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The Remark on the image of an L – fuzzy group

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Abstract

In this short communication we show that if any subset A of a lattice L containing its lowst upper bound can be expressed a direct product of some subchains then A must be an image of an L – fuzzy group, this result answers partly the open problem gave by S. Ovchinnikov.

Key words L – fuzzy subgroup, image, subchain

1. Introduction

The notion of an image of a fuzzy group plays an important role in the theory of fuzzy group. Let L be a lattice and G be a group. $\mu: G \rightarrow L$ is an L – fuzzy subgroup of G: if $\mu(xy^{-1}) \geqslant \mu(x) \wedge \mu(y)$ for all $x, y \in G$, We call $Im\mu = \mu(G)$ the image of μ.

If any sebset A of L satisfying $\sup A \in A$ is an image of an L – fuzzy subgroup, then we say that L satisfies the image property.

In[1], S. Ovchinnikov proved that

Theorem1 L satisfies the image property if and only if L is a chain.

and gived an example which shows that the image of an L – fuzzy group is not nec-

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essarily a sublattice of L. At the same time. An open problem was given: Finding other sufficient conditions that any subset A of L satisfying $\sup A \in A$ is an image of an L-fuzzy subgroup.

2. Main result

Theorem2 Let L be a lattice. $A \subseteq L$ and $\sup A \in A$, If A can be expressed a direct product of some subchains of L, then A must be an image of an L-fuzzy subgroup.

Proof: Suppose that $A \subseteq L$ and $\sup A \in A$. If $A = \prod_{i \in I} C^i$ (C^i is a subchain of L, I is a index set), by theoreml, for every subchain C^i, exists a group G_i and an L-fuzzy subgroup $\mu^i : G^i \to C^i$ of G^i, such that $\mathrm{Im}(\mu^i) = C^i (i \in I)$

Let $G = \prod_{i \in I} G^i = \{g = \{g^i\} \mid g^i \in G^i, i \in I\}$, $\forall g = \{g^i\}_{i \in I}, h = \{h^i\}_{i \in I} \in G$, defining $g \circ h = \{g^i h^i\}_{i \in I}$ $g^i, h^i \in G^i, i \in I$, then $e = \{e^i\}_{i \in I}$ is the unit element of G, where e^i is the unit element of G^i. $G^{-1} = \{(g^i)^{-1}\}_{i \in I}$ is the inverse of g. Thus , $(g, \circ)$ is a group.

According to the order relation in L we can introduce pointwisely the order relation in A: For any $\{\lambda^i\}_{i \in I}, \{\mu^i\}_{i \in I} \in A$,

$$\{\lambda^i\}_{i \in I} = \{\mu^i\}_{i \in I} \Leftrightarrow \lambda^i = \mu^i,$$

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$$\{\lambda^i\} \vee \{\mu^i\} = \{\lambda^i \vee \mu^i\}$$

$$\{\lambda^i\} \wedge \{\mu^i\} = \{\lambda^i \wedge \mu^i\}$$

for all $i \in I$.

Defining $\mu: G \to A$, $\mu(\{g^i\}_{i \in I}) = \{\mu^i(g^i)\}_{i \in I}$, then for any $g = \{g^i\}_{i \in I}$, $h = \{h^i\}_{i \in I} \in G$, we have

$\mu(g \circ h) = \mu(\{g^i\} \circ \{h^i\}) = \mu(\{g^i h^i\}) = \{\mu^i(g^i h^i)\} \geqslant \{\mu^i(g^i) \wedge \mu(h^i)\} = \{\mu^i(g^i)\} \wedge \{\mu(h^i)\} = \mu(g) \wedge \mu(h)$, Similarly, $\mu(g^{-1} \geqslant \mu(g)$, thus μ is L-fuzzy subgroup of G.

It is clear that for any $\lambda = \{\lambda\}_{i \in I} \in A = \prod_{i \in I} C^i$, exists a $g^i \in G^i$, such that $\mu^i(g^i) = \lambda^i$, it implies $\mu(\{g^i\}) = \{\mu^i(g^i)\} = \{\lambda^i\} = \lambda$, so $Im(\mu) = A$ and the proof is completed.

References

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