Stratiform Closed Sets in L-fuzzy Topological Spaces*

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Abstract

Given an L-fuzzy topological space (L^X, δ), for each molecule α of L, *α-closed sets* which called stratiform closed sets are presented,and the properties of the sets are discussed.

Keywords: Fuzzy topology, stratiform set

Can closed sets in L-fuzzy topological spaces be replaced in a manner by something which like closed sets but are not closed sets? This is a question warranting careful consideration.In this paper,the concept of $D\alpha$-closed sets is introduced in L-fuzzy topological spaces.Though these $D\alpha$-closed sets are not,in general,closed sets,but they display some characteristics of closed sets on a certain stratum.Our discussion show that $D\alpha$-closed sets sometime may replace closed sets.

The notions and symbols used in paper follows [1]

Definition 1. *Let* (L^X, δ) *be an L-fts,* $\alpha \in M(L)$ *.The operator* $D_\alpha : L^X \to \delta'$

* *Supported by the Natural Science Foundation of Shandong of China*

is defined as follows: for each $A \in L^X$, $D_\alpha(A) = \wedge\{G \in \delta' : G_{[\alpha]} \supset A_{[\alpha]}\}$.

Example 2. (1) *For L-fts* (L^X, δ), *where* $\delta = \{0_X, 1_X\}$. *Take* $\alpha \in M(L)$ *and* $\alpha \neq 1, A = \alpha_X$. *Then* $D_\alpha(A) = 1_X > A$.

(2) *For L-fts* (L^X, δ), *where* $\delta = \{\alpha_X : \alpha \in L\}$. *Take* $\alpha \in M(L)$, $A = \alpha_X$. *Then* $D_\alpha(A) = \alpha_X = A$.

(3) *Take* $L = X = [0,1]$, *and* $\delta = \{\lambda_X : \lambda \in L\}$, *then* (L^X, δ) *is an L-fts.* $A \in L^X$ *is defined by* $A(x) = x$. *Take* $\alpha = 0.5 \in M(L)$, *then* $D_\alpha(A) = 0.5_X$. *Clearly* $D_\alpha(A) \leq A$ *and* $D_\alpha(A) \geq A$..

Theorem 3. *Let* (L^X, δ) *be an L-fts,* $\alpha \in M(L)$. *The operator* D_α *has the following properties:* $\forall A, B \in L^X$,

(1) $D_\alpha(0_X) = 0_X$;

(2) $D_\alpha(A \vee B) = D_\alpha(A) \vee D_\alpha(B)$;

(3) $A_{[\alpha]} \subset (D_\alpha(A))_{[\alpha]}$;

(4) $D_\alpha(D_\alpha(A)) = D_\alpha(A)$.

Proof. (1) It is clear.

(2) If $A \leq B$, then $A_{[\alpha]} \subset B_{[\alpha]}$. Hence

$$D_\alpha(B) = \wedge\{G \in \delta' : G_{[\alpha]} \supset B_{[\alpha]}\} \geq \wedge\{G \in \delta' : G_{[\alpha]} \supset A_{[\alpha]}\} = D_\alpha(A).$$

From this we get $D_\alpha(A \vee B) \geq D_\alpha(A) \vee D_\alpha(B)$. On the other hand,

$$\begin{aligned} D_\alpha(A) \vee D_\alpha(B) &= (\wedge\{G \in \delta' : G_{[\alpha]} \supset A_{[\alpha]}\}) \vee (\wedge\{H \in \delta' : H_{[\alpha]} \supset B_{[\alpha]}\}) \\ &= \wedge\{G \vee H \in \delta' : G_{[\alpha]} \supset A_{[\alpha]}, H_{[\alpha]} \supset B_{[\alpha]}\} \\ &\geq \wedge\{E \in \delta' : E_{[\alpha]} \supset A_{[\alpha]} \cup B_{[\alpha]}\} \\ &= D_\alpha(A \vee B) \end{aligned}$$

Therefore $D_\alpha(A\vee B)=D_\alpha(A)\vee D_\alpha(B)$.

(3) $(D_\alpha(A))_{[\alpha]}=\bigcap\{G_{[\alpha]}:G\in\delta',G_{[\alpha]}\supset A_{[\alpha]}\}\supset A_{[\alpha]}$.

(4) From $(D_\alpha(A))_{[\alpha]}\supset A_{[\alpha]}$ we get

$$D_\alpha(D_\alpha(A))=\wedge\{G\in\delta':G_{[\alpha]}\supset(D_\alpha(A))_{[\alpha]}\}\geq\wedge\{G\in\delta':G_{[\alpha]}\supset A_{[\alpha]}\}=D_\alpha(A).$$

Also, from $D_\alpha(A)$ is closed and $(D_\alpha(A))_{[\alpha]}\supset(D_\alpha(A))_{[\alpha]}$ we get

$$D_\alpha(D_\alpha(A))=\wedge\{G\in\delta':G_{[\alpha]}\supset(D_\alpha(A))_{[\alpha]}\}\leq D_\alpha(A).$$

Hence $D_\alpha(D_\alpha(A))=D_\alpha(A)$. □

Definition 4. *Let* (L^X,δ) *be an L-fts,* $\alpha\in M(L)$, $A\in L^X$ *is called* $D\alpha-$*closed in* (L^X,δ)*, if* $(D_\alpha(A))_{[\alpha]}=A_{[\alpha]}$. *The set of all* $D\alpha-$*closed sets in* (L^X,δ) *is denoted by* $D_\alpha(\delta)$.

Theorem 5. *Let* (L^X,δ) *be an L-fts,* $A\in L^X$. *If* $A\in\delta'$*, then for each* $\alpha\in M(L), A\in D_\alpha(\delta)$*, i.e.,* $\delta'\subset D_\alpha(\delta)$.

