An Introduction to Intuitionistic Topological Spaces

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Abstract: The purpose of this paper is to construct the basic concepts of the so-called "intuitionistic topological spaces". After giving the fundamental definitions and the necessary examples we introduce the definitions of continuity and compactness, and obtain several preservation properties and some characterizations concerning compactness.

Keywords: Intuitionistic set; intuitionistic topology; intuitionistic topological space; continuity; compactness. **AMS Subject Classification: 54A99.**

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [11] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Krassimir T. Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2,3,4]. Later topological structures in fuzzy topological spaces [5] is generalized to "intuitionistic fuzzy topological spaces" by Çoker in [7], and then the concept "intuitionistic set" is introduced in Çoker [8]. This concept is the discrete form of intuitionistic fuzzy set, and it is one of several ways of introducing *vagueness* in mathematical objects. In this paper we shall give a brief introduction to "intuitionistic topological spaces". Several relevant papers have already appeared in literature, including [6,9,10]. Notice that, in [9,10], intuitionistic sets are renamed as "intuitionistic fuzzy special sets".

2. Preliminaries

Here we shall present the fundamental definitions. The following one is obviously inspired by K. T. Atanassov [3] and is first given in [8]. For the sake of completeness we shall outline the basic facts:

Definition 2.1. [8] Let X be a nonempty fixed set. An **intuitionistic set** (IS for short) A is an object having the form $A =< X, A^1, A^2 >$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

Every crisp set A on a nonempty set X is obviously an IS having the form $< X, A, A^c >$, and one can define several relations and operations between IS's as follows:

Definition 2.2. [8] Let X be a nonempty set, $A =< X, A_1, A_2 >$ and $B =< X, B^1, B^2 >$ be IS's on X, and let $\{A_i : i \in J\}$ be an arbitrary family of IS's in X, where $A^i =< X, A_i^1, A_i^2 >$.

(a) $A \subseteq B$ iff $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$; (b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) $A \sqsubset B$ iff $A^1 \cup A^2 \supseteq B^1 \cup B^2$; (d) $\bar{A} =< X, A^2, A^1 >$;
(e) $\cup A_i =< X, \cup A_i^1, \cap A_i^2 >$; (f) $\cap A_i =< X, \cap A_i^1, \cup A_i^2 >$;
(g) $A - B = A \cap \overline{B}$; (h) $[]A =< X, A^1, (A^1)^c >$;
(i) $<> A =< X, (A^2)^c, A^2 >$; (j) $\emptyset_\sim =< X, \emptyset, X >$ and $X_\sim =< X, X, \emptyset >$.

Here are the basic properties of inclusion and complementation:

Corollary 2.3. [8] Let A, B, C and A_i be IS's in X $(i \in J)$. Then

(a) $A_i \subseteq B$ for each $i \in J \Rightarrow \cup A_i \subseteq B$ (b) $B \subseteq A_i$ for each $i \in J \Rightarrow B \subseteq \cap A_i$
(c) $\overline{\cup A_i} = \cap \overline{A_i}$, $\overline{\cap A_i} = \cup \overline{A_i}$ (d) $A \subseteq B \Leftrightarrow \overline{B} \subseteq \overline{A}$
(e) $\overline{(\overline{A})} = A$ (f) $\overline{\emptyset_\sim} = X_\sim$, $\overline{X_\sim} = \emptyset_\sim$

Now we shall define the image and preimage of IS's. Let X, Y be two nonempty sets and $f : X \to Y$ a function.

Definition 2.4. [8] (a) If $B =< Y, B^1, B^2 >$ is an IS in Y, then the **preimage** of B under f, denoted by $f^{-1}(B)$, is the IS in X defined by $f^{-1}(B) =< X, f^{-1}(B^1), f^{-1}(B^2) >$.

(b) If $A =< X, A^1, A^2 >$ is an IS in X, then the **image** of A under f, denoted by $f(A)$, is the IS in Y defined by $f(A) =< Y, f(A^1), f_-(A^2) >$, where $f_-(A^2) = (f((A^2)^c))^c$.

