A KIND OF INTERVAL-VALUED FUZZY LINEAR PROGRAMMING PROBLEMS

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Abstract Interval-valued fuzzy linear programming problems(IFLP) are presented in this paper. Ranking of triangular interval-valued fuzzy numbers is discussed. Corresponding auxiliary models are obtained in the meaning of different criteria. Algorithm to solve IFLP problems is given.

Keywords Interval-valued fuzzy linear programming(IFLP), triangular interval-valued fuzzy number, auxiliary model

1. Introduction

Fuzzy linear programming(FLP) can be classified into two categories: FLP with fuzzy constraints and FLP with fuzzy coefficients. But it is not easy to determine the membership functions of fuzzy coefficients, especially, when information is not complete, even unable to set up them. However, interval-valued membership function is easy to obtain and it also can reflect the innate character of fuzzy problems[1]. Therefore, many results on interval-valued fuzzy set have appeared. Basic theory for interval-valued fuzzy sets is discussed by [2]. Fuzzy linear programming problems with interval-valued fuzzy coefficients is given by [3]. The definition and operations of interval-valued fuzzy number are put forward by [4].

On the basis of above papers, Ranking of interval-valued fuzzy numbers are discussed in this paper. Then interval-valued fuzzy linear programming problems(IFLP) are presented. After that corresponding auxiliary models are obtained in the meaning of different criteria. And finally algorithm to solve IFLP problems and a numerical example are given.

2. Ranking of interval-valued fuzzy number

Definition 1. If $A=[A^-,A^+]$ is a interval-valued fuzzy set on R and A^-,A^+ are bounded closed fuzzy numbers on R, then A is called a bounded closed interval-valued fuzzy number(BCIFN, for short). The set of all BCIFNs on R is denoted by BC[R].

Definition 2 Let $A \in \mathrm{BC[R]}$ and $A=[A^-,A^+]$. If A^-,A^+ are triangular fuzzy number. Then A is called a triangular interval-valued fuzzy number. The set of all triangular interval-valued fuzzy number is denoted by IFN.

Theorem 1 Let $A,B \in \mathrm{BC[R]}$ and $A=[A^-,A^+], B=[B^-,B^+]$. Then the relation $\leq$ defined by the following

$$A \leq B \Leftrightarrow A \vee B = B \Leftrightarrow A \wedge B = A$$

is a partial order where

$$A \vee B=[A^- \vee B^-, A^+ \vee B^+], A \wedge B=[A^- \wedge B^-, A^+ \wedge B^+].$$

Corollary 1 $A \leq B \Leftrightarrow A^- \leq B^-, A^+ \leq B^+$

In the following, triangular fuzzy number a will be denoted by $a=(a_0,a_1,a_2)$ where a_1 and a_2 denote the lower and upper limits of the support of a fuzzy number with mode a_0.

Theorem 2 If $A=[A^-,A^+], B=[B^-,B^+]$ are two triangular interval-valued fuzzy number and $A^-=(a_0^-,a_1^-,a_2^-)$, $A^+=(a_0^+,a_1^+,a_2^+)$, $B^-=(b_0^-,b_1^-,b_2^-)$, $B^+=(b_0^+,b_1^+,b_2^+)$, then for $\forall k \in R^+$

$$A+B=[A^-+B^-,A^++B^+], A-B=[A^--B^-,A^+-B^+], kA=[kA^-,kA^+].$$

where $A^- \pm B^-=(a_0^- \pm b_0^-, a_1^- \pm b_1^-, a_2^- \pm b_2^-)$, $A^+ \pm B^+=(a_0^+ \pm b_0^+, a_1^+ \pm b_1^+, a_2^+ \pm b_2^+)$, $kA^-=(ka_0^-,ka_1^-,ka_2^-)$, $kA^+=(ka_0^+,ka_1^+,ka_2^+)$.

