ON FUZZY CONTINUOUS FUNCTIONS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

Eftal TAN - Gülhan ASLIM
Ege University,
Department of Mathematics
Bornova, 35100 - İZMİR / TURKEY

Abstract: *The purpose of this paper is to introduce several types of fuzzy continuity between intuitionistic fuzzy topological spaces; namely, fuzzy somewhat continuity, fuzzy almost-somewhat continuity, fuzzy weakly-somewhat continuity.*

Keywords: Intuitionistic fuzzy set, intuitionistic fuzzy topological space, intuitionistic fuzzy regular open set, intuitionistic fuzzy quasi regular space, intuitionistic fuzzy semi-quasi regular space, intuitionistic fuzzy almost-quasi regular space, fuzzy somewhat continuity, fuzzy almost-somewhat continuity, fuzzy weakly-somewhat continuity.

1. Introduction

In [1, 2, 3, 4], Atanassov introduced the fundamental concept of intuitionistic fuzzy set. Later, this concept was generalized to intuitionistic L-fuzzy sets by Atanassov-Stoeva [2, 3]. Çoker [5] introduced the notion of intuitionistic fuzzy topological space, fuzzy continuity and some other related concepts. In this paper we introduce intuitionistic fuzzy quasi regular space, intuitionistic fuzzy semi-quasi regular space and intuitionistic fuzzy almost-quasi regular space. Then we give definitions of several types of somewhat continuity and counter-examples between intuitionistic fuzzy topological spaces.

First we shall give the fundamental definitions given by K. Atanassov:

Definition 1.1. [4] Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element

$x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A \rangle$ for the IFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.2. [4] Let X be a nonempty set and the IFS's A and B be in the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}, B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$$

and let $\{A_i : i \in J\}$ be an arbitrary family of IFS's in X. Then

(a) $A \subseteq B$ iff $\forall x \in X\, [\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)]$;
(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(d) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;
(e) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$;
(f) $\underset{\sim}{0} = \{\langle x, 0, 1\rangle : x \in X\}$ and $\underset{\sim}{1} = \{\langle x, 1, 0\rangle : x \in X\}$.

Now we shall define the image and preimage of IFT's. Let X, Y be two nonempty sets and $f : X \rightarrow Y$ be a function.

Definition 1.3. [5] (a) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}.$$

(b) If $A = \{\langle x, \lambda_A(x), \nu_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f denoted by $f(A)$ is the IFS in Y defined by

$$f(A) = \{\langle y, f(\lambda_A)(y), f_-(\nu_A)(y)\rangle : y \in Y\}$$

where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Now we list the properties of images and preimages, some of which we shall frequently use in the following sections:

Corollary 1.4 [5]. Let A, A_i's ($i \in J$) be IFS's in X, B, B_j's ($j \in K$) IFS's in Y and f: $X \rightarrow Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$;
(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$;
(c) $A \subseteq f^{-1}(f(A))$; [If f is injective, then $A = f^{-1}(f(A))$]
(d) $f(f^{-1}(B)) \subseteq B$; [If f is surjective, then $f(f^{-1}(B)) = B$]
(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$;
(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$;
(g) $f(\bigcup A_i) = \bigcup f(A_i)$;
(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$; [If f is injective, then $f(\bigcap A_i) = \bigcap f(A_i)$]

(i) $f^{-1}(\underset{\sim}{1}) = \underset{\sim}{1}$;

(j) $f^{-1}(\underset{\sim}{0}) = \underset{\sim}{0}$;

(k) If f is surjective, then $f(\underset{\sim}{1}) = \underset{\sim}{1}$;

(l) $f(\underset{\sim}{0}) = \underset{\sim}{0}$;

(m) If f is surjective, then $\overline{f(A)} \subseteq f(\overline{A})$; [If, furthermore, f is injective, then $\overline{f(A)} = f(\overline{A})$]

(n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Definition 1.5. [5] An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFS's in X satisfying the following axioms:

(T_1) $\underset{\sim}{0}$, $\underset{\sim}{1} \in \tau$,

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(T_3) $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and each IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

Definition 1.6. [5] The complement $\overline{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 1.7. [5] Let (X, τ) be an IFTS and $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$ be an IFS in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$\text{cl}(A) = \bigcap \{K : K \text{ is an IFCS in } X \text{ and } A \subseteq K\},$$
$$\text{int}(A) = \bigcup \{G : G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$$

It can be also shown that cl(A) is an IFCS and int(A) is an IFOS in X, and

(a) A is an IFCS in $X \Leftrightarrow \text{cl}(A) = A$,

(b) A is an IFOS in $X \Leftrightarrow \text{int}(A) = A$.

Proposition 1.8. [4] For any IFS A in (X, τ) we have

(a) $\text{cl}(\overline{A}) = \overline{\text{int}(A)}$;

(b) $\text{int}(\overline{A}) = \overline{\text{cl}(A)}$.

Proposition 1.9. [5] Let (X, τ) be an IFTS and A, B be IFS's in X. Then the following properties hold:

(a) $\text{int}(A) \subseteq A$ (a') $A \subseteq \text{cl}(A)$,

(b) $A \subseteq B \Rightarrow \text{int}(A) \subseteq \text{int}(B)$ (b') $A \subseteq B \Rightarrow \text{cl}(A) \subseteq \text{cl}(B)$,

(c) $\text{int}(\text{int}(A)) = \text{int}(A)$

(d) $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$

(e) $\text{int}(\underset{\sim}{1}) = \underset{\sim}{1}$

(c') $\text{cl}(\text{cl}(A)) = \text{cl}(A)$,

(d') $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B)$,

(e') $\text{cl}(\underset{\sim}{0}) = \underset{\sim}{0}$.

Definition 1.10. [6] An IFS A in an IFTS X is called

(a) an intuitionistic fuzzy regular open set of X if $\text{int}(\text{cl}(A)) = A$,

(b) an intuitionistic fuzzy regular closed set of X if $\text{cl}(\text{int}(A)) = A$.

Each intuitionistic fuzzy regular open (closed) set is an intuitionistic fuzzy open (closed) set.

Theorem 1.11. [6] (a) The interior of an IFCS is an intuitionistic fuzzy regular open set,

(b) The closure of an IFOS is an intuitionistic fuzzy regular closed set.

2. Some types of fuzzy continuity in IFTS's

Throughout this section (X, τ), (Y, ϕ) will denote IFTS's and $f: X \to Y$ will denote a function.

Definition 2.1. [5] f is said to be fuzzy continuous if the preimage of each IFS in ϕ is an IFS in τ.

Definition 2.2. [6] A function f is called a fuzzy almost continuous function, if for each intuitionistic fuzzy regular open set A of Y, $f^{-1}(A) \in \tau$.

Theorem 2.3. [6] The following are equivalent:

(a) f is a fuzzy almost continuous function,

(b) $f^{-1}(B)$ is an IFCS, for each intuitionistic fuzzy regular closed set B of Y,

(c) $f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(B))))$, for each IFOS B of Y,

(d) $\text{cl}(f^{-1}(\text{cl}(\text{int}(B)))) \subseteq f^{-1}(B)$, for each IFCS B of Y.

Definition 2.4. [6] A function f is called a fuzzy weakly continuous function if for each IFOS B of Y, $f^{-1}(B) \subseteq \text{int}(f^{-1}(\text{cl}(B)))$.

3. Intuitionistic fuzzy quasi, intuitionistic fuzzy semi-quasi, intuitionistic fuzzy almost-quasi regular spaces

Definition 3.1. An IFTS X is said to be an intuitionistic fuzzy quasi regular space (IFQRS for short) if for any IFOS $A \neq \underset{\sim}{0}$ there exists an IFOS $B \neq \underset{\sim}{0}$ such that $\mathrm{cl}(B) \subseteq A$.