Here we list the properties of images and preimages, some of which we shall frequently use in the following sections:

Corollary 2.5. [8] Let A, A_i $(i \in J)$ be IS's in X, B, B_j $(j \in K)$ IS's in Y, and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$, $\quad B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
(b) $A \subseteq f^{-1}(f(A))$, and if f is injective, then $A = f^{-1}(f(A))$.
(c) $f(f^{-1}(B)) \subseteq B$, and if f is surjective, then $f(f^{-1}(B)) = B$.
(d) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$
(e) $f(\cup A_i) = \cup f(A_i)$; $f(\cap A_i) \subseteq \cap f(A_i)$, and if f is injective, then $f(\cap A_i) = \cap f(A_i)$.
(f) $f^{-1}(Y_\sim) = X_\sim$, $f^{-1}(\emptyset_\sim) = \emptyset_\sim$
(g) $f(\emptyset_\sim) = \emptyset_\sim$, $f(X_\sim) = Y_\sim$, if f is surjective.
(h) If f is surjective, then $\overline{f(A)} \subseteq f(\overline{A})$. If, furthermore, f is injective, then have $\overline{f(A)} = f(\overline{A})$.
(i) $(f^{-1}(\overline{B})) = \overline{f^{-1}(B)}$.

3. Intuitionistic topological spaces

Now we generalize the concept of "intuitionistic fuzzy topological space" to intuitionistic sets:

Definition 3.1. (cf. [7]) An **intuitionistic topology** (IT for short) on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

(T_1) $\emptyset_\sim, X_\sim \in \tau$,
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an **intuitionistic topological space** (ITS for short) and any IS in τ is known as an **intuitionistic open set** (IOS for short) in X.

Example 3.2. Any topological space (X, τ_o) is obviously an ITS in the form $\tau = \{A' : A \in \tau_o\}$, whenever we identify a subset A in X with its counterpart $A' =< X, A, A^c >$ as before.

Example 3.3. Let $X = \{a, b, c, d, e\}$ and consider the family $\tau = \{\emptyset_\sim, X_\sim, A_1, A_2, A_3, A_4\}$, where $A_1 =< X, \{a, b, c\}, \{d\} >$, $A_2 =< X, \{c, d\}, \{e\} >$, $A_3 =< X, \{c\}, \{d, e\} >$, $A_4 =< X, \{a, b, c, d\}, \emptyset >$. Then (X, τ) is an ITS on X.

Example 3.4. Let (X, τ) be a topological space such that τ is not indiscrete. Suppose now that $\tau = \{\emptyset, X\} \cup \{G_i : i \in J\}$. Then we can construct two IT's on X as follows:

(a) $\tau^1 = \{\emptyset_\sim, X_\sim\} \cup \{< X, G_i, \emptyset >: i \in J\}$, $\qquad$ (b) $\tau^2 = \{\emptyset_\sim, X_\sim\} \cup \{< X, \emptyset, G_i^c >: i \in J\}$.

Proposition 3.5. Let (X, τ) be an ITS on X. Then, we can also construct several ITS's on X in the following way:

(a) $\tau_{0,1} = \{[]G : G \in \tau\}$, $\qquad$ (b) $\tau_{0,2} = \{<> G : G \in \tau\}$.

Remark 3.6. Let (X, τ) be an ITS.

(a) $\tau_1 = \{G^1 :< X, G^1, G^2 >\in \tau\}$ is a topological space on X. Similarly, $\kappa_2 = \{G^2 :< X, G^1, G^2 >\in \tau\}$ is the family of all closed sets of the topological space $\tau_2 = \{(G^2)^c :< X, G^1, G^2 >\in \tau\}$ on X.

(b) Since $G^1 \cap G^2 = \emptyset$ for each $G =< X, G^1, G^2) \in \tau$, we obtain $G^1 \subseteq (G^2)^c$ and $G^2 \subseteq (G^1)^c$. Hence, we may conclude that (X, τ_1, τ_2) is a bitopological space.

