Application of the Fuzzy - Stochastic Methodology to Apprising Financial Derivatives - Generalised Sensitivity Analysis

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Abstract

The valuing of the financial derivatives as call and put options for underlying assets (shares, bonds, indices etc.) is now a crucial and very tractable problem of financial decision-making. There are two basic aspects which are traditionally studied, contingent claim feature (payoff function) and risk (stochastic process type). But vagueness is mostly rather neglected. But a combination of risk (stochastic) and vagueness (fuzzy) methodology could be useful because of decision- making and forecasting conditions.

Applicable fuzzy-stochastic methodology under fuzzy numbers (linear T-numbers) is described and proposed. Methodologies of valuing and computing option prices are explained, the Black-Scholes model is mainly described. Input data are in a form of fuzzy numbers, the model is of fuzzy-stochastic type and results are described as possibility-expected value and fuzzy-stochastic distribution. Illustrative example is introduced.

Keywords: Decision support system; Finance; Financial Derivatives, Financial option, Fuzzy sets; Fuzzy-stochastic Model; Pricing; Stochastic processes

1. Introduction

Option pricing theory has many applications in finance. These applications of the option methodology are both in financial markets and in corporate decision-making. A lot of modifications of basic Black-Scholes model were studied, different types of stochastic diffusion process were used, several stochastic variables were applied, see (Black (1973), Duffie (1988). Kariya (1993), Boyle (1995), Siegel (1995), Campbell (1997), Hull (1997), Musiela (1997)). The most of models are considered to be a short-term modelling type but option methodology is also used in long-term decision-making process, survey is possible to find in (Copeland (1988), Dixit (1994), Sick (1995), Trigeorgis (1998)).

In the case of application of contingent claim methodology the estimation of the option value is determined by input data precision. Forecasting is risky and vague and there are problems with data frequency and stochastic validity.

On the other side it is useful to apply distorted stochastic data. Under this framework a risk and vagueness are distinguished. The risk is expressed by random (stochastic) methodology and fuzzy apparatus describes the vagueness. Thus one of the suitable approaches for solving this problem is to apply a fuzzy-stochastic methodology and create a fuzzy-stochastic model.

The valuing process for European call option will be understood as a typical example of an application the proposed fuzzy-stochastic option pricing methodology in the paper.

2. Fuzzy-stochastic models types

Since the fuzzy-stochastic methodology could be understood as a combination of the fuzzy and stochastic approach, it could be useful to characterize basic types of models and describe the typology of models. Fuzzy-stochastic models could be considered, according to purpose of proposed paper and research, the generalization of particular stochastic or fuzzy models.

A fuzzy-stochastic model is determined by (1) type of events, (2) type of expression of events, (3) methods and types of the probability description.

Events can be of crisp boundaries (binary) or fuzzy type. The event expression could be random type or deterministic one. Probability could be stated crisply by a real number or vaguely by a fuzzy set. It is apparent that the combining three characteristics the following fuzzy-stochastic model variants is possible to determine (see Table1).

Table 1 Fuzzy-stochastic models types

	Model type	Event type	Expression event type	Probability expression
A	Deterministic	binary (determ.)	non-stochast. (determ.)	-
B	Stochastic	binary (determ.)	stochastic	non-fuzzy (determ.)
C	Fuzzy	fuzzy	non-stochast. (determ.)	-
D	Fuzzy-stoch. with non-fuzzy probabilities	fuzzy	stochastic	non-fuzzy (determ.)
E	Fuzzy-stoch. With binary events	binary (determ.)	stochastic	fuzzy
F	Fuzzy-stoch. With fuzzy events and probabilities	fuzzy	stochastic	fuzzy

Deterministic model (A) is a basic, simple and trivial case. *Stochastic model* (B), an application of this type of model is very useful and effective for a short-term financial decision-making, for example the firm cash management and capital and money market operations. *Fuzzy model with non-stochastic expression of event* (C) is mainly suitable for dealing with the problems with vaguely stated events. *Fuzzy-stochastic model with non-fuzzy probabilities and fuzzy event* (D) is combination of vaguely expressed events with random events with non-fuzzy probabilities. *Fuzzy-stochastic model with binary events with stochastic expression and fuzzy probability* (E). *Fuzzy-stochastic model with fuzzy probabilities, fuzzy events and stochastic expression* (F).

