Fuzzy Sociogram Analysis Applying Triangular Norms

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In this paper the new approach of measuring and evaluating of human relationship in a group of people is presented. This method is based on use of triangular norm and averaging operator. This approach allows to construct the fuzzy sociogram on data obtained from simple questionnaires. Constructed sociogram is a foundation for analysis of structure of human relationship among the group of people.

Keywords: fuzzy sociogram, triangular norm, averaging operator, dendrogram.

Introduction.

As it is known, sociometry analysis is one of the methods of measurement and evaluation of social structure in minor group by using sociogram [1, 2].

In this paper method of evaluation of social structure, based on the notion of "fuzzy sociogram" which was introduced by H.Yamashita and his colleagues, is developed [2, 3].

In the above mentioned articles, in the capacity of sociological criteria following question for students was used: "Write your friendly members in order". Here they used 12-grade scale. Mathematical methods of fuzzy logic by L.A. Zadeh applied for data processing. Particularly, for evaluation of degree of fuzziness we use Gij, $2/Gij = 1/Fij + 1/Fji$, where Fij is a degree of fuzziness of statement "student Sij prefers student Sj" and on the analogy of this, Fji is a degree of fuzziness of statement "student Sji prefers student Sij " in accordance with sociological criteria.

For Gij is to be obtained we offer to use [4, 6] triangular norm T(x,y) and averaging operator A(x,y), $x, y \in [0, 1]$. In the fuzzy logic T(x,y) models operation of connection and here A(x,y) is a normalizing factor.

Apart from that this method can be used in many other circumstances according to [5]: "in the frames of sociology of minor group some phenomenon and events which are typical for constant and provisional school collectives (like audience, student's group, pupils of additional classes) are traditionally studied. If we are talking about study of formation of out-of -office social relationships, where a logic, procedures, methods of cognition of mechanism of creation and functioning of minor group are apply, then we can define public social movements and informal groups. In different situations they join a lot of people. However, in the center or in the centers of these formations are relatively small groups, which make a face of the whole collective and make a strategy of its behavior.

Developed in the framework of this paper approach is directed to study of social relationships in a specific professional formations like a fire brigades, security rescue teams, who work in an emergency and specific conditions.

2. Analysis method of fuzzy sociogram.

According to [3], let L be a number of respondents, K=(Kij) is obtained data from 12-grade scale of response of respondents for asking sociological criteria. That table is constructed on the basis of questionnaires (table 1).

Table 1.

Personal card of questionnaires.

Rank	Respondent's number	Rank	Respondent's number
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	

From matrix K we have the evaluation matrix R=(Rij), where Rij=N - Kij + 1, Rij>0

$N = \Sigma \max Kij/L+0.5$. Analyzing matrix R we can define fuzzy friendly degrees among members in the group on the asked sociometrical criteria. As a result, we have a fuzzy matrix of preference F=(Fij), Fij= Rij/N, $0 \le Fij \le 1$, where Fij=1, if i=j.

Simultaneously, we have fuzzy graph F=(Fij) which is called fuzzy sociogram.

Nearness Fij to zero indicates a weak preference of Si to Sj; the closer Fij to 1, the stronger is the preference.

Then we define degrees of mutual preference Gij which is rather different from proposed way by H.Yamashita: Gij=T(Fij,Fji)/A(Fij,Fji). Thus, we consider Gij as a degree of fuzziness of statement "respondents Si and Sj have a mutual preference on the asked sociological criteria."

It is known, that $T(Fij,Fji) \ge 0$, $A(Fij,Fji) \ge 0$, $T(Fij,Fji) < A(Fij,Fji)$. From above mentioned, it is correctly to define Gij as following:

$$Gij = \begin{cases} 0, & \text{if } A(Fij,Fji)=0 \\ T(Fij,Fji)/A(Fij,Fji), & \text{else} \end{cases}$$

Obviously, that Gij changes from 0 to 1. If Gij is close to 1, that means presence of strong mutual preference between Si and Sj, if Gij is close to 0, that means a weak mutual preference.

