The Cut-sets of Intuitionistic Fuzzy Sets

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Abstract

In this paper, we defined the cut-sets of intuitionistic fuzzy set in two forms, and gave its main properties.

Keywords: Intuitionistic fuzzy sets, supper cut-sets, lower cut-sets.

1.preparation

Let a set X be fixed and the symbol I denote [0,1], $P(X)$ and $F(X)$ denote the power set and the fuzzy power set respectively.

$\forall a_t \in I, t \in T$, we define $\vee_{t\in T} a_t = \sup\{a_t : t \in T\}$, $\wedge_{t\in T} a_t = \inf\{a_t : t \in T\}$.

Let $L=\{(a, b) \mid a+b\leq 1,\ a, b\in I\}$, $\forall (a_t, b_t)\in L$, $t\in T$. We define the operations on L as follows:

$$\vee_{t\in T}(a_t,b_t) = (\vee_{t\in T} a_t, \wedge_{t\in T} b_t),\ \wedge_{t\in T}(a_t,b_t) = (\wedge_{t\in T} a_t, \vee_{t\in T} b_t)$$

$$(a_t,b_t)' = (b_t,a_t)$$

For any $(a_i, b_i)\in L$, $i=1, 2$, we define the relations as follows:

$$(a_1,b_1) = (a_2,b_2) \Leftrightarrow a_1 = a_2 \ \&\ b_1 = b_2;$$

$$(a_1,b_1) \leq (a_2,b_2) \Leftrightarrow a_1 \leq a_2 \ \&\ b_1 \geq b_2;$$

$$(a_1,b_1) < (a_2,b_2) \Leftrightarrow (a_1,b_1) \leq (a_2,b_2) \ \&\ (a_1,b_1) \neq (a_2,b_2);$$

It is easy to prove that:

Theorem 1.1 Let α, $\alpha_t\in L$, $t\in T$, then

$$\alpha \wedge (\vee_{t\in T} \alpha_t) = \vee_{t\in T}(\alpha \wedge \alpha_t); \qquad \alpha \vee (\wedge_{t\in T} \alpha_t) = \wedge_{t\in T}(\alpha \vee \alpha_t)$$

Theorem 1.2 The system $(L, \leq, \vee, \wedge)$ is a complete lattice with the order-reversing involution " ′ ". And it has maximal element $\tilde{1}=(1, 0)$ and minimal element $\tilde{0}=(0, 1)$.

Definition 1.1[1] An intuitionistic fuzzy set (IFS for short) A in X is an object having the form

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$$

where the functions $\mu_A : X\to I$ and $\nu_A : X\to I$ denote the degree of membership and degree of nonmembership of the element x∈X to set A, which is a fuzzy subset of X, respectively, and for every $x \in X$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

Obviously, every fuzzy set has the form $\{< x, \mu_A(x), 1-\mu_A(x) > | x \in X\}$.

For given IFSs A, B and A_t ($t \in T$), the following relations and operations are valid:

$$A \subseteq B \Leftrightarrow \mu(x)_A \le \mu_B(x) \,\&\, \nu_A(x) \ge \nu_B(x), \forall x \in X;$$
$$A = B \Leftrightarrow A \subseteq B \,\&\, B \subseteq A;$$
$$A' = \{< x, \nu_A(x), \mu(x)_A > | x \in X\};$$
$$\bigcap_{t\in T} A_t = \{< x, \bigwedge_{t\in T} \mu_{A_t}(x), \bigvee_{t\in T} \nu_{A_t}(x) > | x \in X\};$$
$$\bigcup_{t\in T} A_t = \{< x, \bigvee_{t\in T} \mu_{A_t}(x), \bigwedge_{t\in T} \nu_{A_t}(x) > | x \in X\}$$

2.Cut-sets of IFS

Definition 2.1 Let A be an IFS on X, $(\alpha, \beta) \in L$. We define

(1) Supper cut-sets of A:

$$A_{(\alpha,\beta)} = \{x \mid x \in X, \mu_A(x) \ge \alpha \,\&\, \nu_A(x) \le \beta\}$$
$$A_{(\alpha,\beta)\cdot} = \{x \mid x \in X, \mu_A(x) > \alpha \,\&\, \nu_A(x) < \beta\}$$

(2) Lower cut-sets of A:

$$A^{(\alpha,\beta)} = \{x \mid x \in X, \mu_A(x) \le \alpha \,\&\, \nu_A(x) \ge \beta\}$$
$$A^{(\alpha,\beta)\cdot} = \{x \mid x \in X, \mu_A(x) < \alpha \,\&\, \nu_A(x) > \beta\}$$

The cut-sets of IFS above have the following properties

Property 1 Properties of the supper cut-sets.

(1) $(A \cup B)_{(\alpha,\beta)} \supseteq A_{(\alpha,\beta)} \cup B_{(\alpha,\beta)}$, $(A \cup B)_{(\alpha,\beta)\cdot} \supseteq A_{(\alpha,\beta)\cdot} \cup B_{(\alpha,\beta)\cdot}$,
$(A \cap B)_{(\alpha,\beta)} = A_{(\alpha,\beta)} \cap B_{(\alpha,\beta)}$, $(A \cap B)_{(\alpha,\beta)\cdot} = A_{(\alpha,\beta)\cdot} \cap B_{(\alpha,\beta)\cdot}$.