The proposed model is of the E-type.

3. Fuzzy-stochastic model under normal fuzzy sets of T-number type

There are several references concerning combining of fuzzy and stochastic processes see (Dubois (1980), Kacprzyk (1988), Luhandjula (1996), Wang (1993), Viertl (1996)). Further the Wang (1993) approach is mainly followed.

There are several basic fuzzy-stochastic elements, which are very useful from an application point of view (1) fuzzy set, (2) normal and T-number sets, (3) ε-cut, (4) extension principle, (5) approximate ε - cut methodology, (6) fuzzy-random variable, (7) fuzzy-probability function.

Definition 1. A *fuzzy set* (depicted with tilde) is commonly defined by a membership function (μ) as representation from E^n (Euclid n-dimensional space, n > 1) to a set of E^1 specially to the interval of [0;1],

$\tilde{s} \equiv \mu_{\tilde{s}}(x)$, where $\tilde{s}$ is fuzzy set, x is vector and $x \in X \subset E^n$, $\mu_{\tilde{s}}(x)$ is membership function.

It is evident that many fuzzy sets can be created. Most common type of a fuzzy set meeting specified preconditions of normality, convexity and continuity is very well known *normal fuzzy set* see (Dubois (1980)). Set of normal fuzzy sets are depicted $F_N(E^n)$. One of the most widely applied normal fuzzy set types is the *T-number*.

Definition 2. Fuzzy set meeting preconditions of normality, convexity, continuity and closeness and being defined as quadruple $\tilde{s} = (s^L, s^U, s^\alpha, s^\beta)$ where $\phi(x)$ is non-decreasing function and $\psi(x)$ is non-increasing function as follows,

$$\tilde{s} \equiv \mu_{\tilde{s}}(x) = \left\{ \begin{array}{llll} 0 \text{ for} & x \le s^L - s^\alpha; & \phi(x) \text{ for} & s^L - s^\alpha < x < s^L; \\ 1 \text{ for} & s^L \le x \le s^U; & \psi(x) \text{ for} & s^U < x < s^U + s^\beta; \\ 0 \text{ for} & x \ge s^U + s^\beta & & \end{array} \right\},$$

is called *T-number.* Denote set of T-numbers by $F_T(E)$

Definition 3. The ε *-cut* of the fuzzy set $\tilde{s}$, depicted $\tilde{s}^\varepsilon$, is defined as follows,

$$\tilde{s}^\varepsilon = \{x \in E^n; \mu_{\tilde{s}}(x) \ge \varepsilon\} = [{}^-s^\varepsilon, {}^+s^\varepsilon], \text{ where}$$

$${}^-s^\varepsilon = \inf\{x \in E^n; \mu_{\tilde{s}}(x) \ge \varepsilon\}, \quad {}^+s^\varepsilon = \sup\{x \in E^n; \mu_{\tilde{s}}(x) \ge \varepsilon\}.$$

Very useful and powerful instrument, which might be used, for calculating of a function of fuzzy sets is the extension principle see (Zadeh (1965)).

Definition 4. The *extension principle* is derived by sup min composition between fuzzy sets $\tilde{r}_1 \dots \tilde{r}_n$ and $\tilde{s} = f(\tilde{r}_1 \dots \tilde{r}_n)$ as follows. Let $f: E^n \to E^1$, then a membership function of fuzzy set $\tilde{s} = f(\tilde{r}_1 \dots \tilde{r}_n)$ is defined by

$$\mu_{\tilde{s}}(y) \equiv \tilde{s} = \sup_{\substack{x_1, \dots, x_n \\ y = f(x_1, \dots, x_n)}} \min[\mu_{\tilde{r}_1}(x_1) \dots \dots \mu_{\tilde{r}_n}(x_n)], \; x_i, y \in E^1 .$$

There are often such troubles in solving practical and complex problems under fuzzy environment that an analytic solution according to the extension principle is not available. Assuming a fuzzy set is of fuzzy number type (the T-number type as well) it is possible to solve a function of fuzzy numbers $\tilde{s} = f(\tilde{r}_1 \dots \tilde{r}_n)$ in accordance with the extension principle as the approximate procedure of ε-cuts.