Also, we obtain a fuzzy graph G=(Gij), analyzing which we may have dendrogram P. Dendrogram P describes dynamic of clustering in the group.

By summarizing fuzzy graph F and dendrogram P, we can obtain sociogram Un, where n is a level of amicability in a small group, $0<n<1$ **[2, 3].**

Concluding that section we give examples of triangular norms and averaging operators.

Examples of T-norms are:

I) $T(Fij,Fji) = Tp(Fij,Fji) = Fij*Fji$

II) $T(Fij,Fji) = Tw(Fij,Fji) = \begin{cases} Fij, & \text{if } Fji=1; \\ Fji, & \text{if } Fij=1; \\ 0, & \text{else} \end{cases}$

III) $T(Fij,Fji) = Tm(Fij,Fji) = \max(0, Fij+Fji-1)$ and others.

As an averaging operator A(Fij,Fji) we may use following:

I) $A(Fij,Fji) = 0.5(Fij+Fji)$
II) $A(Fij,Fji) = \sqrt{Fij*Fji}$
III) $A(Fij,Fji) = 0.5(1/Fij+1/Fji)$ and others.

3. Case study.

For demonstration of calculation, results of questioning of 16 students are being used with the aim of study structure of relationship in the group **[3]**.

Matrix K is represented in Fig.1. In accordance to the above mentioned formulas, it is easy to obtain evaluation matrix R (Fig.2.) from matrix K. Analyzing matrix R, we have fuzzy matrix F (Fig.3) which is called the fuzzy sociogram F. Then we construct two dendrograms using two different T-norms: $T(x,y) = Tp(x,y)$ and $T(x,y)= Tw(x,y)$. Averaging operator is $A(x,y) = 0.5(x+y)$. Subsequently, we obtain two fuzzy matrixes G (Fig.4, Fig.5) and two dendrograms P (Fig.6, Fig.7). Then using of technology of data processing, as it was described in part two of this paper, we get two sociograms Un, n=0.78. Correspondingly for two cases (Fig.8, Fig.9).

Let us shortly analyze these two cases. In the first one (it was considered in papers **[2,3]**) Gij=0, if at least one of the numbers either Fij or Fji equals to zero. In the second case, demands are more hard, that is why Gij will be different from zero only if one of the numbers either Fij or Fji compulsory equal to 1, and the other one is not equal to zero. That means, that in considered couple of subjects, at least one of them is a leader, and the other one is not indifferent to him in accordance to the sociological criteria. Appropriateness of other cases is provided by possibility of choice of T(x,y) and A(x,y).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	*		2		3	9	4		11		1				10	
2	4	*	1		2			6			3		7			
3		8	*		1	5	2				4					
4				*					4			3		1		
5					*					2	4					
6			3		2	*				1						
7	3	7	1	12	5		*				2		6			
8					1			*				2				3
9					3				*		1	2				
10					1	3				*	4	5				
11	1	2	11		4	8			5	10	*	6	7			
12					2	5			1	3	4	*				
13			8							3	4	1	*			
14				1	3					4	2	8		*		
15	6		1		3		12			2	5	8	9		*	
16							2	1	6	3	4					*

Fig.1. Response matrix K.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	*		6		5	0	4		0		7				0	
2	4	*	7		6			2			5		1			
3		0	*		7	3	6				4					
4				*					4			5		7		
5					*					6	4					
6			5		6	*				7						
7	5	1	7	0	3		*				6		2			
8					7			*				6				5
9					5				*		7	6				
10					7	5				*	4	3				
11	7	6	0		4	0			3	0	*	2	1			
12					6	3			7	5	4	*				
13			0							5	4	7	*			
14				7	5					4	6	0		*		
15	2		7		5		0			6	3	0	0		*	
16							6	7	2	5	4					*

