Polyfactoral analysis of fuzzy sociogram.

O.R. Mukhatdinova

Tashkent State University, Uzbekistan.

In this paper the new approach of measuring and evaluating of group relationship by means of fuzzy logic is proposed. It allows construction of a fuzzy sociogram on the basis of data obtained from simple questionnaire. Constructed sociogram is a base for polyfactoral analysis of links between the members of the group.

Key words: fuzzy sociogram, triangular norm, averaging operator, dendrogram.

1. Introduction.

Sociometrical analysis is one of the methods of measuring and evaluating of social structure of a small group by means of sociogram [1, 2].

In the papers of H. Yamashita and his colleagues [2, 3] where the notion of fuzzy sociogram was introduced, the questions of monofactoral and polyfactoral analysis of sociogram was considered. The influence of every factor (sociometrical criterions) was considered there as equivalent.

In this paper, we propose to take into account the importance of each of sociological criteria Bj, proceeding from the importance of corresponding measuring coefficients Wj, j=1,2,...,m. Measured coefficient is set on the basis of expert estimates. The method of "nondominated alternatives by Orlovsky" [7] is relevant for this.

2. Analysis of fuzzy sociograms.

Let H be the number of respondents, K= (Kij) - response matrix (Fig.1). This matrix is built up from the questionnaire data (Table 1).

Table 1.

Personal questionnaire card.

	B1	B2	...	Bm
S1				
S2				
...				
S3				

From table K we have an evaluation matrix R=(Rij). Since respondent Si makes his selection of respondents Sj in accordance with a set of sociometrical criteria, then Rij means the number of these selections.

Then we obtain corrected evaluation matrix L=(Lij), Lij=Rij*Wj, Wj- measuring coefficients of considered sociometrical criterion. In this paper we don't reveal the procedure of formation of measuring coefficients by the method of nondominated alternatives, you may find it in [8].

Let Fij=Lij/ max Lij, where Lij is a maximal element from all of ones in matrix Lij. Thus, we get a fuzzy matrix F=(Fij), whose elements indicate the grade of a veritable statement: "subject Si prefers subject Sj according to sociometrical criterions".

Here, $0 \le Fij \le 1$, Fij = 1, if i = j.

At the same time, we have fuzzy graph F=(Fij), called fuzzy sociogram [2, 3].

If Fij=1, then the statement "subject Si prefers subject Sj" is veritable. And if Fij is equal to zero, then this statement is false.

The other situations have intermediate grade of verity.

To estimate the statement "subjects Si and Sj have mutual preferences in accordance with the set of sociological criterions", we use Gij:

$$Gij = \begin{cases} 0, & \text{if } A(Fij, Fji) = 0 \\ T(Fij, Fji)/A(Fij, Fji), & \text{else} \end{cases}$$

Thus, for obtaining Gij we introduce [4, 6] triangular norm T(x,y) and averaging operator A(x,y), x,y are changing from 0 to 1. In the fuzzy logic, T(x,y) is modeling an operation of conjunction, A(x,y) is the ordering multiplier.

Obviously, Gij changes from 0 to 1. If Gij is near to 1, that means that Si has strong preference of Sj, and if Gij is near to 0, it means that Si has a week preference of Sj.

We also obtain fuzzy graph G=(Gij), the analysis of which allows us to construct a dendrogram P. Dendrogram describes the dynamic of the clustering in the studied group.

Finally, summarizing information from fuzzy graph F and dendrogram P, we have sociogram Un, where n is the level of mutual preference in the small group, $0 \le n \le 1$ [2, 3].

In the conclusion of this part let us give functional description for Archimed's triangular norm [4] and averaging operator [8].

$T(x,y)= f^{-1}(\min(f(0), f(x)+f(y)))$, where function f: [0,1]—>[0,∞] is an unremitted and strongly decreasing one, f(1)=0. Function f called a generator of T(x,y), f^{-1} - is a reversing function.

$A(x,y)=g^{-1} (d^{-1}(g(1)+dg(x))^{\beta_1} *(g(1)+dg(y))^{\beta_2})$, where function g: [0,1]—>[0,∞] is an unremitted and strongly increasing one, g(0)=0, g - is a reversing function.

In particular, when d=0 $\quad A(x,y)=g^{-1}(\beta_1 g(x)+\beta_2 g(y))$,

when d=∞ $\quad A(x,y)=g^{-1}(g^{\beta_1}(x)*g^{\beta_2}(y))$,

when d=-1 $\quad A(x,y)=g^{-1}(g(1)-(g(1)-g(x))^{\beta_1} * (g(1)-g(y))^{\beta_2})$.

Thus, we see that corresponding choice of functions f and g provides the adequate modeling of Gij.

Example: Let f(x)= -Ln(x); g(x)=x when d=0 $\beta_1=\beta_2=0.5$. Then T(x,y)=x*y, A(x,y)=0.5(x,y) and we have Gij=0.5(1/x + 1/y) [2,3].