Definition 5. ***The approximate procedure of*** ε - *cuts* is defined as follows,

$\mu_{\tilde{s}}(y) = \sup\{\varepsilon; y \in \tilde{s}^{\varepsilon}\}$ for any $y \in E^n$ and $\varepsilon \in [0;1]$, where $\tilde{s}^{\varepsilon} = [{}^{-}s^{\varepsilon}, {}^{+}s^{\varepsilon}]$ is the ε - cut.

Here $(\varepsilon; y \in \tilde{s}^{\varepsilon}) = \begin{Bmatrix} \varepsilon \text{ if } y \in [{}^{-}s^{\varepsilon}, {}^{+}s^{\varepsilon}] \\ 0 \text{ if } y \notin [{}^{-}s^{\varepsilon}, {}^{+}s^{\varepsilon}] \end{Bmatrix}$.

It is apparent that applying the Definition 5 the function of fuzzy numbers could be transformed and solved as several mathematical programming problems for ε by this way.

max (min) $s = {}^{+}s^{\varepsilon}, ({}^{-}s^{\varepsilon})$,

s.t. $s = f(x_1 \ldots x_n)$,

where $x_i \in \left[{}^{-}x_i^{\varepsilon}, {}^{+}x_i^{\varepsilon}\right]$ for $i \in \{1;2 \ldots\ldots n\}$, and $\varepsilon \in [0;1]$.

The crucial category in a fuzzy-stochastic modelling is the fuzzy-random variable.

Definition 6. It is said, $\bar{\tilde{s}} : \Omega \to F_T(E)$ is *fuzzy-random variable* (depicted with tilde and line), if $\bar{\tilde{s}}_w^{\varepsilon} = \{x : x \in E^n, \bar{\tilde{s}}_w \geq \varepsilon\} = [{}^{-}s_w^{\varepsilon}, {}^{+}s_w^{\varepsilon}]$ is random interval (random ε -cut) for every $w \in \Omega$ and $\varepsilon \in]0,1]$, ${}^{-}s_w^{\varepsilon}, {}^{+}s_w^{\varepsilon}$ are two random variables (or finite measurable functions). Let us denote the set of fuzzy-random variables $FR(\Omega, P)$ where $P:\Omega \to [0,1]$ and thus $\bar{\tilde{s}} \in FR(\Omega, P)$.

From definition implies that

(a) $\bar{\tilde{s}} = \bigcup_{\varepsilon} \bar{\tilde{s}}^{\varepsilon}$, because $\forall w \in \Omega, \bar{\tilde{s}}_w = \bigcup_{\varepsilon} \bar{\tilde{s}}_w^{\varepsilon}$. Where $\bar{\tilde{s}}_w$ is a fuzzy set and $\bar{\tilde{s}}_w^{\varepsilon}$ is a random interval.

(b) $\bar{\tilde{s}}$ is the fuzzy-random variable iff $\bar{\tilde{s}}^{\varepsilon}(w)$ is a random interval, it means $\bar{\tilde{s}} = \bigcup_{\varepsilon, w} \bar{\tilde{s}}_w^{\varepsilon}$ for every $w \in \Omega$ and $\varepsilon \in]0,1]$.

The following definition is useful for evaluating and computing the functions of fuzzy-random variables and constructing the fuzzy-probabilities and fuzzy-expected values.

Definition 7. Suppose that $\bar{\tilde{s}} \in FR(\Omega, P)$, $s_w^{\varepsilon} = [{}^{-}s_w^{\varepsilon}, {}^{+}s_w^{\varepsilon}]$, $\varepsilon \in [0;1]$, $x \in E^n$.

Let ${}^{-}g_w^{\varepsilon}$ (resp. ${}^{+}g_w^{\varepsilon}$) be the function of ${}^{-}s_w^{\varepsilon}$ (resp. ${}^{+}s_w^{\varepsilon}$) then let us call $\bar{\tilde{g}}(x) = \bigcup_{\varepsilon} \varepsilon[{}^{-}g^{\varepsilon}(x), {}^{+}g^{\varepsilon}(x)]$ the *fuzzy probability function* of $\bar{\tilde{s}}$ in accordance with Definitions 5 and 6.

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4. Description of the B-S stochastic model of call option

The Black and Scholes were the first who derived explicit formulae for a European call option value see (Black (1973).