For triangular fuzzy number $a=(a_0,a_1,a_2)$ and $b=(b_0,b_1,b_2)$, we use two fuzzy order relations

$$a \leq b \Leftrightarrow a_2 \leq b_1 \text{ and } a \leq b \Leftrightarrow a_0+a_1+a_2 \leq b_0+b_1+b_2$$

Let $A=[A^-,A^+], B=[B^-,B^+]$ are two triangular interval-valued fuzzy number and $A^- = (a_0^-, a_1^-, a_2^-)$, $A^+ = (a_0^+, a_1^+, a_2^+)$, $B^- = (b_0^-, b_1^-, b_2^-)$, $B^+ = (b_0^+, b_1^+, b_2^+)$. We give two ranking methods of triangular interval-valued fuzzy numbers as follows.

$$A \le B \Leftrightarrow a_2^- \le b_1^-, a_2^+ \le b_2^+ \qquad (*)$$

and $A \le B \Leftrightarrow a_0^- + a_1^- + a_2^- \le b_0^- + b_1^- + b_2^-$, $a_0^+ + a_1^+ + a_2^+ \le b_0^+ + b_1^+ + b_2^+$. (**)

3. Fuzzy linear programming problems

Definition 3. If $A = (a_{ij})_{m\times n}$, $\mathrm{b} = (\mathrm{b}_1, b_2, \cdots, b_m)^T$, $c = (c_1, c_2, \cdots, c_n)$, $x = (x_1, x_2, \cdots, x_n)^T$ where $a_{ij}, b_i \in IFN$, $c_j, x_j \in R(i = 1,2,\cdots,m; j = 1,2,\cdots,n)$, then

$$\begin{aligned} \max \quad & z = cx \\ \text{s.t.} \quad & Ax \le b, x \ge 0 \end{aligned} \qquad (1)$$

is said to be a interval-valued fuzzy linear programming problem(IFLP for short).

In the following, the algorithm for solving (1) is given.

Step 1. Transform (1) into (2).

$$\begin{aligned} \max \quad & z = \sum_{j=1}^{n} c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\cdots,m \\ & x_j \ge 0, \quad j = 1,2,\cdots,n. \end{aligned} \qquad (2)$$

Step 2. Let $a_{ij} = (a_{ij}^-, a_{ij}^+)$, $b_i = (b_i^-, b_i^+)$ and $a_{ij}^- = (a_{ij0}^-, a_{ij1}^-, a_{ij2}^-)$, $a_{ij}^+ = (a_{ij0}^+, a_{ij1}^+, a_{ij2}^+)$, $b_i^- = (b_{i0}^-, b_{i1}^-, b_{i2}^-)$, $b_i^+ = (b_{i0}^+, b_{i1}^+, b_{i2}^+)$.

We apply results of Theorem 1-2 in the problem (2) and have

$$\begin{aligned} \max \quad & z = \sum_{j=1}^{n} c_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij}^- x_j \le b_i^-, \ \sum_{j=1}^{n} a_{ij}^+ x_j \le b_i^+, \\ & x_j \ge 0, i = 1,2,\cdots,m; j = 1,2,\cdots,n. \end{aligned} \qquad (3)$$

and $$\max \quad z=\sum_{j=1}^{n} c_j x_j$$

$$\text{s.t.} \quad (\sum_{j=1}^{n} a_{ij0}^- x_j, \sum_{j=1}^{n} a_{ij1}^- x_j, \sum_{j=1}^{n} a_{ij2}^- x_j) \le (b_{i0}^-, b_{i1}^-, b_{i2}^-) \tag{4}$$

$$(\sum_{j=1}^{n} a_{ij0}^+ x_j, \sum_{j=1}^{n} a_{ij1}^+ x_j, \sum_{j=1}^{n} a_{ij2}^+ x_j) \le (b_{i0}^+, b_{i1}^+, b_{i2}^+)$$

$$x_j \ge 0, i=1,2,\cdots,m; j=1,2,\cdots,n.$$

Step 3. From (*) and (**), we have two auxiliary models

$$\max \quad z=\sum_{j=1}^{n} c_j x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij2}^- x_j \le b_{i1}^-, \sum_{j=1}^{n} a_{ij2}^+ x_j \le b_{i1}^+, \tag{5}$$

