$(\in, \in \vee q)$-Fuzzy ideals and $(\in, \in \vee q)$-Fuzzy quotient algebras

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Abstrct: In this paper, the concept of $(\in, \in \vee q)$-fuzzy ideal and $(\in, \in \vee q)$-fuzzy quotient algebra are introduced, and its some elmentary properties are discussed.

Keywords: Belongs to; quasi-coincident; $(\in, \in \vee q)$-fuzzy field; $(\in, \in \vee q)$-fuzzy algebra; $(\in, \in \vee q)$-Fuzzy ideal; $(\in, \in \vee q)$-Fuzzy quotient algebra.

1. Introduction

In 1996, S. K. Bhakat and P. Das[4-5]used relation of "belongs to" and "quasi-coincident" between fuzzy point and fuzzy set, introduced the concepts of an $(\in, \in \vee q)$-fuzzy subgroup and $(\in, \in \vee q)$-fuzzy subrings, and obtained some fundamental results pertaining to these notions. Fuzzy field and Fuzzy algebra over fuzzy field were researched by Nada [1], Gu wenxiang and Lu Tu [2] and Dang [3]. The paper [7] introduces the concept of $(\in, \in \vee q)$-fuzzy algebra over $(\in, \in \vee q)$-fuzzy field. In this paper, $(\in, \in \vee q)$-fuzzy ideal and $(\in, \in \vee q)$-fuzzy quotient algebra are defined, and their some properties are studied.

2. Preliminaries

Let X be any non-empty set.

Definition 2. 1. A map $\lambda: X \to [0, 1]$ is called a fuzzy set of X.

Definition 2. 2. A fuzzy point x_t is said to belong to (resp. be quasi-coincident with) a fuzzy set λ, written as $x_t \in \lambda$ (resp. $x_t q \lambda$) if $\lambda(x) \geqslant t$ (resp. $\lambda(x) + t > 1$). If $x_t \in \lambda$ or $x_t q \lambda$, then we write $x_t \in \vee q \lambda$.

For any $t, r \in [0, 1]$, $M(t, r)$ will denote $\min(t, r)$.

Definition 2. 3. Let X be a field and F a fuzzy set of X. If for all $x, y \in X$ and $t, r \in (0, 1]$, the following conditions hold:

(Ⅰ) $x_t, y_t \in F \Rightarrow (x+y)_{M(t, r)} \in \vee qF$;

(Ⅱ) $x_t \in F \Rightarrow (-x)_t \in \vee qF$;

(Ⅲ) $x_t, y_t \in F \Rightarrow (xy)_{M(t, r)} \in \vee qF$;

(Ⅳ) $x_t \in F\ (x \neq 0) \Rightarrow (x^{-1})_t \in \vee qF$.

we call F an $(\in, \in \vee q)$-fuzzy field of X.

Definition 2. 4. Let F be an $(\in, \in \vee q)$-fuzzy field of the field X, Y a algebra over X, and A a fuzzy set of Y. If for all $x, y \in Y$, $\lambda \in X$ and $t, r \in (0, 1]$, the following conditions hold:

(Ⅰ) $x_t, y_t \in A \Rightarrow (x+y)_{M(t, r)} \in \vee qA$;

(Ⅱ) $x_t \in A, \lambda_r \in F \Rightarrow (\lambda x)_{M(t, r)} \in \vee qA$;

(Ⅲ) $x_t \in A, y_r \in A \Rightarrow (xy)_{M(t, r)} \in \vee qA$;

(Ⅳ) $F(1) \geqslant M(A(x), 0.5)$.

Then we call A an $(\in, \in \vee q)$-fuzzy algebra over $(\in, \in \vee q)$-fuzzy field F.

Proposition 2. 1. The condition (Ⅰ) in definition 2. 4 is equivalent to

(Ⅰ') $A(x+y) \geqslant M(A(x), A(y), 0.5)$, $x, y \in Y$;

The condition (Ⅱ) of Definition 2. 4 is equivalent to

(Ⅱ') $A(\lambda x) \geqslant M(F(\lambda), A(x), 0.5)$, $\lambda \in X$, $x \in Y$;

The condition (Ⅲ) of Definition 2. 4 is equivalent to

(Ⅲ') $A(xy) \geqslant M(A(x), A(y), 0.5)$, $x, y \in Y$.

Proposition 2. 2. Let Y and Z be algebras over the field X, f a algebraic homomorphism of Y into Z and A a fuzzy set of Z. If A an $(\in, \in \vee q)$-fuzzy algebra over an $(\in, \in \vee q)$-fuzzy field F. Then $f^{-1}(A)$ is an $(\in, \in \vee q)$-fuzzy algebra over $(\in, \in \vee q)$-fuzzy field F.

Proposition 2. 3. Let Y and Z be algebras over the field X, f a algebraic homomorphism of Y into Z and A a fuzzy set of Y. If A an $(\in, \in \vee q)$-fuzzy algebra over $(\in, \in \vee q)$-fuzzy field F. Then f (A) is an $(\in, \in \vee q)$-fuzzy algebra over $(\in, \in \vee q)$-fuzzy field F.

3. $(\in, \in \vee q)$-fuzzy ideal and $(\in, \in \vee q)$-fuzzy quotient algebra

Definition 3. 1. A is said to be an $(\in, \in \vee q)$-fuzzy ideal of a algebra Y, If

(i) A is an $(\in, \in \vee q)$-fuzzy algebra of Y;

(ii) $x_t \in A$ and $y \in Y \Rightarrow (xy)_t, (yx)_t \in \vee qA$.