Let us assume that the underlying assets is a market value of the share (S), a call option premium is (F), a strike price is depicted (X) and at maturity (T) the buyer wealth is

$$F_T = \max(S_T - X_T, 0). \quad (1)$$

We suppose that a stock value (S) follows the stochastic diffusion process,

$$dS = \mu_S \cdot S \cdot dt + \sigma_S \cdot S \cdot dz, \quad (2)$$

where dz is Wiener process of the underlying assets value, see for instance (Hull (1997)), parameters μ_S and σ_S are the expected proportional growth and the volatility of an assets value.

Generally, because $F = f(S, t)$ it follows from *Ito's lemma*

$$dF = \left(\frac{\partial F}{\partial S}\mu_S \cdot S + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma_S^2 \cdot S \right) \cdot dt + \frac{\partial F}{\partial S}\sigma_S \cdot S \cdot dz \qquad (3)$$

To eliminate the Wiener process (dz), it is necessary to construct a portfolio (Π) of long assets position and short equity position.

$$\Pi = \frac{\partial F}{\partial S} \cdot S - F \qquad (4)$$

The basic assumption is that the yield of a portfolio Π must be riskless (hedged to assets volatility). It means,

$$\Delta\Pi \equiv r \cdot \Pi \cdot dt = \frac{\partial F}{\partial S} dS - dF, \qquad (5)$$

where r is a continuously compounded risk - free interest rate, assumed to be constant.

Substituting the equations (2) and (3) into (5) and rearranging we gain the Black-Scholes partial differential equation (BS PDE).

$$\frac{\partial F}{\partial t} + \frac{\partial F}{\partial S} \cdot r \cdot S + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma_S^2 \cdot S^2 = r \cdot F \qquad (6)$$

The solution of (6) under boundary condition (1), where the underlying assets value is log-normally distributed, is the B-S pricing formulae for a European call option and $\overline{F}$ expresses an expected equity value.

$$\overline{F} = f(S, X, dt, r, \sigma_S) = S \cdot N(d_1) - e^{-r \cdot dt} \cdot X \cdot N(d_2), \qquad (7)$$

where $d_1 = \dfrac{\ln\left(\dfrac{S}{X}\right) + \left(r + \dfrac{\sigma_S^2}{2}\right) \cdot dt}{\sigma_S \cdot \sqrt{dt}}$, $d_2 = d_1 - \sigma_A \cdot \sqrt{dt}$, N(.) is the normal cumulative distribution function.

It is very interesting to know a probability density function of the option value, h(F), and to compare it with expected value as well. This function is of a log-normal type and is following

$$h(F) = \frac{1}{2\pi} \cdot \exp(-\frac{1}{2} \cdot x1^2) \cdot x2, \qquad (8)$$

where $x1 = (LN\dfrac{F \cdot EXP(r \cdot dt) + X}{S} - (r - \sigma_S^2/2) \cdot dt) / \sigma_S \cdot \sqrt{dt}$, and

$$x2 = \frac{EXP(r \cdot dt)}{\sigma_S \cdot \sqrt{dt} \cdot (F \cdot EXP(r \cdot dt) + S)}.$$

5. Fuzzy-stochastic option valuation model

5.1 Determining of the vagueness of input data

Instantaneous assets value is influenced by various factors. There are also problems with determining market values. It is apparent that the fuzzy estimation of the assets ($\widetilde{S}$) is reasonable.

A similar situation is sometimes in stating an exercise price ($\widetilde{X}$) which might be stated vaguely.

Risk free rate is assumed to be constant, however the yield curves have not often a flat shape therefore the risk free rate ($\tilde{r}$) might be introduced as a fuzzy number.

The underlying assets variance is a crucial problem of valuing options. There is two basic approaches used, *historical* approach and *implied volatility* approach. The first one is based on a real historical data at fixed time intervals. But there are many problems with determining a time length, a statistical validity of estimation and the estimation method type selection (ARCH, GARCH, exponential moving average etc.). The implied volatility method is more suitable for short-term assets but not for long-term assets, which are not regularly traded. Research studies have verified that the variance must be rectified and modified see for instance analysis of (Muller (1993), Hull (1997)). Thus it is reasonable to state variance ($\tilde{\sigma}_S^2$) as a fuzzy number.

