Fuzzy Factor Algebra of F-Algebra

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Abstract: In this paper, we will introduce the concept of fuzzy factor algebra of fuzzy algebra over fuzzy field, and discuss the important properties of it.

Keywords: fuzzy field, F-fuzzy algebra, F-fuzzy ideal, fuzzy homomorphism.

1 Preliminaries

Let X be any set, L a complete distributive lattice with 0,1. A fuzzy subset A on X is characterized by mapping A: X $\rightarrow$L, X^L denotes the set of whole fuzzy subset of X.

Definition 1.1 Let K be a field, $A \in K^L$, the F is called a fuzzy field of K, if for all x, y$\in$K

1) $F(x-y) \geqslant F(x) \wedge F(y)$,

2) $F(xy) \geqslant F(x) \wedge F(y)$,

3) $F(x) \geqslant F(x^{-1})$, if $x \neq 0$.

Definition 1.2 Let Y be an algebra over field K, F a fuzzy field of K, and $A \in Y^L$. Then A is called a fuzzy algebra of Y, if

1) $A(x-y) \geqslant A(x) \wedge A(y)$,

2) $A(xy) \geqslant A(x) \wedge A(y)$,

3) $A(\lambda x) \geqslant A(x) \wedge F(\lambda)$,

for all x, $y \in Y$, and $\lambda \in K$.In brief A is an F-fuzzy algebra.

Unless specially stated, Y only refers to the algebra over field K, F only refers to the fuzzy field of K, A only refers to the F-fuzzy algebra of Y in this paper.

Definition 1.3 Let A be an F-algebra of Y. Then A is called an F-fuzzy left ideal of Y, if $A(xy) > A(x) \vee A(y)$ for all x, $y \in Y$.

Definition 1.4 let B be an F-fuzzy ideal of Y, the fuzzy subset x+B of Y is defined as follows: $(x+B)y=B(x-y)$ for all $y \in Y$.

Definition 1.5 Let B be an F-fuzzy ideal of Y, $Y/B=\{x+B|x+Y\}$. The operation "+", "." and sclar product on Y/B are defined as follows:

$$(x+B)+(y+B)=x+y+B,$$

$$(x+B)(y+B)=xy+B,$$

$$\lambda (x+B)= \lambda x+B.$$

2 Fuzzy factor algebra of the F-fuzzy algebra

Proposition 2.1 Let B be an F-fuzzy ideal of algebra Y, then Y/B is an algebra over field K.

Proposition 2.2 Let B be an F-fuzzy ideal of the algebra Y, and

$G_B=\{x|x\in Y, B(x)=B(0)\}$, then G_B is an ideal of Y, and $Y/G_B \cong Y/B$.

We can easily prove the Proposition 2.1 and Proposition 2.2.

Let Y be an algebra over field K, A an F-fuzzy algebra of Y, B an F-fuzzy ideal of Y. We define a fuzzy set A/B of Y/B as follows:

$$A/B: Y/B \to L \quad \text{and} \quad A/B\,(x+B)=\bigvee_{y+B=x+B} A(y)$$

Theorem 2.3 A/B is an F-fuzzy algebra of Y/B.

Prof. For all $x, y\in Y$, $\lambda\in K$, we have

$$\begin{aligned}
A/B\,((x+B)-(y+B)) &= A/B\,(x-y+B)\\
&= \bigvee_{z+B=x-y+B} A(z)\\
&\geq \bigvee_{\substack{x_1+B=x+B\\ y_1+B=-y+B}} A(x_1+y_1)\\
&\geq \bigvee_{\substack{x_1+B=x+B\\ y_1+B=-y+B}} A(x_1)\wedge A(y_1)\\
&= \Big(\bigvee_{x_1+B=x+B} A(x_1)\Big)\wedge\Big(\bigvee_{y_1+B=-y+B} A(y_1)\Big)\\
&= (A/B(x+B))\wedge(A/B(-y+B))\\
&\geq (A/B(x+B))\wedge(A/B(y+B))
\end{aligned}$$

$$\begin{aligned}
A/B(\lambda(x+B)) &= A/B(\lambda x+B)\\
&= \bigvee_{z=\lambda x} A(z)\\
&\geq \bigvee_{\lambda x_1=\lambda x} A(\lambda x_1)\\
&\geq \bigvee_{\lambda x_1=\lambda x} A(x_1)\wedge F(\lambda)\\
&= F(\lambda)\wedge \bigvee_{\lambda x_1=\lambda x} A(x_1)
\end{aligned}$$

$$=F(\lambda)\wedge A/B(x+B)$$

$$A/B(x+B)(y+B)=A/B(xy+B)$$

$$=\bigvee_{z+B=xy+B} A(z)$$

$$=\bigvee_{\substack{x_1+B=x+B\\ y_1+B=y+B}} A(x_1y_1)$$

$$\geq \bigvee_{\substack{x_1+B=x+B\\ y_1+B=y+B}} A(x_1)\wedge A(y_1)$$

$$=\left(\bigvee_{x_1+B=x+B} A(x_1)\right)\wedge\left(\bigvee_{y_1+B=y+B} A(y_1)\right)$$

$$=(A/B(x+B))\wedge(A/B(y+B)).$$

So,A/B is an F-fuzzy algebra.

Definition 2.4 We call A/B the F-fuzzy factor algebra of A about B.

Definition 2.5 Let Y, Y′ be general sets, f:Y→Y′ a surjective mapping ,and A a fuzzy set of Y. If f(x)=f(y) follows A(x)=A(y), then A is called f-invariant.

Definition 2.6 Let f: Y→Y′ be an algebra homomorphism (isomorphism), A and A′ fuzzy F-algebra of Y and Y′, respectively: If f(A)= A′, then we say A is homomorphic (isomorphic) to A′, which is denoted as A∽ A′ (A≌A′).

Similarly Proposition 2.5, Proposition 2.6, Theorem 2.7, Proposition 2.8, Theorem 2.9 of [1],we have the following Theorems and Propositon.

Proposition 2.7 Let Y be a algebra over field K, A and B as above, then A $\backsim$ A/B.

Proposition 2.8 Let f be an algebra homomorphism from algebra Y into algebra Y´,A the F-fuzzy algebra of Y, and I the ideal of Y. If $G_B \subset$kerf, then A/B $\backsim$ f(A).

Theorem 2.9 Let f:Y→Y´ be an algebra homomorphism, A a fuzzy algebra of Y, B a F-fuzzy ideal of Y and G_B=kerf, then A/B≌f(A).

Proposition 2.10 Let f: Y→Y´ be an algebra homomorphism, B a fuzzy ideal of Y and B be f-invariant. Then Y/B≌Y´/f(B).

Theorem 2.11 Let f: Y→Y´ be an algebra homomorphism, A a fuzzy algebra of Y and B a F-fuzzy ideal of Y. If B is f-invariant , then A/B≌f(A)/f(B).

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