Definition 3.7. Let (X, τ_1), (X, τ_2) be two ITS's on X. Then τ_1 is said to be contained in τ_2 (in symbols, $\tau_1 \subseteq \tau_2$), if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2.

Proposition 3.8. Let $\{\tau_i : i \in J\}$ be a family of IT's on X. Then $\cap_{i\in J}\tau_i$ is an IT on X. Furthermore, $\cap_{i\in J}\tau_i$ is the coarsest IT on X containing all τ_i 's.

Definition 3.9. Let (X, τ) be an ITS on X.

(a) A family $\beta \subseteq \tau$ is called a **base** for (X, τ) iff each member of τ can written as a union of elements of β.

(b) A family $\gamma \subseteq \tau$ is called a **subbase** for (X,τ) iff the family of finite intersections of elements in γ forms a base for (X,τ). In this case the IT τ is said to be generated by γ.

Notice that the IT τ in Example 3.3 is generated by $\{A_1, A_2\}$ if you make use of the equalities $\cap_{i\in\emptyset} G_i = X_\sim$ and $\cup_{i\in\emptyset} G_i = \emptyset_\sim$.

Example 3.10. Consider the set $X = \mathbb{R}$ and take the family $\gamma = \{< \mathbb{R}, (a,b), (-\infty, a] >: a, b \in \mathbb{R}\}$ of IS's in $\mathbb{R}$. In this case γ generates an IT τ on $\mathbb{R}$, which is called the "usual left intuitionistic topology" on $\mathbb{R}$. The base β for this IT can be written in the form $\beta = \{X_\sim\} \cup \gamma$ while τ consists of the following IS's:

$\emptyset_\sim$, $X_\sim$;

$< \mathbb{R}, \cup(a_i, b_i), (-\infty, c] >$, where $a_i, b_i, c \in \mathbb{R}$, $\{a_i : i \in J\}$ is bounded from below, $c < \inf\{a_i : i \in J\}$;

$< \mathbb{R}, \cup(a_i, b_i), \emptyset >$, where $a_i, b_i \in \mathbb{R}$, $\{a_i : i \in J\}$ is not bounded from below.

Similarly one can define the "usual right intuitionistic topology" on $\mathbb{R}$ using an analogous construction.

Example 3.11. Consider again the set $X = \mathbb{R}$ and take the family

$$\gamma = \{< \mathbb{R}, (a,b), (-\infty, a_1] \cup [b_1, \infty) >: a, b, a_1, b_1 \in \mathbb{R}, a_1 \leq a, b \leq b_1\}$$

of IS's in $\mathbb{R}$. In this case γ generates an IT τ on $\mathbb{R}$, which is called the "usual intuitionistic topology" on $\mathbb{R}$. The base β for this IT can be written in the form $\beta = \gamma \cup \{X_\sim\}$. The elements of τ can be easily written down as in the previous example.

Definition 3.12. The complement $\overline{A}$ of an IOS A in an ITS (X,τ) is called an **intuitionistic closed set** (ICS for short) in X.

Now we define closure and interior operations in ITS's:

Definition 3.13. (cf. [7]) Let (X,τ) be an ITS and $A =< X, A^1, A^2 >$ be an IS in X. Then the **interior** and **closure** of A are defined by

$$cl(A) = \cap\{K : K \text{is an ICS in} X \text{ and} A \subseteq K\},\ int(A) = \cup\{G : G \text{is an IOS in} X \text{ and} G \subseteq A\}.$$

It can be also shown that $cl(A)$ is an ICS and $int(A)$ is an IOS in X, and A is an ICS in X iff $cl(A) = A$; and A is an IOS in X iff $int(A) = A$.

Example 3.14. Consider the ITS (X,τ) defined in Example 3.3. If $B =< X, \{a,c\}, \{d\} >$, then we can write down $int(B) =< X, \{c\}, \{d,e\} >$ and $cl(B) =< X, X, \emptyset >= X_\sim$.

