THE DESIGNATED THRESHOLD OF PROFITABILITY FOR MINE WITH USING FUZZY NUBERS IN CONDITIONS OF UNCERTAINTY

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Summary

This article contains considerations concerning the terming of the threshold of extraction profitability in the mine in the case, when coefficients of cost function aren't accurately well-known. By designation this function introduce fuzzy numbers. The defined threshold of profitability as fuzzy set purports risks in the area of profitability. In this case non-linear function expenses define the area as fuzzy set.

1. Introduction

In the modern marked economy, any internal cash generation in a firm should be a remunerative economic unit. The basic gauge of effectiveness depends on financial results arising from the difference between profit and cost [4]. Making decisions in relation to dimensional production must be based on the calculation of economically allowable dimension production and price sales. An important element in the programming of production is the analysis of related costs, production and profit, in order to ascertain dimension a production. This analysis should, above all, answer these questions: what should be the dimension a production, for securing yielded covering costs; what is the optimal dimension a production, which will maximize profit? [4].

By current market capital tax functions some economic categories are: established prices, actions, bonds, drafts, percentages, prices materials, stock, semiproducts, and machines. These cause, that all decision taking ligated with business activity feature large degree risks [3]. Risk also follows with continuous technological progression in the world, with fluctuating markets and products sales. This situation causes, uncertain conditions in making economic decisions.

2. The designated threshold of profitability

In classical savvying, costs of production takes part in fixed costs and in variable costs. It also assumes postulation that variable costs are proportionate to the dimension of production. It also assumes that income from sales is proportionate to the dimension of production. In analysis relation costs – production – profit, allows also a series of other postulations, which are reviewed in labours [1], [2].

Therein the article shall contemplate the finds of the threshold of profitability for the mine caused in conditions of uncertainty with the approach of estimating only singular variable costs. Using the following denotations:

P – dimension of production, extraction in the mine,

K_S – fixed costs,

c – prices,

k – singular variable costs,

$K(P)$ – function of total costs,

$K_Z(P)$ – function of variable costs,

$D(P)$ – function of total incomes.

In calculating these figures, it exists, that singular variable costs aren't accurately known and are represented by the fuzzy number k. The standard threshold of profitability for mine extraction possibilities are shown by equations

$$K(P) = K_Z(P) + K_S = kP + K_S, \tag{1}$$

$$D(P) = cP. \tag{2}$$

The threshold of profitability for mine extractions concerning dimensional production, where market income matches the total costs of production

$$D(P) = K(P), \tag{3}$$

$$cP = kP + K_S, \tag{4}$$

$$P = P_0 = K_S/(c-k). \tag{5}$$

Under these limitations: $K(P)>0$, $D(P)>0$, $K_S>0$, $k>0$, $c>0$ the threshold of profitability, P_0, has a positive value for $c>k$. Only above this number is extraction profitable.

$$D(P) > K(P) \Leftrightarrow P > P_0. \tag{6}$$

3. The designated of areas of risk of profitability

In further considerations the number k shall be regarded as fuzzy number α-β. This number is represented by four parameters (a, b, α, β). Real numbers a and b determinated set with membership = 1. α and β defined left and right breadth of this set. The membership of this fuzzy number is defined

$$\mu_k(x) = \begin{cases} 0 & \text{for} \quad x < a - \alpha, \\ (x - a + \alpha)/\alpha & \text{for} \quad x \in [a - \alpha, a), \\ 1 & \text{for} \quad x \in [a, b], \\ (b + \beta - x)/\beta & \text{for} \quad x \in (b, b + \beta], \\ 0 & \text{for} \quad x > b + \beta. \end{cases} \tag{7}$$

Figure 1 presented graph of this function

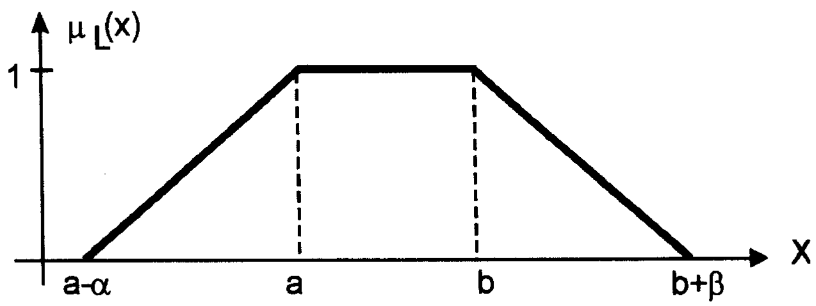

Fig 1. Graph of membership function of fuzzy number α-β

In order to assignations P_0 by formula (5) belongs first estimate difference real number c and fuzzy number k. Next belongs carry out sharing real number K_S by fuzzy number c-k.

