NEW WAY FOR SIMLIFICATION OF FUZZY LOGICAL FORMULA

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Abstract

In this paper, we give way for simplification of fuzzy logical formula using neither the "consensus" concept nor any diagrams such as the Karaugh Diagram, the Fuzzy Diagram and S-diagram. It is easy superior to some other ways in many respects. It is easy to learn, unrestricted by the number of variables. Besides the simplification form conforms to the standard and can be programmed and computerized.

Keywords: Fuzzy Logical Formula, Simplification, Disjunctive Normal Form

1. PRELIMINARIES

Let $\{x_1, x_2, \cdots, x_n\}$ be a set of variables, the fuzzy logical formula

$$F = P_1 + P_2 + \cdots + P_m \quad (1)$$

is called a disjunctive normal form. It often uses P, P_i, P_i' to express phrases or terms, their corresponding sets of literals being L, L_i, L_i'. All the concepts discussed in the references.

Proposition 1 (see [1])

$$P_1 \le P_2 < L_2 \le L_1 \quad (2)$$

Proposition 2 The irredundant single term in the fuzzy logical formula is the additive term in the extremely simplified formula.

Proposition 3 The complementary term in the fuzzy logical formula is either the redundant term in the process of simplification by cancelling literals or the additive term in the simplest formula formed by canceling some exterior complementary literal.

Proposition 4 Given

$$F = yP_0 + P_1 + \cdots P_m \quad (3)$$

where y cannot be cancelled , y still can not be cancelled after F is simplified by cancelled literal.

Proof. Supposing P_{i_0} is simplified into P_{i_0}' after a certain literal in it is cancelled, then $P_{i_0} = zP_{i_0}'$.

If $i_0 = 0$, then

$$F = yP_0' + P_1 + \cdots P_m$$

y can be cancelled, i.e.,

$$F = P_0' + P_1 + \cdots P_m \quad (4)$$

Then

$$\begin{aligned} F = P_0' + P_1 + \cdots P_m &\ge F = zP_0' + P_1 + \cdots P_m \\ &= P_0 + P_1 + \cdots P_m \\ &\ge yP_0 + P_1 + \cdots P_m = F \end{aligned}$$

Therefore $\quad yP_0 + P_1 + \cdots P_m = P_0 + P_1 + \cdots P_m$

The above demonstration shows that inherently can be cancelled, which is contrary to the supposition.

If $i_0 \ne 0$ then

$$F = yP_0 + P_1 + \cdots + P_{i_0 - 1} + P_{i_0}' + P_{i_0 + 1} + \cdots + P_m$$

where y can be cancelled, then

$$\begin{aligned} F &= yP_0 + P_1 + \cdots + P_{i_0 - 1} + P_{i_0}' + P_{i_0 + 1} + \cdots + P_m \\ &= P_0 + P_1 + \cdots + P_{i_0 - 1} + P_{i_0}' + P_{i_0 + 1} + \cdots + P_m \end{aligned}$$

$$\geq P_0 + P_1 + \cdots + P_{i_0-1} + zP_{i_0}' + P_{i_0+1} + \cdots + P_m$$

$$\geq yP_0 + P_1 + \cdots P_m = F$$

Therefore $\quad yP_0 + P_1 + \cdots P_m = P_0 + P_1 + \cdots P_m$

This proves that y can be cancelled, which is contrary to the supposition.

If the above demonstrated conclusion is repeatedly applied, it is evident that y cannot be cancelled after F is simplified by limited number of literal-cancelling. It should be noted that the simplification of F by means of literal-cancelling can be made only a limited number of times. Therefore the proposition is true.

Proposition 5 Given

$$F = yP_0 + P_1 + \cdots P_m \tag{5}$$

where y cannot be cancelled, still cannot be cancelled after some terms in $P_1, \cdots, P_m$ are cancelled from F.