Proposition 3.15. For any IFS A in (X,τ) we have: $cl(\overline{A}) = \overline{int(A)}$, $int(\overline{A}) = \overline{cl(A)}$.

Proposition 3.16. Let (X,τ) be an ITS and A, B be IS's in X. Then the following properties hold:

(a) $int(A) \subseteq A$	(a') $A \subseteq cl(A)$
(b) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$	(b') $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$
(c) $int(int(A)) = int(A)$	(c') $cl(cl(A)) = cl(A)$
(d) $int(A \cap B) = int(A) \cap int(B)$	(d') $cl(A \cup B) = cl(A) \cup cl(B)$
(e) $int(X_\sim) = X_\sim$	(e') $cl(\emptyset_\sim) = \emptyset_\sim$

Proposition 3.17. (cf. [7]) Let (X,τ) be an ITS. If $A =< X, A^1, A^2 >$ is an IS in X, then we have

$$int(A) \subseteq< X, int_{\tau_1}(A^1), cl_{\tau_2}(A^2) >\subseteq A, \quad A \subseteq A < X, cl_{\tau_2}(A^1), int_{\tau_1}(A^2) >\subseteq cl(A),$$

where τ_1 and τ_2 are the topological spaces on X defined in Remark 3.6.

Example 3.18. Take the ITS (X,τ) in Example 3.3. If $B =< X, \{a,c\}, \{d\} >$, then we have $int(B) =< X, \{c\}, \{d,e\} >$. Noting that we have

$$\tau_1 = \{\emptyset, X, \{a,b,c\}, \{c,d\}, \{c\}, \{a,b,c,d\}\}, \quad \tau_2 = \{\emptyset, X, \{a,b,c,e\}, \{a,b,c,d\}, \{a,b,c\}\}$$

and $int_{\tau_1}(\{a,b\}) = \{c\}, cl_{\tau_2}(\{d,e\}) = \{d\}$, we see that the inclusions in Proposition 3.17 may be proper.

4. Intuitionistic continuity

Here come the basic definitions first:

Definition 4.1. (cf. [7]) Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ be a function. Then f is said to be **continuous** iff the preimage of each IS in Φ is an IS in τ.

Definition 4.2. Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ be a function. Then f is said to be **open** iff the image of each IS in τ is an IS in Φ.

Example 4.3. Let (X,τ_o), (Y,Φ_o) be two topological spaces.

(a) If $f : X \to Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of Definition 4.1, too. Here we consider the IT's on X and Y, respectively, as follows:

$$\tau = \{< X, G, G^c >: G \in \tau_o\} \text{ and } \Phi = \{< Y, H, H^c >: H \in \Phi_o\}.$$

In this case we have, for each $< Y, H, H^c > \in \Phi$, $H \in \Phi_o$,

$$f^{-1} < Y, H, H^c > = < X, f^{-1}H), f^{-1}(H^c) > = < X, f^{-1}(H), (f^{-1}(H))^c > \in \tau.$$

(b) Let $f : X \to Y$ be an open function in the usual sense. Then f is also open in the sense of Definition 4.2.

Now we obtain some characterizations of continuity:

Proposition 4.4. $f : (X,\tau) \to (Y,\Phi)$ is continuous iff the preimage of each ICS in Φ is an ICS in τ.

Proposition 4.5. The following are equivalent to each other:

(a) $f : (X,\tau) \to (Y,\Phi)$ is continuous.

(b) $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for each IS B in Y.

(c) $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IS B in Y.

Example 4.6. Let (Y,Φ) be an ITS, X a nonempty set and $f : X \to Y$ a function. In this case $\tau = \{f^{-1}(H) : H \in \Phi\}$ is an IT on X. Indeed, t is the coarsest IT on X which makes the function $f : X \to Y$ continuous. One may call the IT t on X the initial intuitionistic topology with respect to f.