Definition 3.2. An IFTS X is said to be an intuitionistic fuzzy semi-quasi regular space (IFS-QRS for short) if for any IFOS $A \neq \underset{\sim}{0}$ there exists an IFROS $R \neq \underset{\sim}{0}$ such that $R \subseteq A$.

Each intuitionistic fuzzy quasi regular space is an intuitionistic fuzzy semi-quasi regular space, but the converse is not true:

Example 3.3. Let $X = \{a, b, c\}$ and

$$G_1 = \left\langle x, \left(\frac{a}{.3}, \frac{b}{.3}, \frac{c}{.2}\right), \left(\frac{a}{.5}, \frac{b}{.5}, \frac{c}{.7}\right)\right\rangle,\ G_2 = \left\langle x, \left(\frac{a}{.9}, \frac{b}{.6}, \frac{c}{.7}\right), \left(\frac{a}{.1}, \frac{b}{.3}, \frac{c}{.3}\right)\right\rangle.$$

Then the family $\tau = \{\underset{\sim}{1}, \underset{\sim}{0}, G_1, G_2\}$ of IFS's in X is an IFT on X. Since $R = G_1 \subseteq A = G_2$, X is IFS-QRS, but not IFQRS, since $\mathrm{cl}(B) = \mathrm{cl}(G_2) = \underset{\sim}{1} \not\subset A = G_1$.

Definition 3.4. An IFTS X is said to be an intuitionistic fuzzy almost-quasi regular space (IFA-QRS for short) if for any IFROS $R \neq \underset{\sim}{0}$ there exists an IFOS $B \neq \underset{\sim}{0}$ such that $\mathrm{cl}(B) \subseteq R$.

Each intuitionistic fuzzy quasi regular space is an intuitionistic fuzzy almost-quasi regular space, but the converse need not be true.

Example 3.5. Let $X = \{a, b, c\}$ and

$$G_1 = \left\langle x, \left(\frac{a}{.9}, \frac{b}{.7}, \frac{c}{.7}\right), \left(\frac{a}{.1}, \frac{b}{.2}, \frac{c}{.3}\right)\right\rangle,\ G_2 = \left\langle x, \left(\frac{a}{.1}, \frac{b}{.2}, \frac{c}{.3}\right), \left(\frac{a}{.9}, \frac{b}{.8}, \frac{c}{.7}\right)\right\rangle.$$

Then the family $\tau = \{\underset{\sim}{1}, \underset{\sim}{0}, G_1, G_2\}$ of IFS's in X is an IFT on X. Since $\mathrm{cl}(B) = \mathrm{cl}(G_2) = \overline{G_1} \subseteq R = G_1$, X is an IFA-QRS, but not IFQRS, since $\mathrm{cl}(B) = \mathrm{cl}(G_1) = \overline{G_2} \not\subset A = G_2$.

Corollary 3.6. The concepts of IFS-QRS and IFA-QRS are independent.

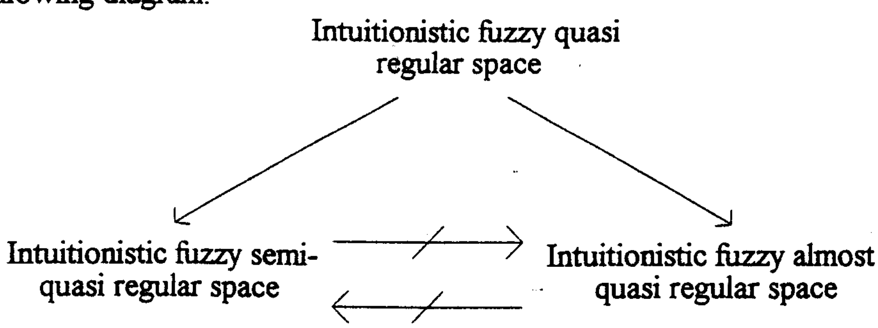
Example 3.7. Refer to Example 3.3. Then X is IFS-QRS, but not IFA-QRS, since $\text{cl}(B) = \text{cl}(G_2) = \underset{\sim}{1} \not\subset R = G_1$.