(2) $(\alpha_1, \beta_1) < (\alpha_2, \beta_2) \Rightarrow A_{(\alpha_1,\beta_1)} \supseteq A_{(\alpha_2,\beta_2)}$, $A_{(\alpha_1,\beta_1)\cdot} \supseteq A_{(\alpha_2,\beta_2)\cdot}$.

(3) $(\bigcup_{t\in T} A^t)_{(\alpha,\beta)} \supseteq \bigcup_{t\in T} A^t{}_{(\alpha,\beta)}$, $(\bigcup_{t\in T} A^t)_{(\alpha,\beta)\cdot} \supseteq \bigcup_{t\in T} A^t{}_{(\alpha,\beta)\cdot}$,
$(\bigcap_{t\in T} A^t)_{(\alpha,\beta)} = \bigcap_{t\in T} A^t{}_{(\alpha,\beta)}$, $(\bigcap_{t\in T} A^t)_{(\alpha,\beta)\cdot} \subseteq \bigcap_{t\in T} A^t{}_{(\alpha,\beta)\cdot}$

(4) $(A')_{(\alpha,\beta)} \subseteq (A_{(\beta,\alpha)\cdot})'$, $((\alpha,\beta) \ne (0,1))$,
$(A')_{(\alpha,\beta)\cdot} \subseteq (A_{(\beta,\alpha)})'$, $((\alpha,\beta) \ne (1,0))$

(5) $A_{\bigvee_{t\in T}(\alpha_t,\beta_t)} = \bigcap_{t\in T} A_{(\alpha_t,\beta_t)}$, $A_{\bigwedge_{t\in T}(\alpha_t,\beta_t)\cdot} \supseteq \bigcup_{t\in T} A_{(\alpha_t,\beta_t)\cdot}$

Property 2 Properties of the lower cut-sets.

$$(1)\ (A\cup B)^{(\alpha,\beta)} = A^{(\alpha,\beta)}\cap B^{(\alpha,\beta)},\ (A\cup B)^{(\alpha,\beta)\cdot} = A^{(\alpha,\beta)\cdot}\cap B^{(\alpha,\beta)\cdot}$$

$$(A\cap B)^{(\alpha,\beta)} = A^{(\alpha,\beta)}\cup B^{(\alpha,\beta)},\ (A\cap B)^{(\alpha,\beta)\cdot} = A^{(\alpha,\beta)\cdot}\cup B^{(\alpha,\beta)\cdot}$$

$$(2)\ (\alpha_1,\beta_1)<(\alpha_2,\beta_2)\Rightarrow A^{(\alpha_1,\beta_1)}\subseteq A^{(\alpha_2,\beta_2)},\ A^{(\alpha_1,\beta_1)\cdot}\subseteq A^{(\alpha_2,\beta_2)\cdot}$$

$$(3)\ (\bigcup_{t\in T}A_t)^{(\alpha,\beta)} = \bigcap_{t\in T}A_t^{(\alpha,\beta)},\ (\bigcup_{t\in T}A_t)^{(\alpha,\beta)\cdot}\subseteq\bigcap_{t\in T}A_t^{(\alpha,\beta)\cdot},$$

$$(\bigcap_{t\in T}A_t)^{(\alpha,\beta)}\supseteq\bigcup_{t\in T}A_t^{(\alpha,\beta)},\ (\bigcap_{t\in T}A_t)^{(\alpha,\beta)\cdot} = \bigcup_{t\in T}A_t^{(\alpha,\beta)\cdot}$$

$$(4)\ (A')^{(\alpha,\beta)}\subseteq(A^{(\beta,\alpha)\cdot})' \qquad ((\alpha,\beta)\neq(1,0)),$$

$$(A')^{(\alpha,\beta)\cdot}\subseteq(A^{(\beta,\alpha)})' \qquad ((\alpha,\beta)\neq(0,1))$$

$$(5)\ A^{\hat{\wedge}_{t\in T}(\alpha_t,\beta_t)} = \bigcap_{t\in T}A^{(\alpha_t,\beta_t)}, \qquad A^{\hat{\wedge}_{t\in T}(\dot{\alpha}_t,\beta_t)\cdot}\subseteq\bigcap_{t\in T}A^{(\alpha_t,\beta_t)\cdot}$$

In [2], K.Atanassov defined the following operators over a fixed IFS A, where $(\alpha,\beta)\in L$:

$$P_{\alpha,\beta}(A)=\{<x,\alpha\vee\mu_A(x),\beta\wedge\nu_A(x)>|\,x\in X\}$$
$$Q_{\alpha,\beta}(A)=\{<x,\alpha\wedge\mu_A(x),\beta\vee\nu_A(x)>|\,x\in X\}$$

Especially, if A is a subset of X, we have

$$P_{\alpha,\beta}(A)=\begin{cases}\{<x,\alpha,\beta>|\,x\in X\}, & x\notin A\\ \{<x,1,0>|\,x\in X\}, & x\in A\end{cases}$$

$$Q_{\alpha,\beta}(A)=\begin{cases}\{<x,\alpha,\beta>|\,x\in X\}, & x\in A\\ \{<x,0,1>|\,x\in X\}, & x\notin A\end{cases}$$

From above, we can get the following decompositions of IFS

Theorem 2.1 Let A be an IFS on X, then

$$(1)\ A=\bigcup_{(\alpha,\beta)\in L}Q_{\alpha,\beta}(A_{(\alpha,\beta)}),$$

$$(2)\ A=\bigcap_{(\alpha,\beta)\in L}(Q_{\beta,\alpha}(A^{(\alpha,\beta)}))'$$

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