

On fuzzy weakly semi-precontinuous multifunctions

Ma Baoguo

(Department of Mathematics and computer science ,Yanan University, Yanan 716000 P.R.China)

Abstract: This paper is devoted to the introduction and study of fuzzy lower and upper weakly semi-precontinuous multifunctions between a general topological space (X, T) and a fuzzy topological space (Y, T_1) . Using the mapping F_* and F^* several equivalent conditions of a lower and upper fuzzy weakly semi-precontinuous multifunction.

Key words: Fuzzy semi-preopen (semi-preclosed) sets; Fuzzy lower and upper semi-precontinuous multifunctions; Fuzzy lower and upper weakly semi-precontinuous multifunctions.

1. Preliminaries

Throughout the paper, by (X, T) or simply by X we will mean a topological space in classical sense, and (Y, T_1) or simply by Y will stand for a fuzzy topological space (fts, for short) as defined by Chang's [1]. Fuzzy sets in Y will be denoted A, B, U, V etc. and interior (semi-interior) and closure (semi-closure) and complement of a fuzzy set A in an fts Y will be denoted by $Int A$ ($SInt A$) $Cl A$ ($SCL A$) and $1 - A$ respectively. A fuzzy set A of fts X is called fuzzy preopen set (fuzzy preclosed set) if $A \leq Int Cl A$ ($A \geq Int Cl A$); A fuzzy set A of fts X is called semi-preopen set (fuzzy semi-preclosed set) if there a fuzzy preopen set U such that $U \leq A \leq Cl U$ ($Int U \leq A \leq U$). Let A be any fuzzy set a fts X , then fuzzy semi-preclosure ($SPcl A$, for short) and fuzzy semi-preinterior ($SPint A$, for short) of A are defined as follows: $SPcl A = \bigcap \{ B \mid B \text{ is fuzzy semi-preinterior and } A \leq B \}$ $SPint A = \bigcup \{ B \mid B \text{ is fuzzy semi-preopen and } B \leq A \}$. when a fuzzy set A is quasi-coincident (q-coincident, for short) with a fuzzy set B in (Y, T_1) , we shall write $A q B$, if A and B are not q-coincident, denoted by

* This work was partially supported by the special fundation of Shaanxi Educational committee.

$A \bar{q} B$. The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd', and 'fts' respectively.

Definition 1.1. Let (X, T) be a topological space in the classical sense and (Y, T_1) be a fuzzy topological space. $F: X \rightarrow Y$ is called a fuzzy multifunction iff for each $x \in X$, $F(x)$ is a fuzzy set in Y .

Definition 1.2. For a fuzzy multifunction $F: X \rightarrow Y$, the $F_*(A)$ and $F^*(A)$ of a fuzzy set A in Y are defined as follows:

$$F_*(A) = \{x \in X : F(x) q A\},$$

$$F^*(A) = \{x \in X : F(x) \leq A\}$$

Theorem 1.3.[4] For a fuzzy multifunction $F: X \rightarrow Y$ we have $F_*(1-A) = X - F^*(A)$, for any fuzzy set A in Y .

Definition 1.4. For a fuzzy multifunction $F: X \rightarrow Y$ is called :

(a) fuzzy lower semi-precontinuous (f.l.s.p.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F_*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U \subset F_*(V)$,

(b) fuzzy upper semi-precontinuous (f.w.s.p.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F^*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U \subset F^*(V)$,

(c) f.l.s.p.c. (f.u.s.p.c.) on X iff it is respectively so at each $x_0 \in X$,

(d) f.s.p.c. on X iff it is f.l.s.p.c. and f.u.s.p.c. .

Definition 1.5. For a fuzzy multifunction $F: X \rightarrow Y$ is called :

(a) fuzzy lower weakly semi-continuous (f.l.w.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F_*(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F_*(SCIV)$,

(b) fuzzy upper weakly semi-continuous (f.u.w.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F^*(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F^*(SCIV)$,

(c) f.l.w.s.c. (f.u.w.s.c.) on X iff it is respectively so at each $x_0 \in X$,

(d) f.w.s.c. on X iff it is f.l.w.s.c. and f.u.w.s.c. .

2. Lower and upper weakly semi-precontinuous fuzzy multifunctions

Definition 2.1. For a fuzzy multifunction $F: X \rightarrow Y$ is called :

(a) fuzzy lower weakly semi-precontinuous (f.l.w.s.p.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F_*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U \subset F_*(SCIV)$,

(b) fuzzy upper weakly semi-precontinuous (f.u.w.s.p.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F^*(V)$, there exists a semi-preopen nbd U of x_0 in X such that $U \subset F^*(SCIV)$,

(c) f.l.w.s.p.c. (f.u.w.s.p.c.) on X iff it is respectively so at each $x_0 \in X$,

(d) f.w.s.p.c. on X iff it is f.l.w.s.p.c. and f.u.w.s.p.c. .

Remark 2.2. It is clear from Definition 2.1 and 1.4,1.5 that every fuzzy semi-continuous (weakly semi-continuous, almost semi-continuous [7], weakly semi-continuous [8], semi-precontinuous [9]) multifunctions is f.w.s.p.c. where as the converse is not true.

