Fuzzy Interval Linear Programming Problems and Their Solutions

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Abstract: Fuzzy linear programming problem with fuzzy interval coefficients is discussed further on the basis of [6] in this paper. The different auxiliary models of the problem are obtained with ranking fuzzy interval numbers in the setting of random sets. Fuzzy solutions of the problem are given.

Keywords: Fuzzy interval linear programming, fuzzy interval number, fuzzy preference relation

1. Introduction

The coefficients in the conventional linear programming problem(LP) is crisp. In real decision –making problems, however, it is usual that coefficients of (LP),where human estimation is used, are inexact. A general model for fuzzy linear programming and a lot of auxiliary models on considered different ranking method to obtain the solution of fuzzy linear programming problem are given in [1,2]. Possibilistic linear programming problems with fuzzy constraints are presented[3]. Since probability theory also deals with uncertain problems and fuzzy sets are viewed as equivalence classes of random sets[4], the combination of both uncertainties has a practical meaning. Ranking fuzzy interval numbers in the setting of random sets is put forward by [5]. Fuzzy linear programming problems with fuzzy interval coefficients and some auxiliary models are given in [6]. In this paper, we further discuss the solution of fuzzy LP problem in different order relation so that decision-maker can make decision according to real problem.

2. Preliminaries

Basic concepts used in the paper can be seen in references.

Let us denote with $FN(L,R)$ the set of all fuzzy interval numbers of the L-R type. Consider two fuzzy preference relations $R_i : i = 1,2$ defined on the set $FN(L,R)$. The membership functions $\mu_i : FN(L,R)^2 \to [0,1], i = 1,2$.

$$\mu_1(A,B) = \Pr ob\{\underline{a} - L^{-1}(Y)\alpha_A \le \underline{b} - L^{-1}(Y)\alpha_B\}$$

$$\mu_2(A,B) = \Pr ob\{\bar{a} - R^{-1}(Y)\beta_A \le \bar{b} - R^{-1}(Y)\beta_B\}$$

where $A,B \in FN(L,R)$, $A = (\underline{a},\bar{a},\alpha_A,\beta_A)_{L-R}$, $B = (\underline{b},\bar{b},\alpha_B,\beta_B)_{L-R}$, the symbol Y stands for a random variable with the uniform distribution on the interval $[0,1]$.

Proposition 1 Let A,B be two fuzzy interval numbers on $FN(L,R)$ and $A = (\underline{a},\bar{a},\alpha_A,\beta_A)_{L-R}, B = (\underline{b},\bar{b},\alpha_B,\beta_B)_{L-R}$. Then

$$\underline{a} - \underline{b} - L^{-1}(1/2)(\alpha_A - \alpha_B) \le 0 \Leftrightarrow \mu_1(A,B) \ge 1/2.$$

Proposition 2 Let A,B be two fuzzy interval numbers on $FN(L,R)$ and $A = (\underline{a},\bar{a},\alpha_A,\beta_A)_{L-R}, B = (\underline{b},\bar{b},\alpha_B,\beta_B)_{L-R}$. Then

$$\bar{a} - \bar{b} + R^{-1}(1/2)(\beta_A - \beta_B) \le 0 \Leftrightarrow \mu_2(A,B) \ge 1/2.$$

3. Fuzzy linear programming problems

Consider fuzzy linear programming problems with fuzzy interval coefficients in the constraints:

$$\max \quad z = \sum_{j=1}^{n} c_j x_j$$

$$s.t. \quad H_i(x) = \sum_{j=1}^{n} A_{ij} x_j \le B_i, \; i = 1,2,\cdots,m, \qquad (1)$$

$$x_j \ge 0, \quad j = 1,2,\cdots,n,$$

where $A_{ij}=(\underline{a}_{ij},\overline{a}_{ij},\alpha_{ij},\beta_{ij})_{L-R}$, $B_i=(\underline{b}_i,\overline{b}_i,\alpha_i,\beta_i)_{L-R}$, $i=1,2,\cdots,m,\ j=1,2,\cdots,n$ are fuzzy interval numbers of the L-R type. It is easy to know that $H_i(x)$ is also a fuzzy interval number of the L-R type.

$$H_i(x)=(\underline{h}_i(x),\overline{h}_i(x),\alpha_{H_i(x)},\beta_{H_i(x)})_{L-R},$$

where

$$\underline{h}_i(x)=\sum_{j=1}^{n}\underline{a}_{ij}x_j\ ,\overline{h}_i(x)=\sum_{j=1}^{n}\overline{a}_{ij}x_j\ ,\alpha_{H_i(x)}=\sum_{j=1}^{n}\alpha_{ij}x_j\ ,\beta_{H_i(x)}=\sum_{j=1}^{n}\beta_{ij}x_j\ .$$

If $\mu_1(A,B)\ge 1/2$ and $\mu_2(A,B)\ge 1/2$ the decision-maker can accept $A\le B$. That is, for any two fuzzy interval numbers $A,B\in FN(L,R)$,

$$A\le B\Leftrightarrow \mu_1(A,B)\ge 1/2\quad and\quad \mu_2(A,B)\ge 1/2. \tag{2}$$

From Proposition 1-2 and (2),the auxiliary model to solve(1) is

$$\max\quad z=\sum_{i=1}^{n}c_jx_j$$

$$s.t.\ \sum_{j=1}^{n}(\underline{a}_{ij}-L^{-1}(1/2)\alpha_{ij})x_j\le \underline{b}_i-L^{-1}(1/2)\alpha_i \tag{3}$$

$$\sum_{j=1}^{n}(\overline{a}_{ij}-R^{-1}(1/2)\beta_{ij})x_j\le \overline{b}_i-R^{-1}(1/2)\beta_i$$

$$x_j\ge 0,\quad i=1,2,\cdots,m,\quad j=1,2,\cdots,n.$$

It is clear that the model (3) is a crisp linear programming problem. We can solve it with simplified method.

4. Numerical example

Solve following fuzzy linear programming problem

$$\max\quad z=3x_1+6x_2$$

$$s.t.\ A_{11}x_1+A_{12}x_2\le B_1,\quad A_{21}x_1+A_{22}x_2\le B_2 \tag{4}$$

$$x_1,x_2\ge 0,$$

where $A_{11} = (3,5,4,2)_{L-R}$, $A_{12} = (6,10,8,12)_{L-R}$,

$A_{21} = (2,3,1,2)_{L-R}$, $A_{22} = (4,7,5,10)_{L-R}$,

$B_1 = (12,20,14,18)_{L-R}$, $B_2 = (18,24,20,18)_{L-R}$,

are fuzzy interval numbers of the L-R type, $L(y) = e^{-y}$, $R(y) = e^{-2y}$. From (3), we obtain the auxiliary model of (4)

$$\max \quad z = 3x_1 + 6x_2$$

$$s.t. \quad 0.2274411x_1 + 0.454823x_2 \le 2.295939$$

$$1.306853x_1 + 0.534264x_2 \le 4.137056$$

$$5.693147x_1 + 14.158883x_2 \le 26.238325$$

$$3.693147x_1 + 10.465738x_2 \le 30.238325$$

$$x_1, x_2 \ge 0,$$

whose optimal solution is $x_1^* = 2.88, x_2^* = 0.69$ and optimal value is $z^* = 12.8117$.

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