Example 6. *Let* $L=[0,1]$ *and X a nonempty crisp set. Take* $\delta=\{0_X,1_X\}$*, then* (L^X,δ) *is a fuzzy topological space. We consider* $A=0.5_X$. *Naturaly A is not closed set in* (L^X,δ)*, but for each* $\alpha\in M(L), A\in D_\alpha(\delta)$.

Li Yongming has presented in [2] the concept of $\alpha-$closed set for L-fts as follows:

Let (L^X,δ) *be an L-fts,* $A\in L^X,\alpha\in M(L)$. *A is called* $\alpha-$*closed set in* (L^X,δ)*, if for any* $x_\alpha\in A^-$ *we have* $x_\alpha\in A$.

The set of all $\alpha-$closed sets in (L^X,δ) is denoted by $L_\alpha(\delta)$.

It is clear that $A\in L_\alpha(\delta)$ *iff* $(A^-)_{[\alpha]}=A_{[\alpha]}$.

Theorem 7. *Let* (L^X,δ) *be an L-fts,* $A\in L^X$. *Then for each* $\alpha\in M(L)$ *we* *have* $L_\alpha(\delta) \subset D_\alpha(\delta)$ *,that is ,* $\alpha-$*closed sets are* $D\alpha-$*closed sets.*

Example 8. *Take* $X = L = [0,1]$*.Let* H_k *denote line segment:* $y = kx$*,where* $x \in [0,1]$*,*k *is the slope and* $0 \le k \le 1$*.Let* $\delta' = \{1_X, H_k : 0 \le k \le 1\}$*,then* δ' *clearly is an L-fuzzy co-topology on X.* $A \in L^X$ *is defined by* $A(x) = 0.25$ *for* $x \in [0,0.5)$ *and* $A(x) = 0.5$ *for* $x \in [0.5,1]$*.Take* $\alpha = 0.5$*.then for* (L^X, δ) *we have*

$$A_{[0.5]} = [0.5,1]; A^- = 1_X; (A^-)_{[0.5]} = X \neq A_{[0.5]}.$$

$$D_{0.5}(A) = \wedge\{G \in \delta' : G_{[0.5]} \supset A_{[0.5]}\} = 1_X \wedge H_1 = H_1 .$$

$$(D_{0.5}(A))_{[0.5]} = [0.5,1] = A_{[0.5]} .$$

Hence A *is* $D0.5-$*closed and not* $0.5-$*closed.This shows that the converse of Theorem 7 is ,in general,false.*

From Theorem 3 (4) we easily see that

Theorem 9. *Let* (L^X, δ) *be an L-fts,* $A \in L^X$*.Then for each* $\alpha \in M(L)$*,* $D_\alpha(A) \in D_\alpha(\delta)$*.*

Theorem 10. *Let* (L^X, δ) *be an L-fts.Then for each* $\alpha \in M(L)$*,* $D_\alpha(\delta)$ *forms an L-fuzzy co-topology on X.*

Theorem 11. *Let* $(L^X, \omega_L(\tau))$ *be an L-fts induced by crisp topological space* (X,τ)*,* $A \in L^X$*,*$\alpha \in M(L)$*.Then* $A \in D_\alpha(\omega_L(\tau))$ *iff* $A_{[\alpha]} \in \tau'$*.*

Proof. Suppose that $A \in D_\alpha(\omega_L(\tau))$,i.e,$(D_\alpha(A))_{[\alpha]} = A_{[\alpha]}$. For each $G \in (\omega_L(\tau))'$ we see that $G_{[\alpha]} \in \tau'$.Hence

$$A_{[\alpha]} = (D_\alpha(A))_{[\alpha]} = (\wedge\{G \in (\omega_L(\tau))' : G_{[\alpha]} \supset A_{[\alpha]}\})_{[\alpha]}$$

$$= \bigcap\{G_{[\alpha]} : G \in (\omega_L(\tau))', G_{[\alpha]} \supset A_{[\alpha]}\}$$

$$\supset \bigcap\{E \in \tau' : E \supset A_{[\alpha]}\} = (A_{[\alpha]})^{-}.$$

This shows that $A_{[\alpha]} = (A_{[\alpha]})^{-}$,and so $A_{[\alpha]} \in \tau'$.

Conversely,assume that $A_{[\alpha]} \in \tau'$.From Theorem 2.11.6 in [1] we have $\chi_{A_{[\alpha]}} \in (\omega_L(\tau))'$.Also,$(\chi_{A_{[\alpha]}})_{[\alpha]} = A_{[\alpha]}$,thus

$$(D_\alpha(A))_{[\alpha]} = \bigcap\{G_{[\alpha]} : G_{[\alpha]} \supset A_{[\alpha]}, G \in (\omega_L(\tau))'\} \subset A_{[\alpha]}.$$

Hence $(D_\alpha(A))_{[\alpha]} = A_{[\alpha]}$,and so $A \in D_\alpha(\omega_L(\tau))$. □

Corollary 12. *Let $(L^X, \omega_L(\tau))$ be an L-fts induced by crisp topological space $(X,\tau), A \in L^X$. Then $A \in (\omega_L(\tau))'$ iff for each $\alpha \in M(L), A \in D_\alpha(\omega_L(\tau))$.*

References

[1] Wang Guojun, The Theory of L-fuzzy Topological Spaces (Shanxi Normal University Press, Xi'an, China,1988) (in Chinese).

[2] Li Yongming, Theory of L-fuzzy closed graph and strong closed graph, *Fuzzy Systems and Math.*2(1991) 30-37 (in Chinese).