For two fuzzy numbers $L_1=(a_1, b_1, \alpha_1, \beta_1)$ and $L_2=(a_2, b_2, \alpha_2, \beta_2)$ is [1]

$$L_1 - L_2 = (a_1 - b_2,\ b_1 - a_2,\ \alpha_1 + \beta_2,\ \beta_1 + \alpha_2), \tag{8}$$

$$L_1 / L_2 = (a_1/ b_2,\ b_1/a_2,\ (a_1\beta_2+b_2\alpha_1)/(b_2(b_2+\beta_2)),\ (b_1\alpha_2+a_2\beta_1)/(a_2(a_2-\alpha_2))), \tag{9}$$

Admitting bequests fuzzy numbers we have

$$k = (a, b, \alpha, \beta), \qquad c = (c, c, 0, 0), \qquad K_S = (K_S, K_S, 0, 0). \tag{10}$$

On the basis (8) and (9) we obtains

$$c - k = (c-b,\ c-a,\ \beta,\ \alpha) \tag{11}$$

$$K_S/(c-k) = (K_S/(c-a),\ K_S/(c-b),\ K_S\alpha/((c-a)(c-a+\alpha)),\ K_S\beta/((c-b)(c-b-\beta)), \tag{12}$$

And so threshold of profitability extractions is fuzzy number of definite of formula (12).

Definition 1.

Areas of risk of profitability L_0 is defined as follows

$$L_0 = (a_0, b_0, \alpha_0, \beta_0). \tag{13}$$

where

$a_0 = K_S/(c-a)$, $b_0 = K_S/(c-b)$, $\alpha_0 = K_S\alpha/((c-a)(c-a+\alpha))$, $\beta_0 = K_S\beta/((c-b)(c-b-\beta))$. (14)

Areas of risk calculated from the defined equations (13) and (14) is an amplification of the notions concerning threshold profitability. Where extraction P is less than $a-\alpha_0$ extraction is unprofitable, however, where extraction P is bigger than $b+\beta_0$ brings extraction profit. In areas of risk of profitability, where ($a-\alpha_0<P<b+\beta_0$) mining can either have a profit or can bear a loss. Accessorily ascertain, that in compartment ($a_0-\alpha_0$, a_0) herewith by decreasing extractions diminishes delineative probability, that mine raised profit. However in compartment (b_0, $b_0+\beta_0$) herewith by increase extractions grows delineative probability, that mine will be profit.

Example 1.

Price of coal is c=115 zloty for one ton. Fixed costs K_S during the year are 320 million zloty. Singular variable costs, which are denominated by fyzzy number k=(24, 26, 2, 3) zloty/ton. Than cost functions and income functions are as follows:

$$K(P) = kP + 320, \qquad D(P) = 115 \tag{15}$$

On the basis of formulas (12) and (14), area of risk of profitability is defined by fuzzy number L_0=(3.516484, 3.595506, 0.075624, 0.125434) million tons. Thus, annual extraction below 3.44 million tons (3.44=3.52-0.08) is unprofitable and extraction above 3.72 million tons (3.72=3.60+0.12) brings profit. The set (3.44, 3.72) million tons may, or may not be because accurate, singular variable costs are not well-known. The discoursed example illustrates figure 2. Costs, incomes and extractions represented by millions.