Fig.2. Evaluating matrix R

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1		0,86		0,71	0	0,57		0		1				0	
2	0,57	1	1		0,86			0,29			0,71		0,14			
3		0	1		1	0,43	0,86				0,57					
4				1					0,57			0,71		1		
5					1					0,86	0,57					
6			0,71		0,86	1				1						
7	0,71	0,14	1	0	0,43		1				0,86		0,29			
8					1			1				0,86				0,71
9					0,71				1		1	0,86				
10					1	0,71				1	0,57	0,43				
11	1	0,86	0		0,57	0			0,43	0	1	0,29	0,14			
12					0,86	0,43			1	0,71	0,57	1				
13			0							0,71	0,57	1	1			
14				1	0,71					0,57	0,86	0		1		
15	0,29		1		0,71		0			0,66	0,43	0	0		1	
16							0,86	1	0,29	0,71	0,57					1

Fig.3. Fuzzy matrix F

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0		0	0	0,63		0		1				0	
2	0	1	0		0		0	0			0,78		0			
3		0	1		0	0,54	0,92				0		0		0	
4				1			0		0			0		1		
5		0	0		1	0	0	0	0	0,92	0,57	0		0		
6			0,54		0	1				0,84	0	0			0	
7	0,63	0	0,92	0	0		1				0		0			0,78
8		0			0			1				0				0
9				0	0				1		0,6	0,92				0
10					0,92	0,84				1	0	0,54	0	0	0	0
11	1	0,78	0		0,57	0	0		0,6	0	1	0,38	0,09	0	0	0
12				0	0	0		0	0,92	0,54	0,38	1	0	0	0	
13		0	0				0			0	0,09	0	1		0	
14				1	0					0	0	0		1		
15	0		0		0		0			0	0	0	0		1	
16							0,78	0	0	0	0					1

Fig.4. Fuzzy matrix G (1st case)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0		0	0	0		0		1				0	
2	0	1	0		0		0	0			0		0			
3	0	0	1		0	0	0,86				0					
4				1					0			0		1		
5	0	0	0		1	0	0	0	0	0,86	0	0		0		
6			0		0	1				0,71		0				
7	0	0	0,86	0	0		1				0		0			0
8		0			0			1				0				0,71
9				0	0				1		0,43	0,86				0
10					0,86	0,71				1	0	0	0	0	0	0
11	1	0	0		0	0	0		0,43	0	1	0	0	0	0	0
12				0	0	0		0	0,86	0	0	1	0	0	0	
13		0	0				0			0	0	0	1		0	
14				1	0					0	0	0		1		
15	0		1		0,71		0			0	0	0	0		1	
16							0	0,71	0	0	0					1

Fig.5. Fuzzy matrix G (2nd case)

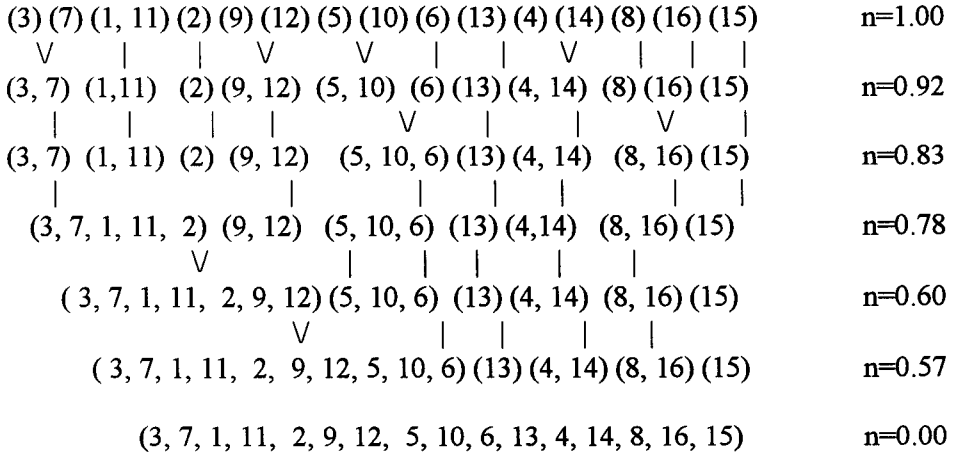

Fig.6. Dendrogram P (1st case)