3. Case study.

For the quantitative calculation we will use the data which is obtained from the test of 17 students with the aim to study structure of relationship of this group [2]. The test was held with account of three sociological criterions: B_1 - "With whom do you want to join quiz program?"; B2 - "With whom do you want to study in group?"; B_3 - "With whom do you want to do volunteer activity?", with corresponding measuring coefficients W=1.0, W=0.8, W=0.3.

Response matrix K is shown in Fig 1. On the basis of the above mentioned formulae it is easy to have fuzzy matrix F (Fig. 2)., which we call also a fuzzy sociogram F [2,3].

Then we will construct two dendrograms, with triangular norm T(x,y)=x*y, but we will be considering two cases. First case with equal factors (sociometrical criterions) [2] and second case with unequal factors (their inequality is represented with measuring coefficients Wi). As an averaging operator we use A(x,y)=0.5(x+y).

Therefore, we have two fuzzy matrixes G (Fig.3, Fig. 4) and two dendrograms P (Fig.5, Fig.6). according the technology of the data processing as it is shown in the second part of this paper, we have two sociograms Un, n=0.71 for two cases (Fig. 7, Fig.8).

	B_1 $W_1=1$	B_2 $W_2=0.8$	B_3 $W_3=0.3$
S01	....	....	....
S02	S10, S13, S14, S17	S03, S09, S12, S17	S04, S17
S03		S06, S12, S17	S06
S04	S09, S11, S12, S13, S17	S02, S07, S10, S12, S13	S02, S06, S07, S09, S10
S05	S02, S06, S10	S04, S06, S16	S04, S16
S06	S09, S10, S17	S03, S07	S09, S14, S17
S07	S09, S14, S17	S04, S12, S14	S04, S09, S17
S08	S02, S06, S07, S09, S11	S09, S11	S04, S09, S10, S11
S09	S07, S08, S10, S11, S14	S07, S08, S11	S07, S08, S11, S04
S10	S02, S11, S12, S13, S14	S02, S11, S12	
S11	S08, S09, S10, S12, S02	S08, S09, S10, S12, S17	S02, S07, S08, S09, S10
S12	S04, S10, S13, S17	S03, S08, S09, S14	S04, S17
S13	S02, S04, S10, S12, S15	S02, S10, S12, S15, S17	S02, S04, S10, S12, S15
S14	S02, S07, S10, S12	S12, S13, S17	S02, S04, S09, S10
S15	....	....	....
S16	S10	S05	S05
S17	S02, S08, S12, S13	S02, S12, S13	S02, S12, S13

Fig.1. Response matrix K.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1																
2		1	0.38	0.42					0.38	0.47		0.38	0.47	0.47			1
3			1			0.52						0.38					0.38
4		0.52		1		0.14	0.52		0.62	0.52	0.47	0.85	0.85				0.47
5		0.47		0.52	1	0.85				0.47						0.52	
6			0.38			1	0.38		0.62	0.47				0.14			0.62
7				0.52			1		0.62			0.38		0.86			0.62
8		0.47		0.14		0.47	0.47	1	1	0.14	1	1					
9				0.14			1	1	1	0.47	1			0.47			
10		0.86								1	0.86	0.86	0.47	0.47			
11		0.62					0.14	1	1	1	1	0.86					
12			0.38	0.62				0.38	0.38	0.47		1	0.47	0.38			0.62
13		1		0.62						1		1	1	1	1		0.38
14		0.62		0.14			0.47		0.14	0.62		0.86	0.38	1			0.38
15															1		
16					0.52					0.47						1	
17		1						0.47				1	1				1

Fig. 2. Fuzzy matrix F.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1																
2		1		0.44						0.44			0.5	0.44			1
3			1			0.44						0.33					
4		0.44		1			0.67		0.44			0.67	0.87				
5					1											0.67	
6			0.44			1											
7				0.67			1		0.8					0.44			
8								1	1		1	0.33					
9				0.44			0.8	1	1		1			0.33			
10		0.44								1	0.8	0.44	0.5	0.44			
11								1	1	0.8	1						
12			0.33	0.67				0.33		0.44		1	0.5	0.44			0.8
13		0.5		0.67						0.5		0.5	1	0.5			0.5
14		0.44					0.44		0.33	0.44		0.44	0.5	1			
15															1		
16					0.67											1	
17		1										0.8	0.5				1

Fig.3. Matrix G (1st case)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1																
2		1		0.44						0.6			0.64	0.53			1
3			1			0.42						0.36					
4		0.44		1			0.52		0.22			0.71	0.71				
5					1											0.52	
6			0.42			1								0.6			
7				0.52			1		0.76								
8								1			1	0.55		0.2			
9				0.22			0.76		1		1			0.54			
10		0.6								1	0.92	0.59					
11										0.92	1			0.51			
12			0.36	0.71				0.55		0.59		1		0.55			0.76
13		0.64		0.71						0.64		0.64	1				0.55
14		0.53					0.6		0.2	0.54		1		1			
15												0.55			1		
16					0.52											1	
17		1										0.55					1