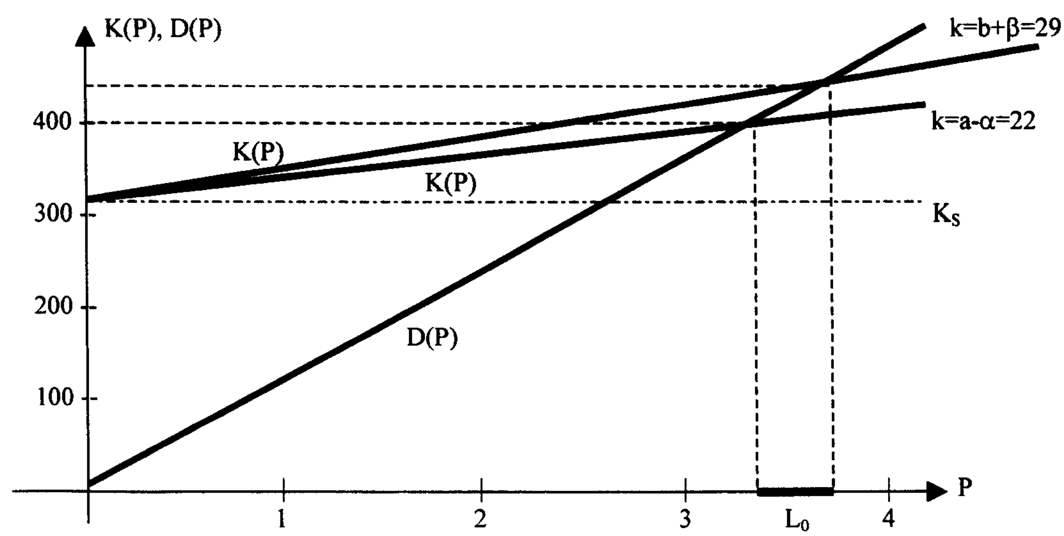

Fig.2. The area of profitability risks in mining (k= (24, 26, 2, 3))

4. Designated area of profitability risks for the non-linear function costs

In the foregoing chapter the usual linear form of function costs ($K(P)=kP+K_S$). It is dependently approached. It can come about, that the accurate dependence between costs and extraction represents the non-linear function. Then for businesses may be made possible some area profitability risks. The consideration of another realization for the function costs purports the polynomial second degree.

Example 2.

The price of coal is c=115 zloty per ton. Fixed costs K_S during the year are 320 million zloty. Singular variable costs are represented, by the fuzzy number k=(24, 26, 2, 3) zloty/ton. On the basis of the tested data in mining from the next 12 months, in has an estimated regression for the function in dependence between total costs K(P) and dimension a production P. This function has the following form

$$K(P) = (0.0426k+18.4562)P^2 + (0.341k-57.3503)P + 320. \quad (16)$$

Incomes function is represented as follows:

$$D(P) = 115P. \quad (17)$$

In the formulas (16) and (17) the costs and incomes are presented in millions of zloty, and production in millions of pieces per year.

For $k=a-\alpha=22$ the equation (16) assumes the form

$$K(P) = 19.3934P^2 - 49.8483P + 320. \quad (18)$$

For k=a=24 the equation (16) assumes the form

$$K(P) = 19.4786P^2 - 49.1663P + 320. \quad (19)$$

For k=b=26 the equation (16) assumes the form

$$K(P) = 19.5638P^2 - 48.4843P + 320. \quad (20)$$

However for $k=b+\beta$ =29 the equation (16) assumes the form

$$K(P) = 19.6916P^2 - 47.4613P + 320. \quad (21)$$

Equalization costs K(P) with incomes D(P):

for the function (18) juts at the production $P_1=3$ and at $P_2=5.5$ million tons,

for the function (19) juts at the production $P_3=3.060911$ and at $P_4=5.367123$ million tons,

for the function (20) juts at the production $P_5=3.128994$ and at $P_6=5.227475$ million tons,

for the function (21) juts at the production $P_7=3.25$ and at $P_8=5$ million tons.

The profit purporting from the difference between incomes D(P) and costs K(P), is positive in the case

for function (18) in extend of production (3, 5.5) million tons,

for function (19) in extend of production (3.060911, 5.3671233) million tons,

for function (20) in extend of production (3.128994, 5.227475) million tons,

for function (21), in extend of production (3.25, 5) million tons.

Thus, these has arisen two areas of risk for profitability extending fuzzy numbers L_1=(3.060911, 3.128994, 0.060911, 0.121006) million tons (α=3.060911-3=0.060911, β=3.25-3.128994=0.121006) and L_2=(5.227475, 5.367123, 0.227475, 0.132877) million tons (α=5.227475–5=0.227475, β=5.5-5.367123=0.132877), in which production may, or may not be cost-effective. Extractio with a yearly production below 3 million tons or above 5.5 million tons is unprofitable. Production levels between (3.25, 5) million tons brings profit. This situation is illustrated in figure 3.