Proposition 3. 1. The condition (ii) in definition 3. 1 is equivalent to A (xy), A (yx) $\geqslant$ M (A (x), 0. 5), $\forall$ x, y $\in$ Y.

Proposition 3. 2. Let Y and Z be algebras over the field X, f a algebraic homomorphism of Y into Z.

(i) If A an $(\in, \in \vee q)$-fuzzy ideal of Y, then f (A) is an $(\in, \in \vee q)$-fuzzy ideal of Z.

(ii) If B an $(\in, \in \vee q)$-fuzzy ideal of Z, ther f^{-1} (B) is an $(\in, \in \vee q)$-fuzzy ideal of Y.

Proof. (i) By proposition 2. 3, f (A) is an $(\in, \in \vee q)$-fuzzy algebra of Z.

For any x, y $\in$ Z.

$$f(A)(xy) = \sup_{f(z)=xy} A(z) \geqslant \sup_{\substack{f(\bar{x})=X \\ f(\bar{y})=y}} A(\bar{x}\bar{y}) \geqslant \sup_{f(\bar{x})=x} M(A(\bar{x}), 0.5)$$

$$= M(\sup_{f(\bar{x})=x} A(\bar{x}), 0.5)$$

$$= M(f(A)(x), 0.5).$$

Similarly, f (A) (yx) $\geqslant$ M (f (A) (x), 0. 5). So f (A) is an $(\in, \in \vee q)$-fuzzy ideal of Z.

(ii) By proposition 2. 2, f^{-1} (B) is an $(\in, \in \vee q)$-fuzzy algebra of Y.

For any x, y $\in$ Y.

$f^{-1}(B)(xy) = B(f(xy)) = B(f(x)f(y)) \geqslant M(B(f(x)), 0.5) = M(f^{-1}(B)(x), 0.5)$

similarly, $f^{-1}(B)(yx) \geqslant M(f^{-1}(B)(x), 0.5)$. So $f^{-1}(B)$ is an $(\in, \in \vee q)$-fuzzy ideal of Y.

Let A be an $(\in, \in \vee q)$-fuzzy algebra of algebra Y, if $A(0) < 0.5$, $x \in Y$, then $A(x) < 0.5$, $x \in Y$. At this time, A is satisfied with the conditions of the article[2]' s defination 2. Henceforth, we suppose $A(o) \geqslant 0.5$ in the following discussion.

Definition 3.2. Let A be an $(\in, \in \vee q)$-fuzzy ideal of Y and $x \in Y$. The fuzzy subest A_x of Y defined by $A_x(g) = M(A(g-x), 0.5)$, $\forall g \in Y$ is called the fuzzy coset determined by x and A.

Proposition 3.3. Let Y be a algebra over field X, for any $(\in, \in \vee q)$-fuzzy ideal A of Y, write $Y/A = \{A_x | x \in Y\}$, definite the operations of the three following over the Y/A.

(i) $A_x + A_y = A_{x+y}$, $x, y \in Y$;

(ii) $A_x \cdot A_y = A_{xy}$, $x, y \in Y$;

(iii) $\lambda A_x = A_{\lambda x}$, $\lambda \in X$ and $x \in Y$.

then Y/A forms algebra about the above three operations.

Definition 3.3. We call Y/A the $(\in, \in \vee q)$-fuzzy quotient algebra of Y w.r.t. A.

Proposition 3.4. $B: Y/A \to [0, 1]$ defined by

$$B(A_x) = A(x), \forall A_x \in Y/A$$

is an $(\in, \in \vee q)$-fuzzy ideal of Y/A.

Proof. Let $x, y \in Y$, $\lambda \in X$. Then

(i) $B(A_x + A_y) = B(A_{x+y}) = A(x+y) \geqslant M(A(x), A(y), 0.5)$
$= M(B(A_x), B(A_y), 0.5)$;

(ii) $B(\lambda Ax) = B(A_{\lambda x}) = A(\lambda x) \geqslant M(F(\lambda), A(x), 0.5)$
$= M(F(\lambda), B(A_x), 0.5)$;

(iii) $F(1) \geqslant M(A(x), 0.5) = M(B(A_x), 0.5)$;

(iV) $B(Ax \cdot Ay) = B(Axy) = A(xy) \geqslant M(A(x), 0.5)$
$= M(B(Ax), 0.5)$.

Similarly $B(A_y \cdot A_x) \geqslant M(B(Ax), 0.5)$. So B is an $(\in, \in \vee q)$-fuzzy ideal of Y/A

Proposition 3. 5. Let A be an $(\in, \in \vee q)$-fuzzy ideal of Y and B be an $(\in, \in \vee q)$-fuzzy ideal of Y/A. Then $\tau: Y \rightarrow [0, 1]$defined by

$$\tau(x) = B(A_x) \ \forall \ x \in Y$$

is an $(\in, \in \vee q)$-fuzzy ideal of Y.

Proof. Straightforward.

Proposition 3. 6. Let Z be algebra and $f: Y \rightarrow Z$ be a algebra homomorphism. Let A and B be $(\in, \in \vee q)$-fuzzy ideals of Y and Z, respectively. Then $\bar{f}: Y/A \rightarrow Z/B$ defined by

$$\bar{f}(A_x) = B_{f(x)}$$

is a homomorphism.

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