$$x_j \ge 0, i=1,2,\cdots,m; j=1,2,\cdots,n.$$

$$\max \quad z=\sum_{j=1}^{n} c_j x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} (a_{ij0}^- + a_{ij1}^- + a_{ij2}^-) x_j \le b_{i0}^- + b_{i1}^- + b_{i2}^- \tag{6}$$

$$\sum_{j=1}^{n} (a_{ij0}^+ + a_{ij1}^+ + a_{ij2}^+) x_j \le b_{i0}^+ + b_{i1}^+ + b_{i2}^+)$$

$$x_j \ge 0, i=1,2,\cdots,m; j=1,2,\cdots,n.$$

Step 4. Solving (5) and (6) with simplex method, then we obtain optimal solutions of (5) and (6). They can be seen as optimal solutions of (1).

4. Numerical example

We consider following IFLP problem

$$\max \quad z=c_1x_1+c_2x_2$$

$$\text{s.t.} \quad a_{11}x_1+a_{12}x_2 \le b_1 \tag{7}$$

$$a_{12}x_1+a_{22}x_2 \le b_2$$

$$x_1, x_2 \ge 0$$

where $a_{11}=[a_{11}^-, a_{11}^+], a_{12}=[a_{12}^-, a_{12}^+], a_{21}=[a_{21}^-, a_{21}^+], a_{22}=[a_{22}^-, a_{22}^+]$, $b_1=[b_1^-, b_1^+], b_2=[b_2^-, b_2^+]$ are all triangular interval-valued fuzzy numbers.

$a_{11}^{-}=(3,1,4)$, $a_{11}^{+}=(3,1,6)$, $a_{12}^{-}=(5,2,6)$, $a_{12}^{+}=(5,1,8)$, $a_{21}^{-}=(4,3,7)$, $a_{21}^{+}=(4,2,7)$, $a_{22}^{-}=(6,3,10)$, $a_{22}^{+}=(6,1,11)$, $b_{1}^{-}=(30,20,40)$, $b_{1}^{+}=(30,18,45)$, $b_{2}^{-}=(56,42,78)$, $b_{2}^{+}=(56,36,89)$, $c_1=6, c_2=8$. From (5) we have

$$\begin{aligned} &\max \quad z=6x_1+8x_2 \\ &\text{s.t.} \quad 4x_1+6x_2\le 20\ ,\ 7x_1+10x_2\le 42 \\ &6x_1+8x_2\le 18\ ,\ 7x_1+11x_2\le 36,\ x_1,x_2\ge 0, \end{aligned} \tag{8}$$

whose optimal solution is $x_1^{*}=0, x_2^{*}=2.25$ and optimal value $z^{*}=18$.

From (6) we have

$$\begin{aligned} &\max \quad z=6x_1+8x_2 \\ &\text{s.t.} \quad 8x_1+13x_2\le 90\ ,\ 14x_1+19x_2\le 176 \\ &10x_1+14x_2\le 93\ ,\ 13x_1+18x_2\le 181,\ x_1,x_2\ge 0, \end{aligned} \tag{9}$$

whose optimal solution is $x_1^{*}=9.30, x_2^{*}=0$ and optimal value $z^{*}=55.80$.

References

[1] B.Gorzafczang. Approximate inference with interval-valued fuzzy sets—an outline. Proc. Polish symp. On intreval and fuzzy math.,Poland,1983,89-95.

[2] Meng Guangwu. Basic theory for interval-valued fuzzy sets. Applied Mathematics, 1993,(2):212-217(in Chinese).

[3] Li Xiaozhong,Shi Kaiquan. Fuzzy linear programming problem with interval-valued coefficients. The Journal of Fuzzy Mathematics, 1998,6(2):327-332.

[4] Xie Shengxian. On interval-valued fuzzy numbers. BUSEFAL,1997,(73).

[5] Luo Chengzhong. An introduction to fuzzy sets(Book one). Beijing Normal University Press, 1989,200-214(in Chinese).

[6] L.Campos,J.L.Verdegay. Linear programming problems and ranking of fuzzy numbers. Fuzzy Sets and Systems,1989,32:1-11.