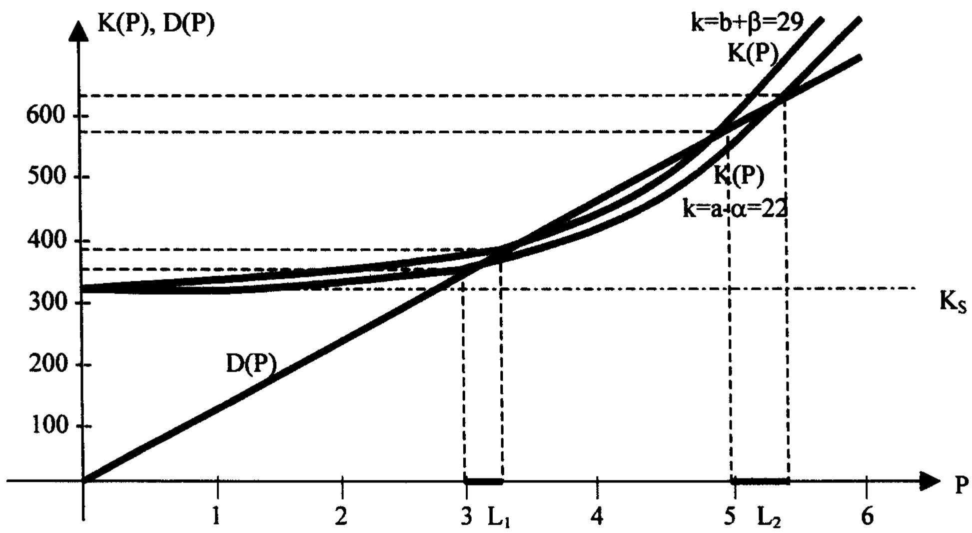

Fig.3. The areas of risk of profitability for non-linear costs function

5. Conclusion

In the associated analysis of scrutinized cost-effective production businesses it is assumed, that function costs and function incomes have linear forms. The threshold of profitability also is shown here. The numbers, based on costs and income is given a value. From the linear form of function costs and the function of income. It follows that only one threshold profitability exists.

The Introduced area of profitability is an extension of the classical notion of the threshold of profitability. This area arises, by effect from inaccurate determinations of singular variable costs, fixed costs, prices or contemporaneously inaccurate determinations of some quantities. Along with the linear function costs juts into one such area, in which one can have profit or incure losses. For non-linear function costs can occur in some such areas.

References

1. Bolc L., Borodziewicz W., Wojcik M.: Bases for porocessing incomplete and uncertain information. PWN, Warszawa 1991.
2. Czogala E., Pedrycz W.: Elements and methods of theory of fuzzy sets. PWN, Warszawa 1985.
3. Drewniak J.: Fuzzy relation calculus. Scientific Works of Silesian University No 1063, Katowice 1989.
4. Drewniak J.: Basis of the theory of fuzzy sets. Handbook of Silesian University No 347, Katowice 1987.
5. Dubois D., Prade H.: Operations on fuzzy numbers. Int. J. Syst. Sci., vol. 9, 1978.
6. Horngren Ch.T., Introduction to Management Accounting, Prentice-Hall International, London 1990.
7. Kacprzyk J.: Fuzzy sets in system analysis. PWN, Warszawa 1983.
8. Kaplan R.S., Advansed Management Accounting, Prentice-Hall International, London 1982.
9. Kopiński A., Obszar symulacyjno-analityczny jako podstawowy element systemu mierników kondycji ekonomicznej przedsiębiorstw, Badania Operacyjne i Decyzje 2/1992, Wrocław 1993.
10. Kowalik S.: Decision making in mining in conditions of uncertainty. Scientific Works of Silesian Technical University, Mining, No 228, Gliwice 1996.
11. Nowak E., Relacja koszty-produkcja-zysk w rachunku decyzyjnym, Badania Operacyjne i Decyzje 2/1992, Wrocław 1993.
12. Zadeh L. A.: Fuzzy sets. Information and Control, vol. 8, 1965.