Example 3.8. Refer to example 3.5. Then X is IFA-QRS, but not IFS-QRS, since $R = G_1 \not\subset A = G_2$.

Theorem 3.9. An IFTS X is IFQRS iff it is IFS-QRS and IFA-QRS.

Proof: ($\Rightarrow$) Obvious.
($\Leftarrow$) Let $A \neq \underset{\sim}{0}$ be an IFOS. By the definition of an IFS-QRS there exists an IFROS $R \neq \underset{\sim}{0}$ such that $R \subseteq A$ and by the definition of an IFA-QRS there exists an IFOS $B \neq \underset{\sim}{0}$ such that $\text{cl}(B) \subseteq R \subseteq A$. Thus X is IFQRS.

The relations among types of IFQRS considered in this section are shown in the following diagram:



4. Some types of fuzzy somewhat continuity in IFTS's

Throughout this section (X, τ), (Y, ϕ) will denote IFTS's and $f : X \to Y$ will denote a function.

Definition 4.1. A function f is said to be fuzzy somewhat continuous if for any IFOS A in Y for which $f^{-1}(A) \neq \underset{\sim}{0}$ we have $\text{int}(f^{-1}(A)) \neq \underset{\sim}{0}$.

A fuzzy continuous function is always fuzzy somewhat continuous. But the converse is not true.

Example 4.2. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$ and

$$G_1 = \left\langle x, \left(\frac{a}{.4}, \frac{b}{.4}, \frac{c}{.5}\right), \left(\frac{a}{.4}, \frac{b}{.4}, \frac{c}{.4}\right)\right\rangle, \; G_2 = \left\langle x, \left(\frac{a}{.2}, \frac{b}{.3}, \frac{c}{.4}\right), \left(\frac{a}{.5}, \frac{b}{.5}, \frac{c}{.5}\right)\right\rangle,$$

$$U_1 = \left\langle y, \left(\frac{1}{.5}, \frac{2}{.4}, \frac{3}{.5}\right), \left(\frac{1}{.4}, \frac{2}{.4}, \frac{3}{.3}\right)\right\rangle,\ U_2 = \left\langle y, \left(\frac{1}{.4}, \frac{2}{.2}, \frac{3}{.4}\right), \left(\frac{1}{.5}, \frac{2}{.4}, \frac{3}{.5}\right)\right\rangle.$$

Then the family $\tau = \{\underset{\sim}{1}, \underset{\sim}{0}, G_1, G_2\}$ of IFS's in X is an IFT on X and the family $\phi = \{\underset{\sim}{1}, \underset{\sim}{0}, U_1, U_2\}$ of IFS's in Y is an IFT on Y. If we define the function $f: X \to Y$ by $f(\mathrm{a}) = 2, f(\mathrm{b}) = 3, f(\mathrm{c}) = 1$, then

$$f^{-1}(U_1) = \left\langle x, \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.5}\right), \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.3}, \frac{\mathrm{c}}{.4}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(U_1)) = G_1 \neq \underset{\sim}{0}$$

$$f^{-1}(U_2) = \left\langle x, \left(\frac{\mathrm{a}}{.2}, \frac{\mathrm{b}}{.4}, \frac{\mathrm{c}}{.4}\right), \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.5}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(U_2)) = G_2 \neq \underset{\sim}{0}.$$

Thus f is fuzzy somewhat continuous, but not fuzzy continuous since

$$f^{-1}(U_2) = \left\langle x, \left(\frac{\mathrm{a}}{.2}, \frac{\mathrm{b}}{.4}, \frac{\mathrm{c}}{.4}\right), \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.5}\right)\right\rangle \notin \tau.$$

Definition 4.3. A function f is said to be fuzzy almost somewhat continuous if for any IFOS A in Y for which $f^{-1}(A) \neq \underset{\sim}{0}$ we have $\mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(A)))) \neq \underset{\sim}{0}$.

A fuzzy somewhat continuous function is always fuzzy almost somewhat continuous. But the converse is not true in general.