Interval to maturity could be stated in some cases vaguely as a fuzzy set because time to maturity is difficult to estimate precisely.

5.2 Formulation of the fuzzy-stochastic model of valuing a call option

Input data for the B-S model are determined in equation (7). From practical and implementation point of view these parameters are random or deterministic. But, it is often difficult to have crisp or statistically valid data into disposal for computing and forecasting. More realistic is to have blurred data and to solve problem under this assumption.

We assume that input data are stated vaguely and are of the fuzzy numbers type. Under these assumptions the fuzzy expected (fuzzy-random) value is described in the context by the function $\tilde{\bar{F}} = f(\tilde{S}, \tilde{X}, d\tilde{t}, \tilde{r}, \tilde{\sigma}_S)$. Now applying the Definition 7 on the equation (7) the B-S model for computing a fuzzy expected European call option value is determined as follows.

(Problem P1)

$$\tilde{\bar{F}} = \mu_{\tilde{\bar{F}}}(\bar{F}) = \sup_{\substack{\bar{F} = S\cdot N(d_1) - e^{-r\cdot dt}\cdot X\cdot N(d_2) \\ d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma_S^2}{2}\right)\cdot dt;}{\sigma\cdot\sqrt{dt}} \\ d_2 = d_1 - \sigma\cdot\sqrt{dt} \\ S, X, \sigma_S, r, dt}} \min\left[\mu_{\tilde{S}}(S), \mu_{\tilde{X}}(X), \mu_{\tilde{\sigma}_S}(\sigma_S), \mu_{\tilde{r}}(r), \mu_{d\tilde{t}}(dt)\right] \tag{9}$$

The problem could be understood to be the fuzzy function with β-interactive coefficients and fuzzy-random variables, in detail see (Dubois (1980)), and solved by the approximate ε-cut procedure (Definition 5).

6. Illustrative example

We now give the example of calculating a European call option value. Described application is based on the problem P1, equation (9). Since an analytic solution is not available the approximate ε-cut procedure (Definition 5) is used. It is supposed in the example of calculating the option value that input data are introduced vaguely as the fuzzy numbers of the linear T-number type.

Definition 8. The linear T-number is defined as the T-number (Definition 2), where functions $\phi(x)$ and $\psi(x)$ are linear, and is depicted as quadruple $\tilde{s} = (s^L, s^U, s^\alpha, s^\beta)$, membership function is as follows,

$$\tilde{s} \equiv \mu_{\tilde{s}}(x) = \left\{ \begin{array}{lllll} 0 & \text{for} & x \le s^L - s^\alpha; & \dfrac{x - \left(s^L - s^\alpha\right)}{s^\alpha} \quad \text{for} & s^L - s^\alpha < x < s^L; \\ 1 & \text{for} & s^L \le x \le s^U; & \dfrac{(s^U + s^\beta) - x}{s^\beta} \quad \text{for} & s^U < x < s^U + s^\beta; \\ 0 & \text{for} & x \ge s^U + s^\beta & & \end{array} \right\}.$$

Input data $\tilde{r}$, $\tilde{dt}$, $\tilde{S}$, $\tilde{X}$, $\tilde{\sigma}_S$, are the linear T-numbers and are introduced in Table 2. There are in Table 3 displayed resulting values of option component (underlying asset, premium) via ε-cuts (Definition 3), results also shows Fig. 1.

Table 2 Input fuzzy data for valuing equity as a call option

Linear T-number (Characteristics)	**Input data** $\tilde{r}$	$\tilde{dt}$	$\tilde{S}$	$\tilde{X}$	$\tilde{\sigma}_S$
s^L	0,070	0,5	75	70	0,25
s^U	0,070	0,5	77	71	0,25
s^α	0,001	0,0011	3,0	1,0	0,03
s^β	0,001	0,002	2,0	2,0	0,02

Table 3 Values of call option in ε-cuts approximation

	ASSETS- $\tilde{S}$		**PREMIUM-** $\tilde{\tilde{F}}$	
ε	$^{-}S^{\varepsilon}$	$^{+}S^{\varepsilon}$	$^{-}F^{\varepsilon}$	$^{+}F^{\varepsilon}$
0	72,00	80,00	1,88	12,41
0,25	72,75	79,25	2,88	11,66
0,5	73,50	78,50	4,05	10,91
0,75	74,25	77,75	5,21	10,16
1	75,00	77,00	6,44	9,40