Proposition 4.7. Let $f : (X,\tau) \to (Y,\Phi)$ be a continuous function. Then the functions

(a) $f : (X,\tau_1) \to (Y,\Phi_1)$, (b) $f : (X,\tau_2) \to (Y,\Phi_2)$

are also continuous, where τ_1, Φ_1, τ_1, Φ_2 are the topological spaces defined in Remark 3.6. [In other words, $\tau_1 = \{G^1 : G \in \tau\}$, $\Phi_1 = \{H^1 : H \in \Phi\}$, $\tau_2 = \{(G^2)^c : G \in \tau\}$, $\Phi_2 = \{(H^2)^c : H \in \Phi\}$. where $G = < X, G^1, G^2 >$ and $H = < Y, H^1, H^2 >$.]

5. Intuitionistic compactness

First we present the basic concepts:

Definition 5.1. (cf. [7]) Let (X,τ) be an ITS.

(a) If a family $\{< X, G_i^1, G_i^2 >: i \in J\}$ of IOS's in X satisfies the condition $\cup\{< X, G_i^1, G_i^2 >: i \in J\} = X_\sim$, then it is called an **open cover** of X. A finite subfamily of an open cover $\{< X, G_i^1, G_i^2 >: i \in J\}$ of X, which is also an open cover of X, is called a **finite subcover** of $\{< X, G_i^1, G_i^2 >: i \in J\}$.

(b) A family $\{< X, K_i^1, K_i^2 >: i \in J\}$ of ICSs in X satisfies the **finite intersection property** (FIP for short) iff every finite subfamily $\{< X, K_i^1, K_i^2 >: i = 1, 2, ..., n\}$ of the family satisfies the condition $\cap < X, K_i^1, K_i^2 > \neq \emptyset_\sim$.

Definition 5.2. An IFTS (X,τ) is called **compact** iff each open cover of X has a finite subcover.

Example 5.3. (a) Let $X = \mathbb{N}$ and consider the IS's A_n given below:

$A_1 = < X, \{2,3,4,...\}, \emptyset >, A_2 = < X, \{3,4,5,...\}, \{1\} >, A_3 = < X, \{4,5,6,...\}, \{1,2\} >, ...$

$A_n =< X, \{n+1, n+2, n+3, ...\}, \{1, 2, 3, ..., n-1\} >, ...$
Then $\tau = \{\emptyset_\sim, X_\sim\} \cup \{A_n : n = 1, 2, 3, ...\}$ is an IT on X and (X, τ) is compact.

(b) Let $X = (0,1)$ and take the IS's $A_n =< X, (\frac{1}{n}, \frac{n-1}{n}), (0, \frac{1}{n}) >$, $n = 3, 4, 5, ...$ in X. In this case $\tau = \{\emptyset_\sim, X_\sim\} \cup \{A_n : n = 3, 4, 5, ...\}$ is an IT on X which is not compact.

Now we give a proposition stating that compactness in (X, τ) indeed identical to compactness in $(X, \tau_{0,1})$:

Proposition 5.4. Let (X, τ) be an ITS on X. Then, (X, τ) is compact iff the ITS $(X, \tau_{0,1})$ is compact.

Proof. ($\Rightarrow$:) Let (X, τ) be compact, and consider an open cover $\{[]G_j : j \in K\}$ of X in $(X, \tau_{0,1})$. Since $\cup([]G_j) = X_\sim$, we obtain $\cup G_j^1 = X$, and hence, by $G_j^2 \subseteq (G_j^1)^c \Rightarrow \cap G_j^2 \subseteq (\cup G_j^1)^c = \emptyset \Rightarrow \cap G_J^2 = \emptyset$, we deduce $\cup G_j = X_\sim$. Since (X, τ) is compact, $\exists G_1, G_2, ..., G_n$ such that $\cup_{i=1}^n G_i = X_\sim$ from which we obtain $\cup_{i=1}^n G_i^1 = X$ and $\cap_{i=1}^n (G_i^2) = \emptyset$, i.e. (X, τ) is compact.