Example 4.4. Let $X = \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$, $Y = \{1, 2, 3\}$ and

$$G_1 = \left\langle x, \left(\frac{\mathrm{a}}{.5}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.4}\right), \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.3}, \frac{\mathrm{c}}{.4}\right)\right\rangle,\ G_2 = \left\langle x, \left(\frac{\mathrm{a}}{.5}, \frac{\mathrm{b}}{.35}, \frac{\mathrm{c}}{.4}\right), \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.5}\right)\right\rangle,$$

$$U_1 = \left\langle y, \left(\frac{1}{.5}, \frac{2}{.6}, \frac{3}{.5}\right), \left(\frac{1}{.4}, \frac{2}{.4}, \frac{3}{.3}\right)\right\rangle,\ U_2 = \left\langle y, \left(\frac{1}{.4}, \frac{2}{.5}, \frac{3}{.4}\right), \left(\frac{1}{.5}, \frac{2}{.4}, \frac{3}{.5}\right)\right\rangle.$$

Then the family $\tau = \{\underset{\sim}{1}, \underset{\sim}{0}, G_1, G_2\}$ of IFS's in X is an IFT on X and the family $\phi = \{\underset{\sim}{1}, \underset{\sim}{0}, U_1, U_2\}$ of IFS's in Y is an IFT on Y. If we define the function $f: X \to Y$ by $f(\mathrm{a}) = 1, f(\mathrm{b}) = 3, f(\mathrm{c}) = 2$, then

$$f^{-1}(U_1) = \left\langle x, \left(\frac{\mathrm{a}}{.5}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.6}\right), \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.3}, \frac{\mathrm{c}}{.4}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(U_1)))) = \underset{\sim}{1} \neq \underset{\sim}{0}$$

$$f^{-1}(U_2) = \left\langle x, \left(\frac{\mathrm{a}}{.4}, \frac{\mathrm{b}}{.4}, \frac{\mathrm{c}}{.5}\right), \left(\frac{\mathrm{a}}{.5}, \frac{\mathrm{b}}{.5}, \frac{\mathrm{c}}{.4}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(U_2)))) = \underset{\sim}{1} \neq \underset{\sim}{0}.$$

Thus f is fuzzy almost somewhat continuous, but not fuzzy somewhat continuous, since

$$\text{int}(f^{-1}(U_1)) = G_1 \neq \underset{\sim}{0}$$
$$\text{int}(f^{-1}(U_2)) = \underset{\sim}{0}.$$

Corollary 4.5. Every fuzzy almost continuous function is also fuzzy almost somewhat continuous.

Proof: Let A be an IFOS of Y such that $f^{-1}(A) \neq \underset{\sim}{0}$. Since f is fuzzy almost fuzzy continuous by Theorem 2.3. $f^{-1}(A) \subseteq \text{int}(f^{-1}(\text{int}(\text{cl}(A))))$. On the other hand, we obtained $\text{int}(f^{-1}(\text{int}(\text{cl}(A)))) \neq \underset{\sim}{0}$ from $f^{-1}(A) \neq \underset{\sim}{0}$. This shows that f is fuzzy almost-somewhat continuous.

It is shown in the following example that the converse of the above corollary is not true, in general.

Example 4.6. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$ and

$$G_1 = \left\langle x, \left(\frac{a}{.4}, \frac{b}{.4}, \frac{c}{.5}\right), \left(\frac{a}{.4}, \frac{b}{.4}, \frac{c}{.4}\right)\right\rangle, G_2 = \left\langle x, \left(\frac{a}{.2}, \frac{b}{.4}, \frac{c}{.3}\right), \left(\frac{a}{.5}, \frac{b}{.5}, \frac{c}{.5}\right)\right\rangle,$$

$$U_1 = \left\langle y, \left(\frac{1}{.3}, \frac{2}{.2}, \frac{3}{.4}\right), \left(\frac{1}{.3}, \frac{2}{.35}, \frac{3}{.4}\right)\right\rangle, U_2 = \left\langle y, \left(\frac{1}{.3}, \frac{2}{.2}, \frac{3}{.5}\right), \left(\frac{1}{.2}, \frac{2}{.2}, \frac{3}{.4}\right)\right\rangle.$$