Proof Given the first k terms in $P_1, \cdots, P_m$ are cancelled, i.e.

$$F = yP_0 + P_1 + \cdots P_m = yP_0 + P_{k+1} + \cdots P_m \tag{6}$$

where y can be cancelled, i.e.

$$F = yP_0 + P_1 + \cdots P_m = P_0 + P_{k+1} + \cdots P_m \tag{7}$$

then

$$(yP_0 + P_1 + \cdots P_m) + (P_1 + \cdots P_k)$$

$$= (P_0 + P_{k+1} + \cdots P_m) + (P_1 + \cdots P_k)$$

i.e. $\quad yP_0 + P_1 + \cdots P_m = P_0 + P_1 + \cdots P_m$

This shows inherently can be cancelled, which is contrary to the supposition.

The judgement of the cancellation of the literals. (see,[1])

Given $F = yP_0 + P_1 + \cdots P_m$ is an irredundant of the disjunctive normal form, then $\bar{y}$ cannot be cancelled if yP is a monomial; y cannot be cancelled if yP is a term with one order and y is in P; y cannot be cancelled if P_i (i =1,2,...,m) contains y; y cannot be cancelled if yP is a complementary term and $\bar{y} \in \bar{L}$, $\exists i \in \{1, \cdots, m\}, \bar{y} \in L_i, L_i - \{\bar{y}\} \subseteq L$.

Given P is a complementary term, where $P_1, \cdots, P_m$ contain terms without y, the judgment can be made as follows:

(1) Pick out all the phrases without y, which are supposed to be $\{P_1, \cdots, P_t\}$. Then delete the literal in P and $\bar{y}$ from every P_i (i =1,2,...,m), getting $\{P_1', \cdots, P_t'\}$. If $\exists i \in \{1, \cdots, t\}, L_i' = \Phi$, y can be cancelled. If not, go on with the 2nd step.

(2) Pick out from $P_1', \cdots, P_t'$ the monomials, which are separated from each other, thus getting $\{P_{i_1}', \cdots, P_{i_k}'\}$. Given $L_{i_1}' \cap \cdots \cap L_{i_k}' \neq \Phi$ or there are not complementary couples in $L_{i_1}' \cup \cdots \cup L_{i_k}'$, y cannot be cancelled. If not, go on with the 3rd step.

(3) $yP_0 + P_1 + \cdots P_m = P_0 + P_1 + \cdots P_m \Leftrightarrow P_{i_1}' + \cdots + P_{i_k}'$ is Boole always true $\Leftrightarrow \forall (y_1, \cdots y_k) \in L_{i_1}' \times \cdots \times L_{i_k}'$, where there are complementary couples in $y_1, \cdots y_k$

2 WAY FOR SIMPLIFICATION AND IT'S THEORETICAL BASIS

2.1 Way for Simplification

Given F is a standard formula of product and sum, simplify it as follows:

Step 1. Cancel the redundant terms in F according to proposition 1 and get $F=F'$, then turn to step 2.

Step 2. First arrange the terms in F' from the first to the last according to their progressively increased orders. As for the terms with the same order the arrangement is made in accordance with the progressively increased literals, while the arrangement for the terms with both the same order and the same number of literals can be made at random. Then, starting from the first terms with the lowest order other that the zero one, check up in the given order the cancelling possibility of the literal of the non-complementary couple of the term according to the way to judge the cancelling possibility of literal. Cancel the new redundant terms coming from the same kind of terms and the terms with high orders if a literal is cancelled. After simplifying the non-complementary literal of the term, deal with the literal of the non-complementary couple of the next term and so on until the last term, the sum of the remaining terms being F''.

Step 3. Simplify the complementary terms in F'' so as to get the simplest from of F, which is the sum of all the remaining terms.

Note. The way to judge the cancelling possibility of literal suggests that if the orders are different of judging the cancelling possibility of the literal in the non-complementary couple in the complementary terms as well as the complementary same. This conforms to the characteristics of the simplest formula, which has more that one form.

