Fuzzy Implication Ideals in N(2,0) Algebras

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Abstract: In this paper we gave fuzzy implication ideals of N(2,0) algebras and studied basic properties of its

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1. Preliminaries

Definition 2.1 An algebraic system(S, $*$, 0) of type (2,0) is called N(2,0) algebras if it satisfies the following conditions for any $x,y,z\in S$

(N_1) $x*(y*z)=z*(x*y)$; (N_2) $0*x=x$.

Remark Let (S, $*$, 0) be a N(2,0) algebra, then the following identities hold for any $x,y,z\in S$

(1) $x*y=y*x$; (2) $(x*y)*z=x*(y*z)$;

(3) $x*(y*z)=y*(x*z)$, $(x*y)*z=(x*z)*y$;

(4) 0 is unit element.

Then N(2,0)algebra is a monoid semigroups.

Definition 2.2 Let S be a N(2,0) algebra, a nonempty subset I of S is called a ideal of S if it satisfies the following conditions

(I_1) $0\in \mathbf{I}$; (I_2) $x\in \mathbf{I}, x*y\in \mathbf{I}\rightarrow y\in \mathbf{I}$ for any $x,y\in S$

Definition 2.3 Let S be a N(2,0) algebra, a nonempty subset I of S is called a implication ideal of S if it satisfies the following conditions

(I_1) $0 \in \mathbf{I}$; (IM) $z * ((x * y) * x) \in \mathbf{I}, z \in \mathbf{I} \rightarrow x \in \mathbf{I}$, for any $x, y, z \in S$

Definition 2.4 Let S be a set, A fuzzy set in S is a map

$$\mu: \rightarrow [0,1]$$

Definition 2.5 Let μ be a fuzzy set in **X**. For any $t \in [0,1]$, the set $\mu_t = \{ x \in \mathbf{X}: \mu(x) \geqslant t\}$ is called a level subset of μ.

Definition 2.6 Let S be a N(2,0) algebra, a fuzzy set μ in S is called a fuzzy ideal of S if it satisfies the following conditions

(F_1) $\mu(0) \geqslant \mu(x), x \in S$,

(F_2) $\mu(x) \geqslant \min\{\mu(y * x), \mu(y)\}$, for any $x, y \in S$

Definition 2.7 Let S be a N(2,0) algebra, a fuzzy set μ in S is called a fuzzy N-ideal of S if it satisfies the following conditions for all $x, y \in S$

$$\mu(x * (x * y)) \geqslant \mu(x), x \neq 0, \ \mu(0) \geqslant \mu(x).$$

2. Fuzzy Implication Ideals in N(2,0) Algebras

Definition 3.1 Let S be a N(2,0) algebra, a fuzzy subset μ in S is called fuzzy implication ideal of S if it satisfies the following conditions

(F_1) $\mu(0) \geqslant \mu(x), x \in \mathbf{S}$,

(F-IM) $\mu(x) \geqslant \min\{\mu(z * ((x * y) * x)), \mu(z)\}$, for all $x, y, z \in \mathbf{S}$.

Theorem 3.1 Let S be a N(2,0) algebra, for any $x \in S$, if $x * x = 0$, then any fuzzy implication ideal in S is fuzzy ideal of S.

Proof Let $z = y$ and $y = x$ in the Definition 3.1, by (F-IM) we have

$$\mu(x) \geqslant \min\{\mu(y * ((x * x) * x)), \mu(y)\},$$

on the other hand by $x*x=0, 0*x=x$, we have

$$\mu(x) \geqslant \min\{\mu(y*x), \mu(y)\},$$

Therefore(F_2) hold, then μ is fuzzy ideal of S .

Theorem 3.2 Let S be a N(2,0) algebra, if μ is a fuzzy N-ideal of S, then μ is also a fuzzy implication ideal of S.

Proof Because μ is a fuzzy N-ideal of S, then

$$\mu(z*((x*y)*x)) = \mu(x*(x*(z*y))) \geqslant \mu(x),$$

hence $$\mu(x) \geqslant \min\{\mu(z*((x*y)*x)), \mu(z)\}.$$

Therefore μ is also a fuzzy implication ideal of S.

Theorem 3.3 Let S be a N(2,0) algebra, then a fuzzy subset μ of S is a fuzzy implication ideal of S if and only if for any $t \in [0,1]$, $\mu(t)$ is a implication ideal of S or $\mu(t)$ is empty.

Proof Assume μ is a fuzzy implication ideal of S,

by (F_1) we have $\mu(0) \geqslant \mu(x)$, for any $x \in S$.

Therefore $\mu(0) \geqslant \mu(x) \geqslant t, x \in \mu_t$,

thus $0 \in \mu_t$. let $z*((x*y)*x) \in \mu_t$ and $z \in \mu_t$. Then

$$\mu(z*((x*y)*x)) \geqslant t \text{ and } \mu(z) \geqslant t.$$

Since μ is implication ideal of S, hence

$$\mu(x) \geqslant \min\{\mu(z*((x*y)*x)), \mu(z)\} \geqslant t,$$

then $x \in \mu_t$, therefore μ_t is a implication ideal of S.

Conversely only proof (F_1) and (F-IM) is true.

If (F_1)is not true, then there exists $x' \in S$ such that $\mu(0) < \mu(x')$.

If take $t' = (\mu(0) + \mu(x'))/2$, then

$$\mu(0) < t' \text{ and } 0 \leqslant t' < \mu(x') \leqslant 1. \quad (1)$$

Where $x' \in \mu_{t'}$ and $\mu_{t'} \neq \varnothing$. Since μ_t is a implication ideal of S.

We have $0 \in \mu_{t'}$ and $\mu(0) \geqslant t'$, (2)

Thus, the inequality (2) and (1) are contradiction . Now assume (F-IM) is not true, then there exists $x', y', z' \in S$, such that

$\mu(x') < \min\{\mu(z' * ((x' * y') * x')), \mu(z')\}$. let

$t' = (\mu(x') + \min\{\mu(z' * ((x' * y') * x')), \mu(z')\})/2$, then

$\mu(x') < t'$ and $0 \leqslant t' < \min\{\mu(z' * ((x' * y') * x')), \mu(z')\} \leqslant 1$. (3)

Therefore $\mu(z' * ((x' * y') * x')) > t'$ and $\mu(z') > t'$ then

$z' * ((x' * y') * x') \in \mu_{t'}$, $z' \in \mu_{t'}$, since $\mu_{t'}$ is a implication ideal of S, thus

$x' \in \mu_{t'}$ and $\mu(x') \geqslant t'$. (4)

Thus, the inequality (4) and (3) are contradiction . Then (F_1) and (F-IM) all hold, hence μ is a fuzzy implication ideal of S.

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