mu'}$.

Hence ν is a epimorphism from A to $\bar{A}$ and $\bar{A}$ is the homomorph of A

(2) For any $(aH)_{\mu} \in \bar{A}$, let $\bar{\phi}((aH)_{\mu}) = \phi(b_{\mu})$ $, b \in D_{\mu}(aH)$.

Firstly we prove that $\bar{\phi}$ is a one-valued mapping. If there is another $c \in D_{\mu}(aH)$, then

$$bH = aH = cH,\ b_{\mu} = c_{\mu}h_{\mu},\ h_{\mu} \in H.$$

And $\phi(b_{\mu}) = \phi(c_{\mu}h_{\mu}) = \phi(c_{\mu})\phi(h_{\mu}) = \phi(c_{\mu})$, hence $\bar{\phi}$ is a one-valued mapping.

If $\bar{\phi}$ isn't a monomorphism, then there exists $(aH)_{\mu}$, $(bH)_{\nu} \in \bar{A}$ such that $(aH)_{\mu} \neq (bH)_{\nu}\ (a \in D_{\mu}(aH), b \in D_{\nu}(bH))$,

but $\bar{\phi}((aH)_{\mu}) = \bar{\phi}((bH)_{\nu})$ .hence $\phi(a_{\mu}) = \phi(b_{\nu})$

Since

$$\phi(a_{\mu}b_{\nu}^{-1}) = \phi(a_{\mu})\phi(b_{\nu}^{-1}) = \phi(b_{\nu})\phi(b_{\nu}^{-1}) = \phi(b_{\nu}b_{\nu}^{-1}) = \phi(e_{\nu}) = e'_{\nu'} \subset e'_{\lambda'},$$

So $a_{\mu}b_{\nu}^{-1} = h_l \in H$, $a_{\xi}b_{\xi} = h_{\xi}$, $\xi = \min(\mu, \nu, l)$,

Hence $aH = bH$, and $(aH)_{\mu} \neq (bH)_{\nu}$

Then $\mu \neq \nu$;

But by (1') we have that the values of $\phi(a_{\mu})$ and $\phi(b_{\nu})$ is equal and according to Definition 2.4 $\mu = \nu$ is true, there has a contradiction. Hence $\bar{\phi}$ is a monomorphism.

Since ϕ is a homomorphism from A onto A', then for arbitrary $a'_{\mu'} \in A'$, $a'_{\mu''} \subset a'_{\mu'}$, $\mu'' \neq \mu'$, there always exists $a_{\mu} \in A$ satisfying

$$\phi(a_{\mu}) = a'_{\mu''}.$$

But $\bar{A}(aH) = \sup A(x) \geq A(a), x \in \mathrm{supp}(aH)$.

So we know there is a $(aH)_{\mu} \subset (aH) \in \bar{A}$ fitting

$\bar{\phi}((aH)_{\mu}) = \phi(a_{\mu}) = a'_{\mu''}$ hence $\bar{\phi}$ is a epimorphism.

The other conditions proving $\bar{\phi}$ is an isomorphism are easy to get.

(3) For arbitrary $a_{\mu} \in A$, we have

$$(\bar{\phi}\nu)(a_{\lambda}) = \bar{\phi}(\nu(a_{\lambda})) = \bar{\phi}((aH)_{\lambda}) = \phi(a_{\lambda}),$$

hence $\bar{\phi}\nu = \phi$.

References

1 A.Rosenfeld, Fuzzy groups, J.Math.Anal.Appl.35(1971)512-517.
2 QI Zhen-kai, Pointwise fuzzy groups,Fuzzy Math.2(1981)29-36.
3 Gu Wen-Xiang, Pointwise fuzzy groups with operator, Fuzzy Math.3(1983)17-24.
4 Liu Rong-bin, The equivalents of pointwise fuzzy groups,Fuzzy Math.1(1984)111-114.
5 Gu Wen-Xiang, Relations between the M-subgroups of homomorphic fuzzy gooups with operator,Fuzzy Math.2(1984)41-46.
6 Gu Wen-Xiang.The isomorphism theorem of M-pointwise fuzzy groups.Fuzzy Math.4(1985)83-86.
7 Qi Zhen-kai,The direct products of fuzzy groups and L-pro-ducts,L-fuzzy point and L-fuzzy group,4(1983)107-108.