Then the family $\tau = \{\underset{\sim}{1}, \underset{\sim}{0}, G_1, G_2\}$ of IFS's in X is an IFT on X and the family $\phi = \{\underset{\sim}{1}, \underset{\sim}{0}, U_1, U_2\}$ of IFS's in Y is an IFT on Y. If we define the function $f: X \rightarrow Y$ by $f(a) = 2, f(b) = 3, f(c) = 1$, then

$$f^{-1}(U_1) = \left\langle x, \left(\frac{a}{.2}, \frac{b}{.4}, \frac{c}{.3}\right), \left(\frac{a}{.35}, \frac{b}{.4}, \frac{c}{.3}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\text{int}(f^{-1}(\text{int}(\text{cl}(U_1)))) = G_2 \neq \underset{\sim}{0}$$

$$f^{-1}(U_2) = \left\langle x, \left(\frac{a}{.2}, \frac{b}{.5}, \frac{c}{.3}\right), \left(\frac{a}{.2}, \frac{b}{.4}, \frac{c}{.2}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\text{int}(f^{-1}(\text{int}(\text{cl}(U_2)))) = \underset{\sim}{1} \neq \underset{\sim}{0}.$$

Thus f is fuzzy almost somewhat continuous, but not fuzzy almost continuous, since for $U_1 \subset Y$ IFROS

$$f^{-1}(U_1) = \left\langle x, \left(\frac{a}{.2}, \frac{b}{.4}, \frac{c}{.3}\right), \left(\frac{a}{.35}, \frac{b}{.4}, \frac{c}{.3}\right)\right\rangle \notin \tau.$$

Definition 4.7. A function f is said to be fuzzy weakly somewhat continuous if for any IFOS A in Y for which $f^{-1}(A) \neq \underset{\sim}{0}$ we have $\text{int}(f^{-1}(\text{cl}(A))) \neq \underset{\sim}{0}$.

A fuzzy almost somewhat continuous function is always fuzzy weakly somewhat continuous, but the converse is not true.

Example 4.8. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$ and

$$G_1 = \left\langle x, \left(\frac{a}{.4}, \frac{b}{.5}, \frac{c}{.5}\right), \left(\frac{a}{.3}, \frac{b}{.4}, \frac{c}{.4}\right)\right\rangle, G_2 = \left\langle x, \left(\frac{a}{.5}, \frac{b}{.5}, \frac{c}{.5}\right), \left(\frac{a}{.2}, \frac{b}{.3}, \frac{c}{.1}\right)\right\rangle,$$

$$U_1 = \left\langle y, \left(\frac{1}{.5}, \frac{2}{.4}, \frac{3}{.5}\right), \left(\frac{1}{.4}, \frac{2}{.4}, \frac{3}{.3}\right)\right\rangle, U_2 = \left\langle y, \left(\frac{1}{.4}, \frac{2}{.2}, \frac{3}{.4}\right), \left(\frac{1}{.5}, \frac{2}{.4}, \frac{3}{.5}\right)\right\rangle.$$

Then the family $\tau = \{\underset{\sim}{1}, \underset{\sim}{0}, G_1, G_2\}$ of IFS's in X is an IFT on X and the family $\phi = \{\underset{\sim}{1}, \underset{\sim}{0}, U_1, U_2\}$ of IFS's in Y is an IFT on Y. If we define the function $f : X \to Y$ by $f(a) = 1, f(b) = 3, f(c) = 2$, then

$$f^{-1}(U_1) = \left\langle x, \left(\frac{a}{.5}, \frac{b}{.5}, \frac{c}{.4}\right), \left(\frac{a}{.4}, \frac{b}{.3}, \frac{c}{.4}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(\mathrm{cl}(U_1))) = \underset{\sim}{1} \neq \underset{\sim}{0}$$

$$f^{-1}(U_2) = \left\langle x, \left(\frac{a}{.4}, \frac{b}{.4}, \frac{c}{.2}\right), \left(\frac{a}{.5}, \frac{b}{.5}, \frac{c}{.4}\right)\right\rangle \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(\mathrm{cl}(U_2))) = G_2 \neq \underset{\sim}{0}.$$