($\Leftarrow$:) Suppose that $(X, \tau_{0,1})$ is compact and consider an open cover $\{G_j : j \in K\}$ of X in (X, τ). Since $\cup G_j = X_\sim$, we obtain $\cup G_j^1 = X$ and $\cap (G_j^1)^c = \emptyset$. Since $(X, \tau_{0,1})$ is compact, $\exists G_1, G_2, ..., G_n$ such that $\cup_{i=1}^n([]G_i) = X_\sim$, i.e. $\cup_{i=1}^n G_i^1 = X$ and $\cap (G_i^1)^c = \emptyset$. Hence $G_i^1 \subseteq (G_i^2)^c \Rightarrow X = \cup_{i=1}^n G_i^1 \subseteq (\cap_{i=1}^n G_i^2)^c \Rightarrow \cap_{i=1}^n G_i^2 = \emptyset$. Thus $\cup_{i=1}^n G_i = X_\sim$ follows, i.e. (X, τ) is compact. □

Proposition 5.4 implies (Notice that we have $\tau_1 = \{G^1 :< x, G^1, G^2 > \in \tau\}$.):

$$(X, \tau) \text{ is compact iff } (X, \tau_{0,1}) \text{ is compact iff } (X, \tau_1) \text{ is compact.}$$

In view of this proposition we can obtain the following results easily:

Corollary 5.5. An ITS (X, τ) is compact iff every family $\{< X, K_i^1, K_i^2 >: i \in J\}$ of ICS's in X having the FIP has a nonempty intersection.

Corollary 5.6. Let (X, τ), (Y, Φ) be ITS's and $f : X \to Y$ a continuous surjection. If (X, τ) is compact, then so is (Y, Φ).

Since compactness of an ITS (X, τ) is identical to the compactness of the topological space (X, τ_1), we must define the compactness of any IS in (X, τ) as follows:

Definition 5.7. Let (X, τ) be an ITS and A an IS in X.

(a) If a family $\{< X, G_i^1, G_i^2 >: i \in J\}$ of IOS's in X satisfies the condition $A \subseteq \cup\{< X, G_i^1, G_i^2 >) : i \in J\}$, then it is called an **open cover** of A. A finite subfamily of an open cover $\{< X, G_i^1, G_i^2 >: i \in J\}$ of A, which is also an open cover of A, is called a **finite subcover** of $\{< X, G_i^1, G_i^2 >: i \in J\}$.

(b) An IS $A =< X, A^1, A^2 >$ in an ITS (X, τ) is called **compact** iff every open cover of A has a finite subcover.

Corollary 5.8. An IS $A =< X, A^1, A^2 >$ in (X, τ) is compact iff for each family $\mathcal{G} = \{G_i : i \in J\}$, where $G_i =< X, G_i^1, G_i^2 >$, $i \in J$, of IOS's in X with the properties $A^1 \subseteq \cup_{i \in J} G_i^1$ and $A^2 \supseteq \cap_{i \in J} G_i^2$, there exists a finite subfamily $\{G_i : i = 1, 2, ..., n\}$ of $\mathcal{G}$ such that $A^1 \subseteq \cup_{i=1}^n G_i^1$ and $A^2 \supseteq \cap_{i=1}^n G_i^2$.

Example 5.9. Let (X, τ_o) be a topological space and $A \subseteq X$ a compact set in X in the usual sense. We can construct an ITS τ on X as in Example 3.2. Now the IFS $A' =< X, A, A^c >$ is also compact in (X, τ).

Example 5.10. Consider the usual left IT τ on $X = \mathbb{R}$ defined in Example 3.10. The IS $A =< X, [0,1], (-\infty, 0) >$ is compact, but the following IS's B and C are not compact:

$$B =< \mathbb{R}, [0,1], (-1,0) >, C =< \mathbb{R}, [0,1], (1,2) > .$$

Corollary 5.11. Let (X, τ), (Y, Φ) be ITS's and $f : X \to Y$ a continuous function. If A is compact in (X, τ), then so is $f(A)$ in (Y, Φ).

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