Fig.4. Matrix G (2nd case)

```
(5) (16) (3) (6) (14) (4) (13) (7) (8, 9, 11) (10) (12) (2, 17)      n=1.00
 |    |   |   |    |   |    |                          V
(5) (16) (3) (6) (14) (4) (13)  (7, 8, 9, 11, 10)  (12, 2, 17)       n=0.80
   V      |   |    |
(5,16)  (3) (6) (14) (4, 13, 7, 8, 9, 11, 10, 12, 2, 17)             n=0.67
   |     |   |        V
(5,16)  (3) (6) (14, 4, 13, 7, 8, 9, 11, 10, 12, 2, 17)              n=0.50
   |       V                |
(5,16)  (3, 6) (14, 4, 13, 7, 8, 9, 11, 10, 12, 2, 17)               n=0.44
   |       V
(5,16)  (3, 6, 14, 4, 13, 7, 8, 9, 11, 10, 12, 2, 17)                n=0.33
   |  V
(5,16, 3, 6, 14, 4, 13, 7, 8, 9, 11, 10, 12, 2, 17)                  n=0.00
```

Fig.5. Dendrogram P (1st case)

(5) (16) (3) (6) (8) (14) (7) (9) (2) (10) (11) (13) (4) (12) (17)	n=1.00
(5) (16) (3) (6) (8) (14) (7,9) (2) (10,11) (13) (4) (12,17)	n=0.76
(5) (16) (3) (6) (8) (14) (7,9) (2) (10,11) (13, 4,12,17)	n=0.71
(5) (16) (3) (6) (8) (14) (7,9) (2,10,11,13, 4,12,17)	n=0.64
(5) (16) (3) (6) (8) (14,7,9) (2,10,11,13, 4,12,17)	n=0.6
(5,16) (3) (6) (8,14,7,9, 2,10,11,13, 4,12,17)	n=0.59
(5,16) (3,6) (8,14,7,9, 2,10,11,13, 4,12,17)	n=0.36
(5,16) (3,6, 8,14,7,9, 2,10,11,13, 4,12,17)	n=0.32
(5,16,3,6, 8,14,7,9, 2,10,11,13, 4,12,17)	n=0.00

Fig.6. Dendrogram P (2nd case)

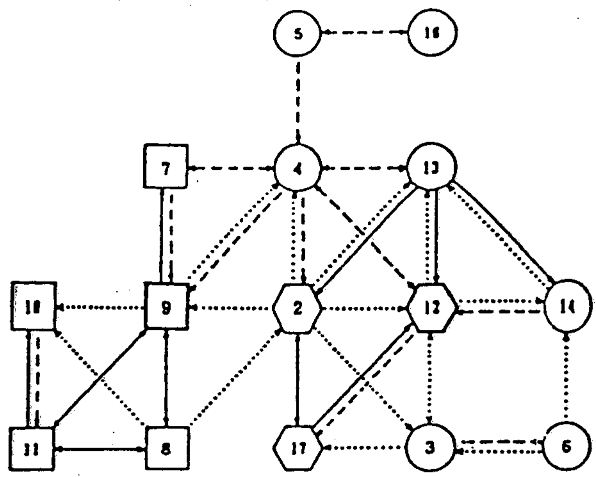

Fig.7. Sociogram Un, n=0.71 (1st case)

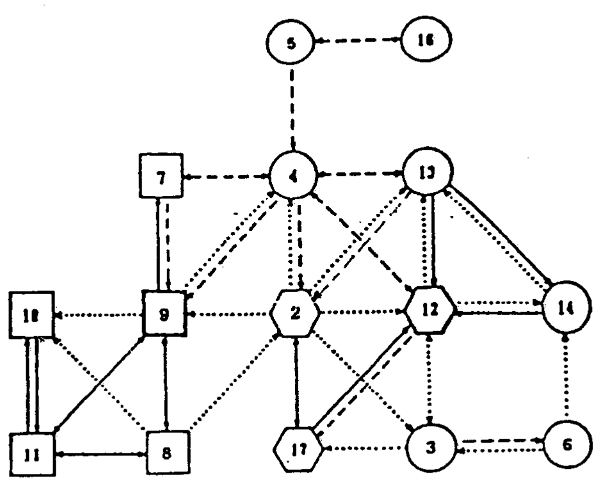

Fig.8. Sociogram Un, n=0.71 (second case)

Let's briefly analyze these two cases. It is obvious, that the process of clustering in the second case is different in its dynamic (there are ten levels for dendrogram of the second case and there are seven for the first dendrogram). The quantitative meaning of grades of preference inside the clusters will be corrected. This correction is a result of the influence of coefficients of sociological criterions on the structure of relationships. This enables us to analyze the structure of links inside the group while first criteria is dominat but the other criteria also taken into account.

In accordance with [2], selected students in the cluster: S_{07}, S_{08}, S_{09}, S_{10}, S_{11} - are the members of a tennis club and they are the friends. Lets analyze the links of their relationships. From the first sociogram we see, that S_{08}, S_{09} and S_{11} possess a strong mutual preference. Then, when we use measuring coefficients, which emphasize the factor of study, we find that the grades of mutual preferences among the students are increasing, except for student S_{08}. Apparently he is in this cluster only because of his athletic achievements. Thus, the usage of measuring coefficients enables us to make deeper analysis of the links among the members of the group.

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