Thus f is fuzzy weakly somewhat continuous, but not fuzzy almost somewhat continuous, since

$$\mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(U_1)))) = \underset{\sim}{1} \neq \underset{\sim}{0}$$

$$\mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(U_2)))) = \underset{\sim}{0}.$$

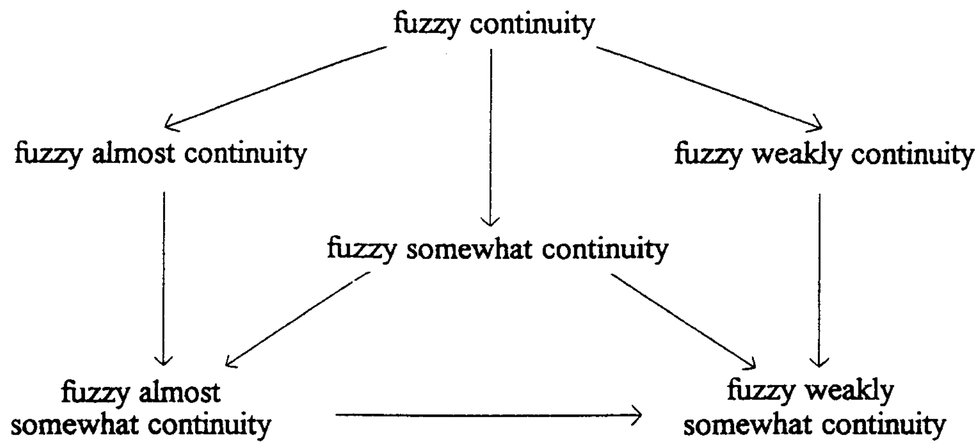
Corollary 4.9. Every fuzzy weakly continuous function is also fuzzy weakly somewhat continuous.

Proof: Let A be an IFOS of Y such that $f^{-1}(A) \neq \underset{\sim}{0}$. Since f is fuzzy weakly continuous by definition 2.4. we have $f^{-1}(A) \subseteq \mathrm{int}(f^{-1}(\mathrm{cl}(A)))$. On the other hand, we obtained $\mathrm{int}(f^{-1}(\mathrm{cl}(A))) \neq \underset{\sim}{0}$ from $f^{-1}(A) \neq \underset{\sim}{0}$. This shows that f is fuzzy weakly somewhat continuous.

It is shown in the following example that the converse is not true, in general.

Example 4.10. Refer to example 4.8. Then f is fuzzy weakly somewhat continuous, but not fuzzy weakly continuous, since $f^{-1}(U_2) \not\subset \mathrm{int}(f^{-1}(\mathrm{cl}(U_2)))$.

We have the following diagram between these types of fuzzy continuity in IFTS's:



References

1. Atanassov, K., *Intuitionistic Fuzzy Sets*, VII ITKR's Session, Sofia, 1989, (Bulgaria).

2. Atanassov, K., Stoeva, S., *Intuitionistic Fuzzy Sets*, Polish Symposium on Interval and Fuzzy Mathematics, Poznan, 1983, 23-26.

3. Atanassov, K., Stoeva, S., *Intuitionistic L-Fuzzy Sets*, Cybernetics and System Research (1984), 539-540.

4. Atanassov, K., *Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems 20 (1986), 87-96.

5. Çoker, D., *An Introduction to Intuitionistic Fuzzy Topological Spaces*, to appear in Fuzzy Sets and Systems.

6. Gürçay; H., Eş, A.H., Çoker, D., *On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces*, to appear in The Journal of Fuzzy Mathematics.