2.2 Theoretical Basis

Step 1. Having been carried out, F is simplified into a non-redundant standard formula of product and sum. The aim of arranging the terms in order is both to avoid making mistakes and to speed the process of simplification because when a literal in the preceding term is cancelled, more new emerged redundant terms will occur so that more succeeding terms can be cancelled. According to proposition 2, it is not necessary to simplify the monomoials in F'. So the simplification should be made from the term of none order. According to proposition 3, all one has to do is to examine the possibility of the literal in the non-complementary couple in the complementary terms. According to proposition 4, 5, the only thing for one to do is to examine the exterior complementary couple in order. It is known that when a certain literal in a certain term is cancelled, there may be new emerged terms. These can only be the same kind of terms or terms of high order covered by the term. Therefore, one only has to find out new emerged redundant terms ofas to cancel them. Notice again that the process of cancelling literals and new emerged redundants discussed her corresponds to the process of integration and cancellation of the covered terms dealt with in the C-algorithm presented in the reference article. Thus, the completion of the simplification corresponds to the ending of C-algorithm. So, the sum of the remaining terms after making the simplification is the simplest formula.

2.3. Examples

Ex. 1. Simplification

$$F = \bar{x}_2 x_3 x_4 + x_1 \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_1 x_2 \bar{x}_4 + x_1 \bar{x}_1 x_2 x_3 x_4 + x_1 \bar{x}_2 x_3 x_4$$

Solution. (a) Cancel the redundant terms and get

$$F = \bar{x}_2 x_3 x_4 + x_1 \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_1 x_2 \bar{x}_4 + x_1 \bar{x}_1 x_2 x_3 x_4$$

(b) The above equation meets the requirement for the arrangement in order.

The couple, $x_1 \bar{x}_1 x_2 \bar{x}_3$:

Examine x_2 and know that x_2 cannot be cancelled according to the judgement of the possibility of the cancellation of the literal.

Examine $\bar{x}_3$ and know in the same way that $\bar{x}_3$ can be cancelled.

Cancel the newly emerged redundant terms $x_1\bar{x}_1x_2\bar{x}_4, x_1\bar{x}_1x_2x_3x_4$. Now, the 2nd step is gone through, thus

$$F = \bar{x}_2x_3x_4 + x_1\bar{x}_1x_2$$

which is the simplest formula.

Ex. 2. Simplification

$$F = \overline{x_2x_4} + x_1x_2\bar{x}_3 + \bar{x}_1x_2x_4 + x_1\bar{x}_2x_3x_4 + \overline{x_1x_1x_2}x_3x_4$$

Solution. Evidently there are no redundant terms. The couple: $x_1\bar{x}_1x_2x_3x_4$.

First examine $\bar{x}_2$ and know that $\bar{x}_2$ can be cancelled. No new redundant terms. Finally examine x_4 and know x_4 cannot be cancelled. Thus the simplest equation is

$$F = \overline{x_2x_4} + x_1x_2\bar{x}_3 + \bar{x}_1x_2x_4 + x_1\bar{x}_2x_3x_4 + x_1\bar{x}_1x_4$$

If look at $\bar{x}_3$ first, then examine x_4, finally consider x_2, you can get the simplest equation is

$$F = \overline{x_2x_4} + x_1x_2\bar{x}_3 + \bar{x}_1x_2x_4 + x_1\bar{x}_2x_3x_4 + x_1\bar{x}_1x_2$$

If look at $\bar{x}_2$ first, then examine x_4, finally consider x_3, you can get the simplest equation is

$$F = \overline{x_2x_4} + x_1x_2\bar{x}_3 + \bar{x}_1x_2x_4 + x_1\bar{x}_2x_3x_4 + x_1\overline{x_1x_3}$$

The example proves that literal-cancelling in different order results in different simplest equations.

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