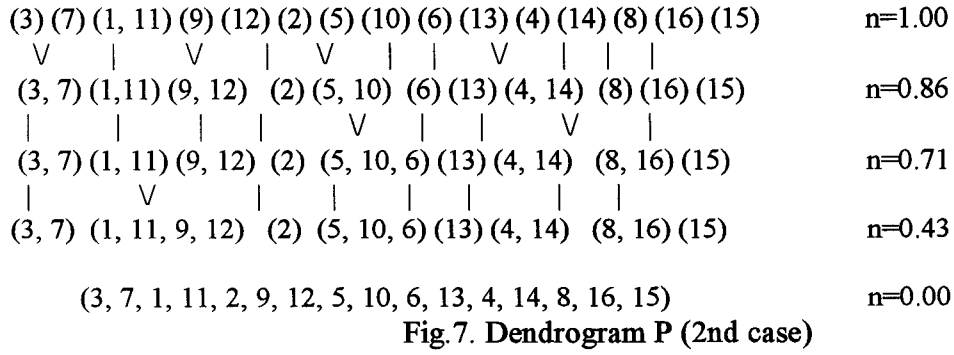

Fig.7. Dendrogram P (2nd case)

For identification of different clusters we use triangles, squares, pentagons, hexagons and circles in Fig.8, 9.

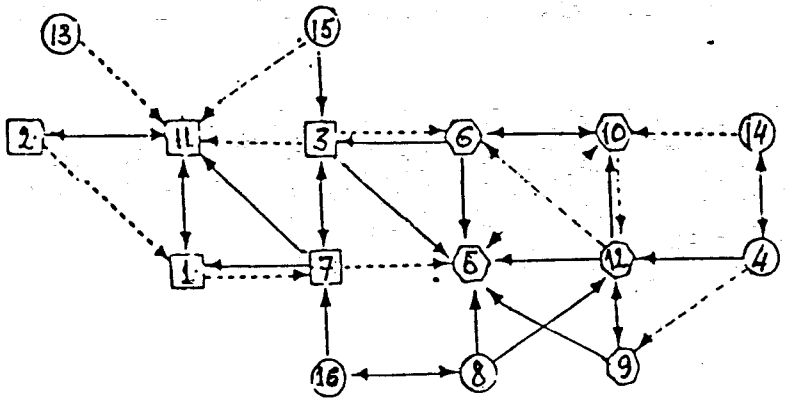

Fig.8. Sociogram U (1st case)

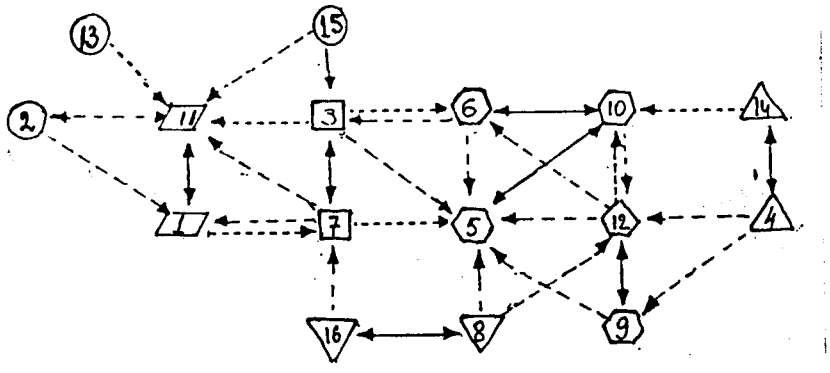

Fig.9. Sociogram U (2nd case)

Comparing the obtained sociograms, we see that in the second case, some groups with high degree of mutual preferences in accordance to sociological criteria are appeared. Thus, using the second way of sociological analysis is useful for special kind of occupations. For example, for fire brigades and rescue teams and others who work in emergency.

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