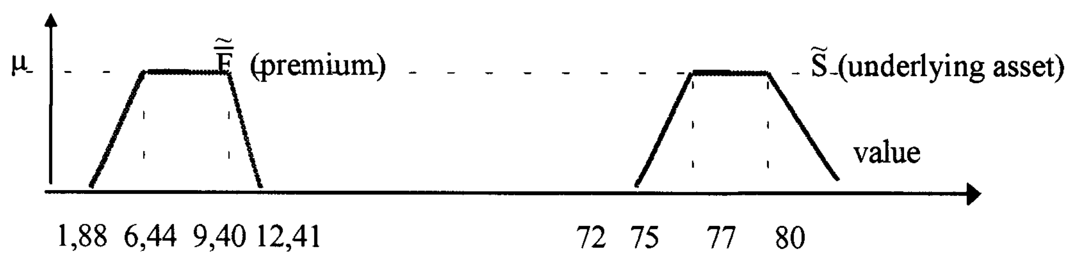

Fig. 1. Values of call option

It is necessary to remark that the call option is in the money. It is apparent the whole range of option value is from 1,88 to 12,41 currency unit and the very possible value (membership function is 1) is from 6,44 to 9,40 currency unit.

The fuzzy-probability density function can give useful and new information about an option value probability-possibility distribution. By a fuzzyfication process of formulae (8), applying Definition 7 under the assumption of fuzzy input data in accordance with Table 2 and applying the approximate ε-cut procedure (Definition 5) it is possible to make up of a fuzzy-probability distribution, which is shown on Fig.2.

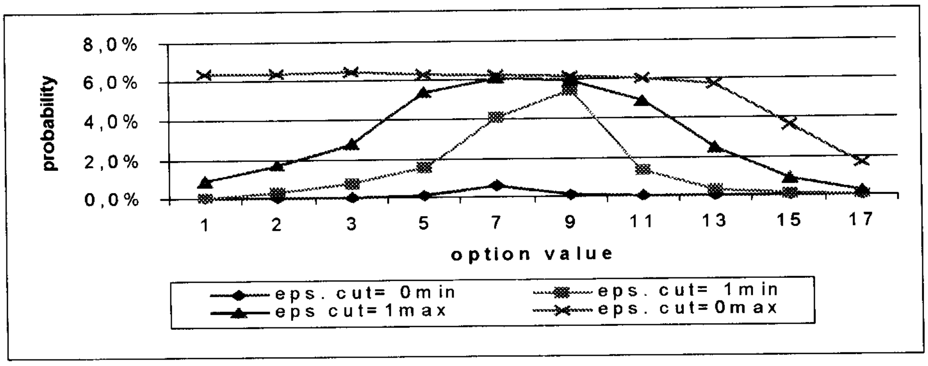

Fig.2 Graph of fuzzy-probability call option value (F) in ε-cut description

This result could be understood as a flexibility decision-making space for financial manager in the apprising process. For a particular decision there is into disposal several defuzzyfication processes.

7. Conclusion

Apprising of derivatives contains aspects of contingent claim, risky (randomness) and indeterminacy (fuzziness). It is apparent that price estimation is determined by input data precision. The valuation is usually made under deterministic or stochastic environment but uncertainty (vagueness) is mostly neglected and not considered. But it is often not easy to get statistically valid and quality data in financial decision-making. Data might be often considered to be introduced as a distorted probability. It is pity on the other hand not to use information in distorted probability. Thus utilisation the fuzzy-stochastic methodology is one of the suitable approaches for solving a problem. Purpose of the paper was to propose and verify one approach for dealing with the problem. Application of the fuzzy-stochastic apparatus could be seen as one of the suitable means for computing and estimate the option values. The described model might be considered to be an advanced method and a generalised sensitivity analysis of estimating the derivative values under contingent claim conditions. The terms of developing the described model application are often characteristic for a financial short-term and mainly long-term decision-making. Therefore development and verifying of the fuzzy-stochastic models type might be useful. In this respect the fuzzy value as a result of fuzzy-stochastic approach could give global information for decision-maker. And this feature was a basic intention and motivation of utilisation the described methodology.

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