Theorem 2.3. Let $F: X \rightarrow Y$ is f.l.w.s.p.c.(f.u.w.s.p.c.) iff for every fuzzy open set V in Y , $F_*(V) \subset SP \text{ int } F_*(SCIV)$ (respectively, $F^*(V) \subset SP \text{ int } F^*(SCIV)$).

Proof. we prove the theorem for the case of f.l.w.s.c. of F only; there other case is quite similar.

Let F be f.l.w.s.p.c. and V be f.open in Y . If $x \in F_*(V)$, then there exists semi-preopen nbd U of x in X such that $U \subset F_*(SCIV)$ and hence $U \subset SP \text{ int } F_*(SCIV)$.

Conversely , let $x \in X$ and V be fuzzy open set in Y such that $x \in F_*(V)$. Then $x \in F_*(V) \subset SP \text{ int } F_*(SCIV) = U$ (say). Thus U is a semi-preopen nbd of x such that $U \subset F_*(SCIV)$ and consequently, F is f.l.w.s.p.c. .

Theorem 2.4. If $F: X \rightarrow Y$ is f.l.w.s.p.c., for every fuzzy pre-semiopen set V in Y , $F_*(V) \subset SP \text{ int } F_*(SCIV)$.

Proof. Let F be f.l.w.s.p.c. and V be any fuzzy pre-semiopen set in Y . For any $x \in F_*(V)$, then there exists semi-preopen nbd U_x of x in X such

that $U_x \subset F_*(SCInt SCI V) \subset F_*(SCI V)$ and hence $F_*(V) \subset \cup \{U_x : x \in F_*(V)\} \subset F_*(SCI V)$, and hence $F^*(V) \subset SPint F^*(SCI V)$.

Theorem 2.5. If $F: X \rightarrow Y$ is f.u.w.s.p.c., for every fuzzy pre-semiopen set V in Y , $F^*(V) \subset SPint F^*(SCI V)$.

Proof. Similar to Theorem 2.4. and omitted.

Theorem 2.6. If a fuzzy multifunction $F: X \rightarrow Y$ is f.l.w.s.p.c. on X , then $SPcl F_*(V) \subset F_*(SCI V)$ for any fuzzy open set V in Y .

Proof. Let $x \notin F_*(SCI V)$, where V is a fuzzy open set in Y . Then $F(x) \not\subseteq SPcl V$ so $F(x) \leq 1 - SPcl V$. By fuzzy lower weakly semi-precontinuity of F , there exists a semi-preopen nbd U of x in X such that $F(z) \leq SPcl(1 - SCI V) \leq 1 - V$ for all $z \in U$. Thus $F(z) \not\subseteq V$, i.e. $z \notin F_*(V)$, for all $z \in U$, hence $U \cap F_*(V) = \emptyset$. Since U is a semi-preopen nbd of x , it then follows that $x \notin F_*(SCI V)$.

Theorem 2.7. If a fuzzy multifunction $F: X \rightarrow Y$ is f.u.w.s.p.c. on X , then $SPcl F^*(V) \subset F^*(SCI V)$ for any fuzzy open set V in Y .

Proof. Similar to Theorem 2.6. and omitted.

Theorem 2.8. Let $F: X \rightarrow Y$ is fuzzy multifunctions then following are equivalent:

- (a) F is f.l.w.s.p.c.,
- (b) for any fuzzy open set V in Y , $F_*(V) \subset SPint F_*(SCI V)$ and $F^*(V) \subset F^*(SCI V)$.

Proof. Evident.

Theorem 2.9. Let $F: X \rightarrow Y$ is fuzzy lower weakly semi-precontinuity if for each fuzzy open set V , $F_*(CI V)$ is fuzzy semi-preopen set.

Proof. Straightforward.

References

- [1] C.L.Chang, Fuzzy topological spaces, *J.Math.Anal.Appl* 24(1968) 182-190.
- [2] N.S.Papageorgiou, Fuzzy topology and fuzzy multifunction, *J.Math.Anal. Appl.* 109(1985) 397-425.
- [3] M.N.Mukherjee and S.Malaker, On almost continuous and weakly continuous fuzzy multifunctions, *Fuzzy set and systems* , 41(1991) 113-125.
- [4] B.G. Ma, On fuzzy semi-continuous and irresolute multifunction, © ICIK95 August 21-25, 1995, Dalian, P.R.China.
- [5] S.Dang, A.Behera. S.Nanda, On fuzzy weakly semi-continuous functions, *Fuzzy set and systems* , 67(1994) 239-245.
- [6] L.A.Zadeh, Fuzzy sets, *Inform and Control* 8(1965) 338-353.
- [7] B.G. Ma, Lower and upper almost semi-continuous fuzzy multifunction, *Fuzzy Sets Theory and Applications* (Hebei University Press 1998.8, Baoding P.R.China) 369-372 .
- [8] B.G. Ma, Lower and upper weakly semi-continuous fuzzy multifunction, *BUSEFAL(in France)* , Vol .78(1999) 8-11.
- [9] B.G. Ma, Fuzzy semi-preopen sets and fuzzy semi-precontinuous multifunctions, *Journal of yan'an university (in chinese)*, 2(1999) .