On The Complex Fuzzy Derivative

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As we known, the complex derivative theory is a very important subject in mathematics. In this note, we begin to study the complex fuzzy derivative and give a few basic properties.

Definition 1. Let E be a vector space over the field K of real or complex numbers, (E, T) be a fuzzy topological space, if the two mappings

(i) $\sigma : E\times E\to E, (x, y)\to x+y$

(ii) $\pi : K\times E\to E,(\alpha , x)\to \alpha x$

where K is the induced fuzzy topology of the usual nom, are fuzzy continuous. Then (E, T) is said to be a fuzzy topological vector space over the field K.

Definition 2. Let C be the field of complex numbers and (C, T) be a fuzzy topological vector space over the field C. f: C$\to$C, $z_0\in C$, the function f is said to be fuzzy differentiable at the point z_0 if there is a function ϕ that is fuzzy continuous at the point z_0, and have

$$f(z)-f(z_0)= \phi(z)(z-z_0)$$

for all $z \in C$. $\phi(z_0)$ is said to be the fuzzy derivative of f at z_0 and denote $f'(z_0)=\phi(z_0)$.

Definition 3. Let C be the field of complex numbers and (C, T) be a fuzzy topological vector space over the field C. $f: C \to C$, $z_0 \in C$, if f is fuzzy differentiable at each point z and $f'(z)$ is also fuzzy differentiable at the point z_0, then f is said to be 2 –order fuzzy differentiable, and the 2 –order fuzzy derivative is denoted by $f''(z_0)$. Similar, we can define n –order fuzzy derivative ($n \geqslant 1$).

Our main results are as follows:

Theorem 1. If f is fuzzy differentiable at the point z_0 and g is fuzzy differentiable at the point $f(z_0)$, then h=gof is also fuzzy differentiable at the point z_0 and $h'(z_0)=g'(f(z_0))f'(z_0)$.

Theorem 2. If f is fuzzy differentiable at the point z_0 and g is also fuzzy differentiable at z_0, then

$$(f+g)'(z_0) = f'(z_0)+g'(z_0),$$
$$(fg)'(z_0) = f'(z_0)\, g(z_0)+f(z_0)g'(z_0)$$

If $g(z_0) \neq 0$, we have

$$(\frac{f}{g})'(z_0)= \frac{f'(z_0)g(z_0)-g'(z_0)f(z_0)}{g^2(z_0)}$$

Theorem 3. If f: $C\to C$, $f(z)=u(x, y)+iv(x, y)$, then f is fuzzy differentiable at $z_0=a+bi$ if and only if $u(x, y)$ and $v(x, y)$ is fuzzy differentiable at (a, b) and

$$\left.\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}\right|_{(a,b)}, \left.\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}\right|_{(a,b)}$$

Theorem 4. If f: $C\to C$ is fuzzy differentiable at z_0, then for each $n\in$ N, f is also n-order fuzzy differentiable.

Theorem 5. If f: $C\to C$ is fuzzy differentiable on C and for each $z\in C$, $f'(z)=0$, then $f(z)=z_0$.

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